

$P(s)$ = Hurwitz

10/04/06
EE 610

no zeros in $\sigma > 0$, simple ones
only on $j\omega$ axis

if none, call strictly Hurwitz

$$P(s) = E_v P(s) + O_d P(s) \\ = E_v P(s) \left[1 + \frac{O_d P(s)}{E_v P(s)} \right] = O_d P(s) \left[1 + \frac{E_v P(s)}{O_d P(s)} \right]$$

if Hurwitz (for $P(s)$) then $Y(s) = \frac{O_d P(s)}{E_v P(s)}$ & $\frac{1}{Y(s)}$
are lossless admittances (note $Y(-s) = -Y(s)$)

1st look at passive admittances $Y(s)$

- 1) $Y(s)$ is real for real $s > 0$. (real elements)
- 2) $Y(s)$ has no singularities in $\sigma > 0$ (stable)
- 3) $\operatorname{Re} Y(s) \geq 0$ in $\sigma > 0$ (passive)

any function with these three properties is
called "positive real", if rational designate
it as PR, (also PR means physically realizable)
 $\Rightarrow$ a PR $Y(s)$ can be synthesized
by a finite number of positive R's,
L's & C's or a short if $Y(s) \equiv 0$

chapter 8 of Peikari

Lossless 1-ports: $\operatorname{Re} Y(j\omega) = 0$ i.e. take no
power in
sinusoidal steady
state

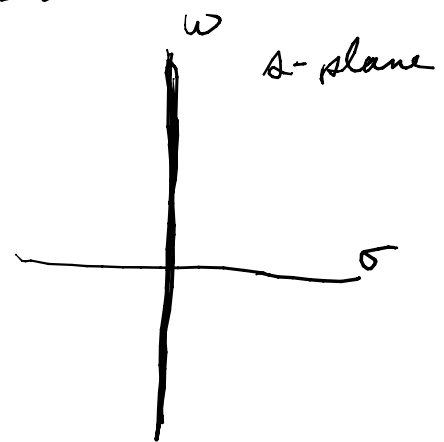
$$P_{in}(j\omega) = \operatorname{Re} V^* I = \operatorname{Re} [V^* y(j\omega) V] = |V|^2 \operatorname{Re} y(j\omega)$$

$= 0$ if lossless

$$\operatorname{Re} y(j\omega) = \frac{y(j\omega) + y(-j\omega)}{2} \Big|_{s=j\omega} = 0$$

$$= \frac{y(s) + y(-s)}{2} \Big|_{s=j\omega} = 0$$

$$s = \sigma + j\omega$$

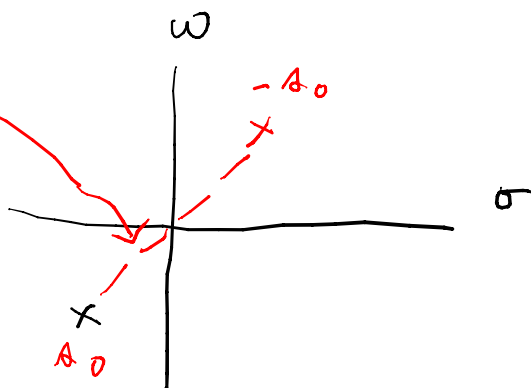


Theorem of complex variable theory: if $f(s) = 0$ on a dense set ^{on a boundary} in a region ~~not~~ then it is zero everywhere in the region

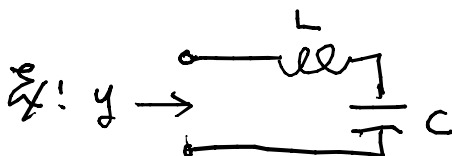
$0 = y(s) + y(-s)$ for all s if $y(s)$ is rational

$$y(-s) = -y(s)$$

If PR & lossless



$\therefore$ no poles of PR lossless $y(s)$ anywhere but on the $j\omega$ axis (also as $\frac{1}{y(s)}$ in PR & lossless there are no zero anywhere but on the $j\omega$ axis)

