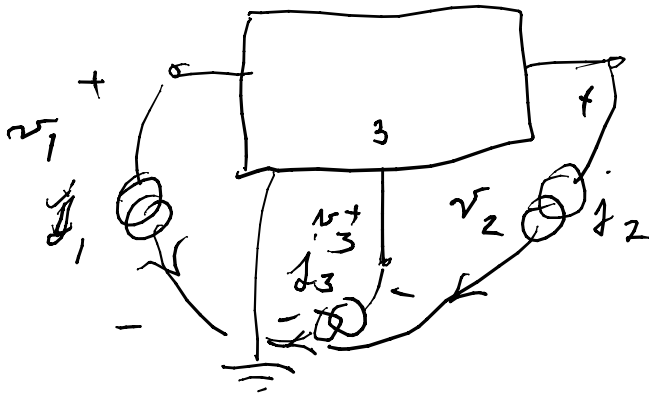


09/25/06
EE610



$$\begin{bmatrix} i_1 \\ i_2 \\ 0 \end{bmatrix} = Y_{\text{nodal}} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$= \begin{bmatrix} \underbrace{y_{11}}_t & \underbrace{y_{12}}_t \\ \underbrace{y_{21}}_t & \underbrace{y_{22}}_t \end{bmatrix} \begin{bmatrix} v_t \\ v_r \end{bmatrix}$$

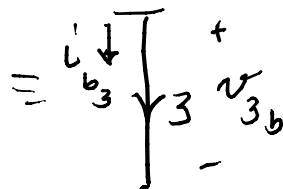
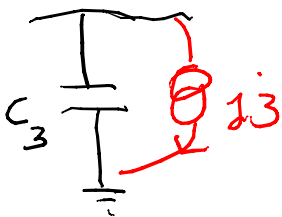
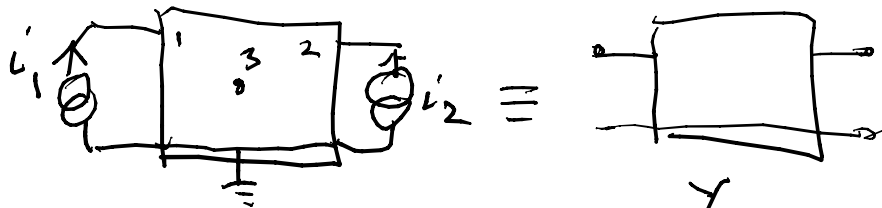
t r n

$$0 = y_{21} v_t + y_{22} v_r$$

$$v_r = -y_{22}^{-1} y_{21} v_t$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_t \\ -y_{22}^{-1} y_{21} v_t \end{bmatrix} = \begin{bmatrix} y_{11} - y_{12} y_{22}^{-1} y_{21} \\ -y_{22}^{-1} y_{21} \end{bmatrix} v_t$$

