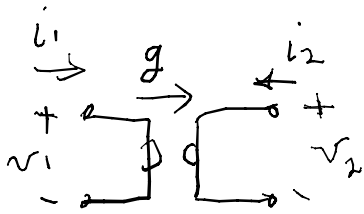


gyrator

09/20/06
EE610



$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

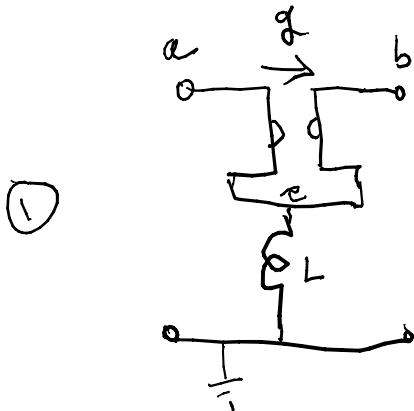
g = gyration conductance

$$P_{in} = v^T i = [v_1 \ v_2] \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = [-g v_2, g v_1] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$= -g v_2 v_1 + g v_1 v_2 = 0$$

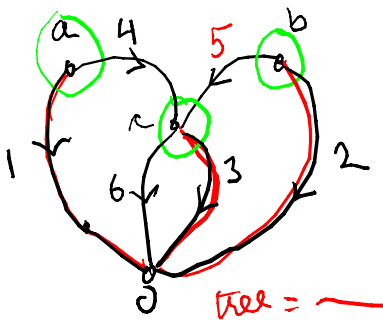
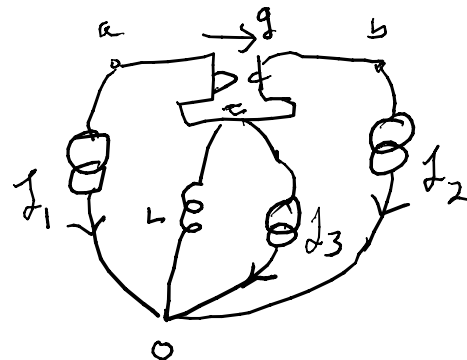
$\therefore$ Power in on left
= Power out on right

$\therefore$ lossless



②

$\Rightarrow$



$$e: \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 3 & 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix}; v = e i_b$$

$$i = Y_{b \times b} v$$

$$Y_{b \times b} = \begin{bmatrix} 0_3 & 0_3 \\ 0_3 & \begin{bmatrix} 0 & g \\ -g & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix}$$

$b=6$

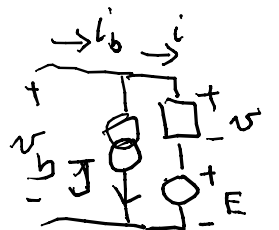
as current sources in branches 1, 2, 3

$$i_b = i + J$$

$$v_b = v + E$$

here $E=0$

$$J = [j_1, j_2, j_3 | 0 \ 0 \ 0]^T$$



$$i = Y_{b \times b} v = i_b - J \Rightarrow e i = e Y_{b \times b} v = e i_b - e J = 0 - e J$$

$$\Rightarrow e Y_{b \times b} v = -e J \quad \text{now use } v_b = e^T v_t = v + E$$

$$\therefore v_b = v = e^T v_t \quad "0"$$

$$e Y_{b \times b} e^T v_t = -e J$$

here

$$e Y_{b \times b} = \left[\begin{array}{c|ccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & -1 & 1 & 1 \end{array} \right] \left[\begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] = \left[\begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$(e Y_{b \times b}) e^T = \left[\begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \left[\begin{array}{c} 1 \\ 0 \\ 0 \\ 1 \end{array} \right] = \left[\begin{array}{ccc} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 1 & -1 & 1 \end{array} \right] = Y_{\text{nodal}} = e Y_{b \times b} e^T$$

$$\begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = -e J = - \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = - \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \Rightarrow 0 \text{ for the 2-port}$$

these are the node equations

Now set $i_3 = 0$ to get the 2-port equations

the last is $i_3 = 0 = [0 \quad -1] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + [1] v_3$ solve for v_3

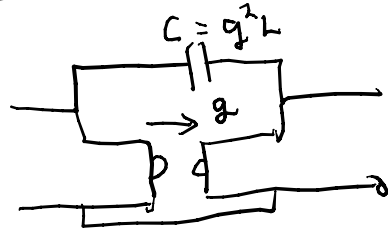
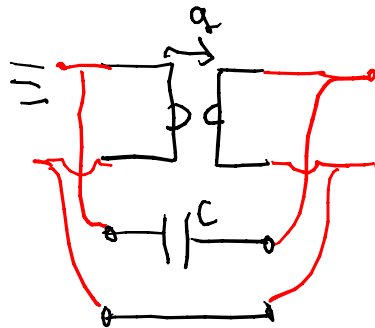
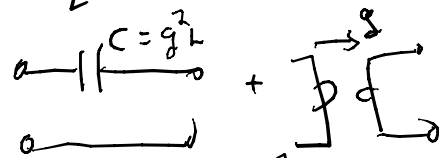
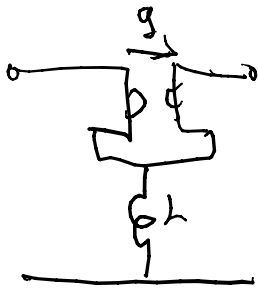
$$\therefore v_3 = -1 \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} -g \\ g \end{bmatrix} v_3 = \begin{bmatrix} -i_1 \\ -i_2 \end{bmatrix}$$

