

10/02/06
EE610

$$\begin{array}{c} + \\ v_1 \\ - \end{array} \begin{array}{c} \boxed{Y} \end{array} \begin{array}{c} + \\ y_L v_2 \\ - \end{array} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ -y_L v_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

last eqs: $-y_L v_2 = y_{21} v_1 + y_{22} v_2$

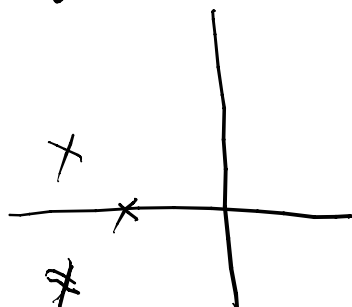
$$(-y_L - y_{22}) v_2 = y_{21} v_1$$

if a 2-port $\frac{v_2}{v_1} = \frac{-y_{21}}{y_{22} + y_L}$ if $y_L = 1$ & $y_{21} = \frac{n_{21}}{d_{21}}$
 $y_{22} = \frac{n_{22}}{d_{22}}$

$$= \frac{-n_{21}/d_{21}}{\frac{n_{22}}{d_{22}} + 1}$$

$$= \frac{-n_{21} (d_{22}/d_{21})}{n_{22} + d_{22}}$$

if the 2-port is passive, no poles of $n_{22} + d_{22}$ in $\sigma \geq 0$

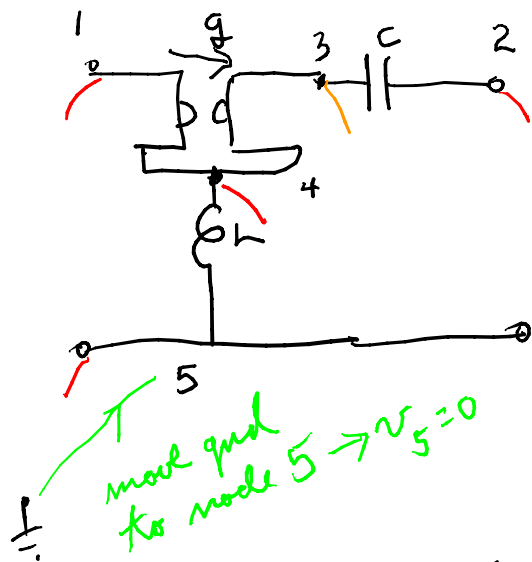


therefore

$P(s) = n_{22}^{(s)} + d_{22}^{(s)}$ is an Hurwitz polynomial (actually is strictly Hurwitz) i.e. no zeros in the closed right half plane.

Ex. for indefinite admittance:

$$Y_{ind} = 5 \times 5$$



for gyrator

$$i_1 = g(v_3 - v_4)_{gy}$$

$$i_3 = -g(v_1 - v_4)_{gy}$$

$$i_4 = -(i_1 + i_3)_{gy}$$

$$Y_{ind_{gy}} = \begin{bmatrix} 0 & 0 & g & -g & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -g & 0 & 0 & g & 0 \\ +g & 0 & -g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Y_{ind_{cap}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \Delta C & -\Delta C & 0 & 0 \\ 0 & -\Delta C & +\Delta C & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Y_{ind_L} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/\Delta L & -1/\Delta L \\ 0 & 0 & 0 & -1/\Delta L & +1/\Delta L \end{bmatrix}$$

$$Y_{ind} = Y_{ind_{gy}} + Y_{ind_{cap}} + Y_{ind_L}$$

as these are all in parallel (i.e. currents are added at the nodes, with the all the same node voltages)

$$Y_{ind} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 0 & g & -g & 0 \\ 2 & 0 & \Delta C & -\Delta C & 0 & 0 \\ 3 & -g & -\Delta C & \Delta C & g & 0 \\ 4 & g & 0 & -g & 1/\Delta L & -1/\Delta L \\ 5 & 0 & 0 & 0 & -1/\Delta L & 1/\Delta L \end{bmatrix}$$

remove as
of no interest
when qnd node 5

as
 $v_5 = 0$
when move
qnd to node 5

by add column
entries to zero

define the part 2×2 Y matrix $\Rightarrow$ force i_3 & i_4 from
outside to be zero.

ex: 1st choose $i_4 = 0$ & then later choose $i_3 = 0$

for $i_4 = 0$ allows elimination of v_4

$$0 = \begin{bmatrix} g & 0 & -g \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + \begin{bmatrix} 1 \\ \Delta L \end{bmatrix} v_4 \Rightarrow v_4 = (\Delta L) \begin{bmatrix} -g & 0 & g \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$Y_{3 \times 3} = \begin{bmatrix} 0 & 0 & g \\ 0 & \Delta C & -\Delta C \\ -g & -\Delta C & \Delta C \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + \begin{bmatrix} -g \\ 0 \\ g \end{bmatrix} (\Delta L) \begin{bmatrix} -g & 0 & g \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$= \left\{ \begin{bmatrix} 0 & 0 & g \\ 0 & \Delta C & -\Delta C \\ -g & -\Delta C & \Delta C \end{bmatrix} + \Delta L \begin{bmatrix} g^2 & 0 & -g^2 \\ 0 & 0 & 0 \\ -g^2 & 0 & g^2 \end{bmatrix} \right\} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$Y_{3 \times 3} = \begin{bmatrix} \Delta L g^2 & 0 & g - \Delta L g^2 \\ 0 & \Delta C & -\Delta C \\ -g - \Delta L g^2 & -\Delta C & \Delta C + \Delta L g^2 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = Y_{3 \times 3} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