this eliminates node 3 so don't see it by i_1 & i_2

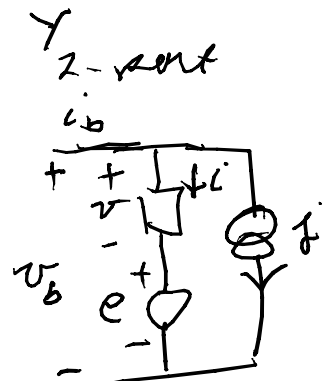


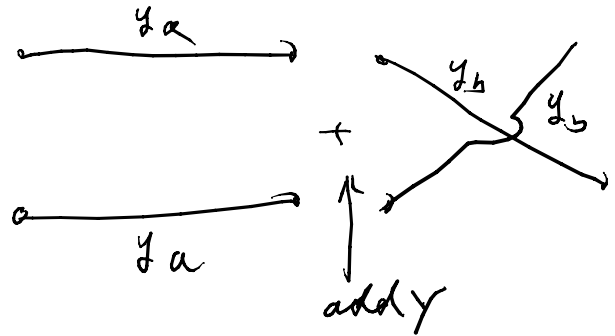
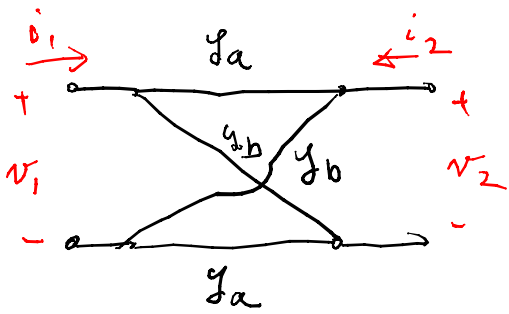
$$v_{3b} = v_3$$

$$i_{3b} = i_3$$

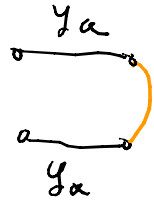
$$i_{3b} = i_3 = C_3 \frac{dv_3}{dt}$$

$$= C_3 \frac{dv_{3b}}{dt} = C_3 \frac{dv_{3b}}{dt}$$





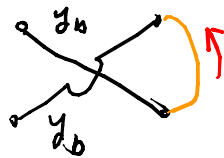
$$Y_a = \frac{1}{2} \begin{bmatrix} y_a - y_a \\ -y_a \ y_a \end{bmatrix}$$



$$\Rightarrow 2Y_a = \frac{2}{y_a} = 2i_{in}, \quad y_{in} = \frac{1}{2i_{in}} = \frac{y_a}{2}$$

$$Y_b = \frac{1}{2} \begin{bmatrix} y_b + y_b \\ y_b - y_b \end{bmatrix}$$

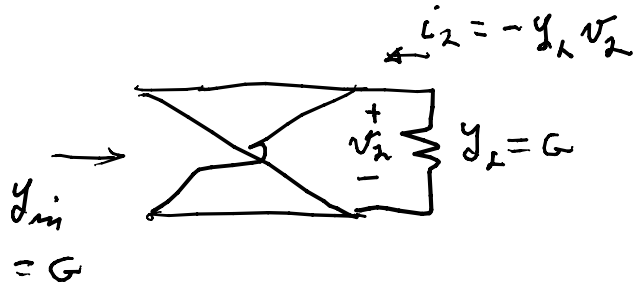
$$y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0}$$



$$Y = Y_a + Y_b$$

$$= \frac{1}{2} \begin{bmatrix} y_b + y_a & y_b - y_a \\ y_b - y_a & y_b + y_a \end{bmatrix}$$

$$y_{in} \text{ when } y_L = G = 1/R$$



$$\begin{bmatrix} i_1 \\ i_2 = -y_L v_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} y_b + y_a & y_b - y_a \\ y_b - y_a & y_b + y_a \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$(-y_L - y_b - y_a) v_2 = (y_b - y_a) v_1$$

$$i_1 = \left[\frac{1}{2} (y_b + y_a) + \frac{\frac{1}{2} (y_b - y_a) (y_b - y_a) / 2}{-2y_L - y_b - y_a} \right] v_1 = G v_1$$

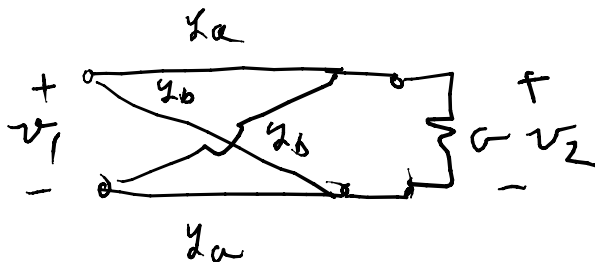
const R
↓

$$= \frac{1}{2} \left\{ \frac{(y_b + y_a)G + \overbrace{(y_b + y_a)^2 - (y_b - y_a)^2}^{= 4y_a y_b}}{2G + (y_b + y_a)} \right\}$$