$$= \begin{bmatrix} +g^2 L & g - g^2 L \\ -g - g^2 L & g^2 L \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

port port

$$Y = g^2 L \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$$



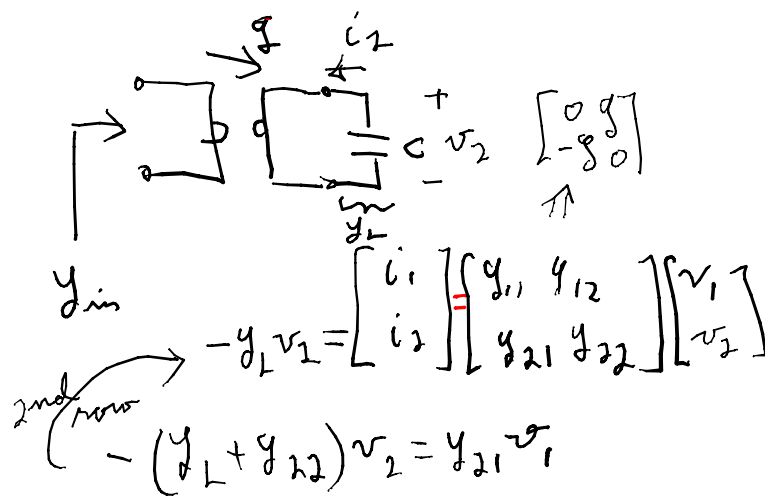
$$L = C/g^2$$

if make gyrator with CMOS, $g = g_m$; $i_D = \frac{K_P W}{2 L} (V_{GS} - V_{TO})^2$

$$g_m = \frac{\partial i_D}{\partial V_{GS}} = \frac{2 K_P W}{2 L} (V_{GS} - V_{TO}) = \frac{2 K_P W}{2 L} \cdot \frac{(V_{GS} - V_{TO})^2}{(V_{GS} - V_{TO})} = 2 \frac{i_D}{(V_{GS} - V_{TO})}$$

if $I_D = 1 \text{ mA}$, $V_{GS} - V_{TO} = 2$ then $g_m = 10^{-3}$

then $L = 10^6 \times C$, if $C = 1 \text{ pF}$, $L = 1 \text{ H}$

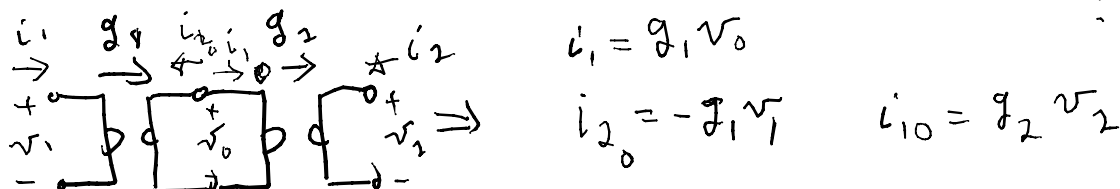
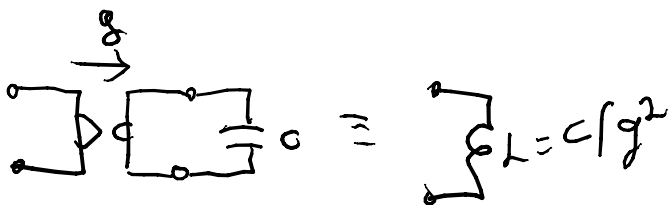


$$y_m = \frac{\Delta y + y_{11} g_L}{y_{22} + y_L}$$

$$y_m = \frac{\Delta y + 0}{0 + \Delta C} = \frac{g^2}{\Delta C}$$

$$= \frac{1}{\Delta L} \Rightarrow L = C/g^2$$

$$y_m = y_{11} - \frac{y_{12} y_{21}}{y_{22} + y_L} = \frac{\Delta y + y_{11} y_L}{y_{22} + y_L}$$



$$\text{but } i_{20} = -i_{10} \Rightarrow -g_1 v_1 = -g_2 v_2$$

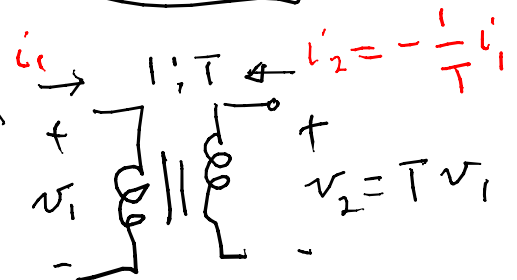
$$v_2 = \frac{g_1}{g_2} v_1$$

$$i_1 = g_1 v_0$$

$$i_2 = -g_2 v_0 \Rightarrow v_0 = -\frac{1}{g_2} i_2$$

$$\Rightarrow \boxed{i_1 = -\frac{g_1}{g_2} i_2}$$

$\Rightarrow$ ideal transformer



$$= -\frac{i_1}{T} i_2$$

$$i_1 = -T i_2$$

$$0 = P_m = v_1 i_1 + v_2 i_2$$

$$= v_1 i_1 + T v_1 \left(-\frac{1}{T}\right) i_2$$

$$T = \text{Turns ratio} = g_1/g_2$$