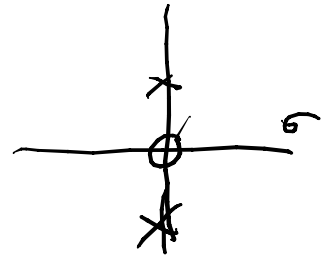


$$y(s) = \frac{1}{z(s)} = \frac{1}{sL + \frac{1}{Cs}} = \frac{Cs}{1 + LCs^2}$$

poles: = zeros of denominator $1 + LCs^2 = 0$

$$s^2 = -\frac{1}{LC} \Rightarrow j\omega = \frac{\pm 1}{\sqrt{LC}}$$

zeros: $Cs = 0 \rightarrow s = 0$



$$y(s) = \frac{Cs}{1 + LCs^2}, \quad y(-s) = \frac{-Cs}{1 + LCs^2} \quad ; C, L > 0$$

(its poles are on $j\omega$ axis)

$$2\text{Ev}y = y(s) + y(-s) = \frac{Cs - Cs}{1 + LCs^2} \equiv 0$$

$$2\text{Od}y = y(s) - y(-s) = \frac{Cs - (Cs[-s])}{1 + LCs^2} = \frac{2Cs}{1 + LCs^2} = 2y(s)$$

$\therefore$ for a lossless PR function all poles are on the $j\omega$ axis; these are simple poles & alternate with simple zeros. At any pole the residue is real and positive.

$$\text{Ex: } y(s) = \frac{3(s^2 + 4)(s^2 + 6)}{s(s^2 + 5)} \quad \text{a possible lossless } y(s)$$

make a partial fraction expansion

$$= \frac{r_0}{s} + \left(\frac{r_1}{s - j\sqrt{5}} + \frac{r_1^*}{s + j\sqrt{5}} \right) + 3s$$

$$r_0: \left. \frac{3(s^2+4)(s^2+6)}{s^2+5} \right|_{s=0} = \left[\frac{V_0}{s} + \frac{A r_1}{s-j\sqrt{5}} + \frac{A r_1^*}{s+j\sqrt{5}} + 3s^2 \right] \Big|_{s=0}$$

$$= \frac{3(4)(6)}{5} = r_0$$

$$\therefore r_0 = \frac{72}{5}$$

$$\text{use } r_1^* = r_1$$

$$y(s) = \frac{\frac{72}{5}}{s} + 3s + \frac{2s r_1}{s^2+5}$$

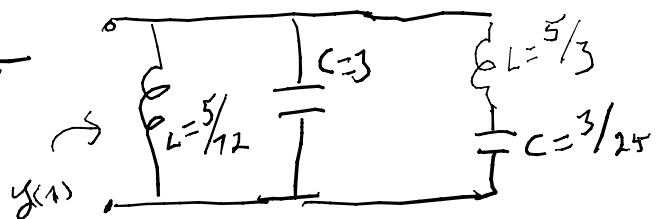
$$\left. (s^2+5)y(s) \right|_{s^2=-5} = \left. (s^2+5) \left[\frac{\frac{72}{5}}{s} + 3s \right] \right|_{s^2=-5} + 2s r_1 \Big|_{s^2=-5}$$

$$= \left. \frac{3(s^2+4)(s^2+6)}{s} \right|_{s^2=-5} = \left. 2s r_1 \right|_{s^2=-5} \Rightarrow \left. \frac{3(s^2+4)(s^2+6)}{s^2} \right|_{s^2=-5} = 2r_1$$

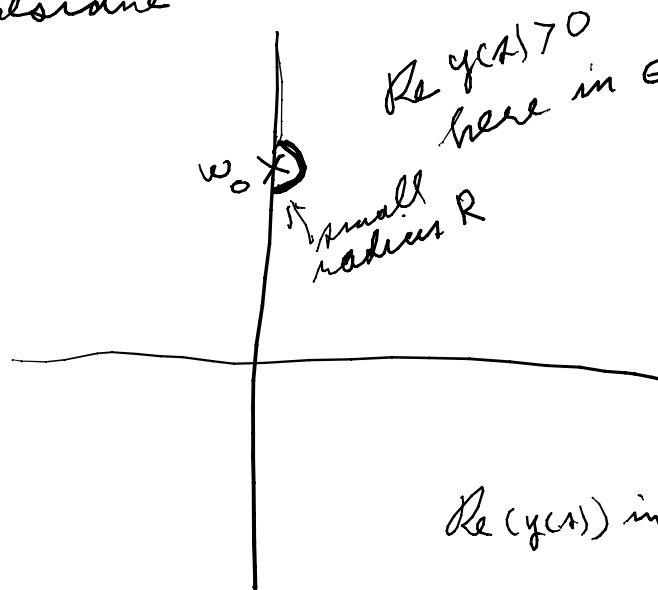
$$\frac{(3)(-1)(1)}{-5} = \frac{3(-5+4)(-5+6)}{-5} = \frac{3}{5} = 2r_1$$