$$2G = \frac{(y_b + y_a)G + 4y_a y_b}{2G + y_b + y_a} \Rightarrow 4G^2 + 2G(y_b + y_a) = G(y_b + y_a) + 4y_a y_b$$

$$\Rightarrow 4G^2 + 2G(y_b + y_a) - 4y_a y_b = 0 \quad \left(\begin{array}{l} \text{should get} \\ y_a y_b = G^2 \end{array} \right)$$

$$\therefore y_a y_b = G^2$$



from above

$$T(a) = \left. \frac{v_2}{v_1}(a) \right|_{y_a = G} = - \left(\frac{y_b - y_a}{2G + y_b + y_a} \right) = \frac{y_a - \frac{G^2}{y_a}}{2G + \frac{G^2}{y_a} + y_a} = \frac{y_a^2 - G^2}{2Gy_a + G^2 + y_a^2}$$

$$= \frac{(y_a - G)(y_a + G)}{(y_a + G)(y_a + G)} = \frac{y_a - G}{y_a + G} = T(a) = \frac{v_2}{v_1}(a)$$

to look at all-pass design using this:

$$P(a) = \sum_{i=0}^p a_i a^i = \underbrace{\sum_{\text{even powers}} P(a)}_{\text{even powers}} + \underbrace{\sum_{\text{odd powers}} P(a)}_{\text{odd powers}}$$

$$Ex: a^5 + 3a^4 + 2a^3 + 4a + 6 = P(a)$$

$$Ev P(a) = 3a^4 + 0 \cdot a^2 + 6$$

$$Od P(a) = a^5 + 2a^3 + 4a = a(\text{even}(a))$$

$$\text{Ev } P(a) = \frac{P(a) + P(-a)}{2}, \quad \text{od } P(a) = \frac{P(a) - P(-a)}{2}$$

$$\begin{aligned} 2 \text{od } P(a) &= (+a^5) + 3(+a)^4 + 2(+a)^3 + 4(+a) + 6 \\ &\quad - ((-a)^5 + 3(-a)^4 + 2(-a)^3 + 4(-a) + 6) \\ &= \frac{2a^5 + 0 + 4a^3 + 8a}{2} = 2(a^5 + 2a^3 + 4a) \end{aligned}$$

$$\text{Look at } \frac{P(-a)}{P(a)} = \frac{P_{ev_1}(a) - a P_{ev_2}(a)}{P_{ev_1}(a) + a P_{ev_2}(a)} \quad \left(= T(a) = \frac{v_2}{v_1}(a) \right)$$

for real coefficients



$x = \text{poles}$

$$a = \sigma + j\omega$$

$$\begin{aligned} |T(j\omega)| &= \frac{|P(-j\omega)|}{|P(j\omega)|} \\ &= \text{product of} \end{aligned}$$

$$P(a) = (a - p_1)(a - p_2) \dots$$

$$|P(j\omega)| = |j\omega - p_1| \cdot |j\omega - p_2| \dots$$

$$P(j\omega) = (j\omega - p_1)(j\omega - p_2) \dots$$

$\therefore$ for every pole there is a zero with the same distance to $j\omega$ axis $\frac{|j\omega - p_1|}{|j\omega - p_1|} = 1$