general rule to eliminate internal current

$$Y_{k \times n} \Rightarrow \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 = 0 \end{bmatrix}$$

$$I_1 = Y_{11} v_1 - Y_{12} Y_{22}^{-1} Y_{21} v_1 \Rightarrow Y_{\text{reduced}} = Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}$$

here

$$Y_{2 \text{ port}} = \begin{bmatrix} \Delta L g^2 & 0 \\ 0 & \Delta C \end{bmatrix} - \begin{bmatrix} g - \Delta L g^2 \\ -\Delta C \end{bmatrix} \frac{1}{\Delta C + \Delta L g^2} \begin{bmatrix} -g - \Delta L g^2 & -\Delta C \end{bmatrix}$$

$$= \begin{bmatrix} \Delta L g^2 + \frac{(g - \Delta L g^2)(g + \Delta L g^2)}{\Delta C + \Delta L g^2} & \frac{\Delta C (g - \Delta L g^2)}{\Delta C + \Delta L g^2} \\ -\frac{\Delta C (g + \Delta L g^2)}{\Delta C + \Delta L g^2} & \Delta C - \frac{\Delta C^2}{\Delta C + \Delta L g^2} \end{bmatrix}$$

$$\rightarrow \frac{\Delta^2 C L g^2 + (\Delta L g^2)^2 + g^2 - (\Delta L g^2)^2}{\Delta C + \Delta L g^2}$$

$$\frac{\Delta^2 L g^2 C}{\Delta C + \Delta L g^2} = \frac{\Delta L g^2}{1 + \Delta L g^2 / C}$$

$$Y_{2port} = \begin{bmatrix} \frac{s^2 LC g^2 + g^2}{sC + g^2 L} & \frac{g - sLg^2}{1 + Lg^2/C} \\ \frac{-(g + sLg^2)}{1 + Lg^2/C} & \frac{sLg^2}{1 + C/Lg^2} \end{bmatrix}$$

$$= \frac{1}{1 + Lg^2/C} \begin{bmatrix} sLg^2 + \frac{1}{s} g^2/C & g - sLg^2 \\ -g - sLg^2 & sLg^2 \end{bmatrix}$$

$$= \frac{1}{1 + Lg^2/C} \left\{ sLg^2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{g^2/C}{s} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \right\}$$

alternate: force $i_3 \neq i_4$ to be 0 simultaneously

$$Y_{1port} = Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}$$

use Y_{22} is lower 2×2 matrix
in Y_{ind} node 5 and

$$Y_{22} = \begin{bmatrix} sC & g \\ -g & 1/sL \end{bmatrix}$$

$$Y_{12} = \begin{bmatrix} g & -g \\ -sC & 0 \end{bmatrix}, \quad Y_{21} = \begin{bmatrix} -g & -sC \\ g & 0 \end{bmatrix}$$

$$Y_{22}^{-1} = \frac{1}{\frac{1}{sL} + g^2} \begin{bmatrix} 1/sL & -g \\ g & sC \end{bmatrix}$$

$$Y_{11} = \begin{bmatrix} 0 & 0 \\ 0 & sC \end{bmatrix}$$

$$y_{\text{port}} = \begin{bmatrix} 0 & 0 \\ 0 & LC \end{bmatrix} - \begin{bmatrix} g & -g \\ -LC & 0 \end{bmatrix} \frac{L}{C+g^2L} \begin{bmatrix} 1/L & -g \\ g & LC \end{bmatrix} \begin{bmatrix} -g & -LC \\ g & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & LC \end{bmatrix} - \frac{L/C}{1+g^2L/C} \begin{bmatrix} \frac{g}{L} - g^2 & -g^2 - LgC \\ -\frac{C}{L} & + LgC \end{bmatrix} \begin{bmatrix} -g & -LC \\ g & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & LC \end{bmatrix} - \frac{L/C}{1+g^2L/C} \begin{bmatrix} -\frac{g^2}{L} + \cancel{g^3} - \cancel{g^3} - LgC^2 & -gC/L + Lcg^2 \\ \frac{gC}{L} + Lcg^2 & LC^2/L \end{bmatrix}$$

$$= \frac{1}{1+g^2L/C} \left\{ \begin{bmatrix} \frac{g^2}{LC} + Lg^2 & +g - Lg^2 \\ -g - Lg^2L & LC + Lg^2L - LC \end{bmatrix} \right\}$$

which agrees with the above!