$$r_1 = \frac{3}{10}$$

$$y(s) = \frac{72/5}{s} + 3s + \frac{\frac{3}{5}s}{s^2+5}$$



Proof that poles on $j\omega$ axis are simple and have a positive residue



near the pole

$$y(s) \sim \frac{k}{(s - j\omega_0)^m}$$

$$\sim \frac{|k| e^{j\angle k}}{|s - j\omega_0|^m} e^{j\angle(s - j\omega_0)m}$$

$\text{Re}(y(s))$ in $\sigma > 0$

$$y(s) \sim \frac{|k| e^{j\angle k}}{R^m e^{j\angle m(s - j\omega_0)}}$$

here as in $\sigma > 0$

$$-\frac{\pi}{2} \leq \angle(s - j\omega_0) \leq \frac{\pi}{2}$$

$$\text{Re } y(s) \sim \frac{|k|}{|R|^m} \cos[\angle k - \angle m(s - j\omega_0)] \quad \text{this is } < 0 \text{ if } \angle k \neq 0, m \neq 1$$

$\therefore$ the residue is $k > 0$ & the pole is simple as $m=1$

lossless PR

$$\left[y(s) = k_\infty s + \frac{k_0}{s} + \sum_{i=1}^m \frac{k_i s}{s^2 + \omega_i^2}, \quad k_i > 0 \right] \quad \text{partial fraction expansion of } y(s)$$

let $s = j\omega$ divide by j

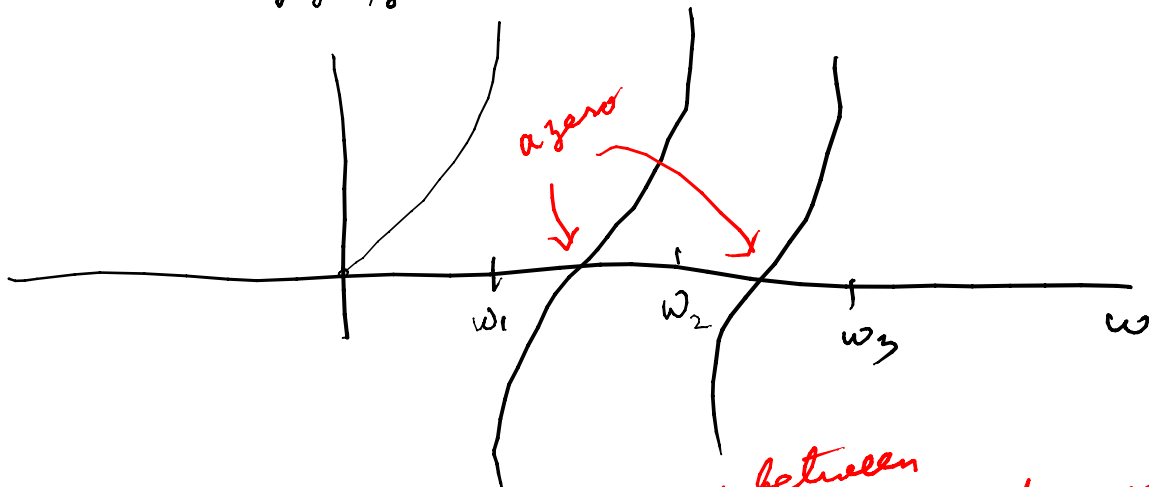
$$\frac{1}{j} y(j\omega) = k_\infty \omega - \frac{k_0}{\omega} + \sum_{i=1}^m \frac{k_i \omega}{\omega_i^2 - \omega^2}$$

the derivative versus ω (is always positive)

$$\frac{d \frac{y(j\omega)}{j}}{d\omega} = k_{\infty} + \frac{k_0}{\omega^2} + \sum_{i=1}^m \left[\frac{k_i}{\omega_i^2 - \omega^2} - \frac{k_i \omega [-2\omega]}{(\omega_i^2 - \omega^2)^2} \right] \geq 0$$

$$= \sum_{i=1}^m \frac{k_i(\omega_i^2 + \omega^2)}{(\omega_i^2 - \omega^2)^2}$$

$y(j\omega)/j$



The zeros are between the poles $\Rightarrow$ poles and zeros alternate