$$\therefore \left| \frac{P(-j\omega)}{P(j\omega)} \right| = 1$$

$$\therefore T(a) = \frac{P(-a)}{P(a)} \text{ is all-pass}$$

Ex: $P(s) = s^2 + as + b$

real a & $b > 0$

$$P(-s) = s^2 - as + b$$

$$T(s) = \frac{s^2 - as + b}{s^2 + as + b}$$

is all pass degree 2

derive $= \frac{y_a - G}{y_a + G} \Rightarrow T \times (y_a + G) = y_a - G$

$$(T-1)y_a = -G - TG$$

$$y_a = \frac{G(1+T)}{1-T}$$

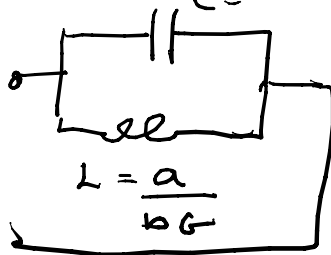
$\therefore$ for this $T(s)$

$$y_a = G \left(\frac{1 + \frac{s^2 - as + b}{s^2 + as + b}}{1 - \frac{s^2 - as + b}{s^2 + as + b}} \right) = G \frac{(s^2 + as + b) + (s^2 - as + b)}{s^2 + as + b - s^2 + as + b}$$

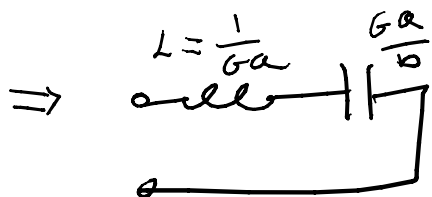
$$= G \left(\frac{2s^2 + 2b}{2as} \right) = G \frac{s^2 + b}{as} = y_a$$

$C = G/a$

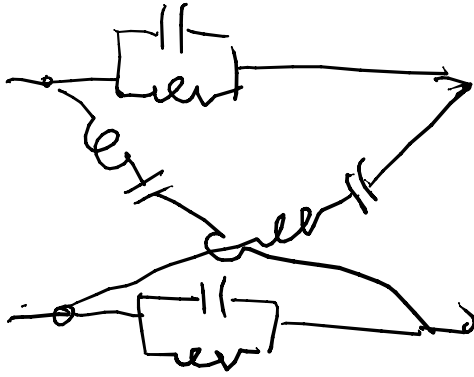
$\therefore y_a = \frac{G}{a}s + \frac{Gb}{as} \Rightarrow$



$$y_b = \frac{G^2}{y_a} = \frac{G(as)}{s^2 + b} \Rightarrow \frac{1}{y_b} = z_b = \frac{s^2 + b}{Ga s} = \frac{s}{Ga} + \frac{1}{\frac{Ga}{b}s}$$

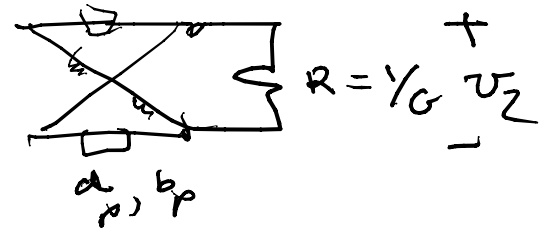
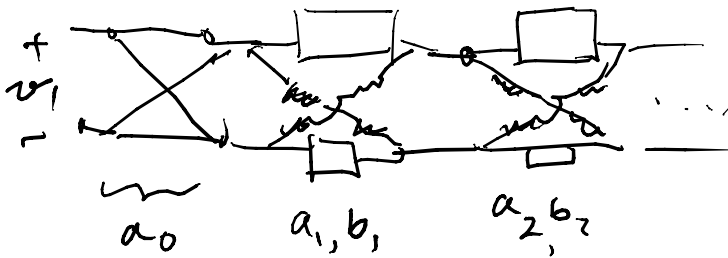


(dual circuits)



∴ for all-pass synthesis

$$T(s) = \frac{P(-s)}{P(s)} = \frac{(a_0 - s) \prod_{i=1}^p (s^2 - a_i s + b_i)}{(a_0 + s) \prod_{i=1}^p (s^2 + a_i s + b_i)}$$



gives $T(s)$

useful for phase correction

$$T(s) \Big|_{s=j\omega} = \frac{1}{e^{j\angle P(j\omega)}} \cdot \frac{e^{j\angle P(-j\omega)}}{e^{j\angle P(j\omega)}} = e^{j\angle P(-j\omega) - j\angle P(j\omega)}$$