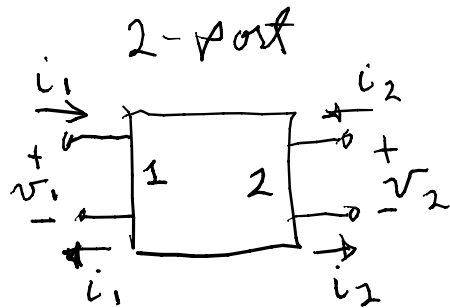
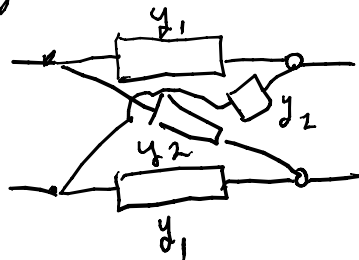


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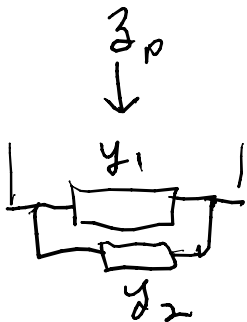
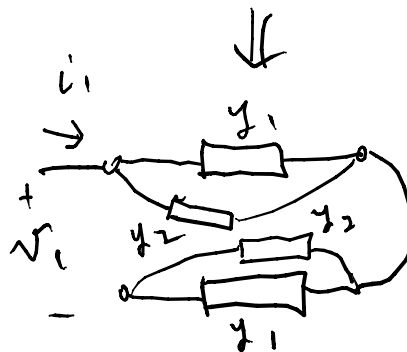


$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Ex: symmetric lattice



$y_{11} = i_1/v_1 \Big|_{v_2=0}$ = input admittance at port 1 when port 2 is shorted

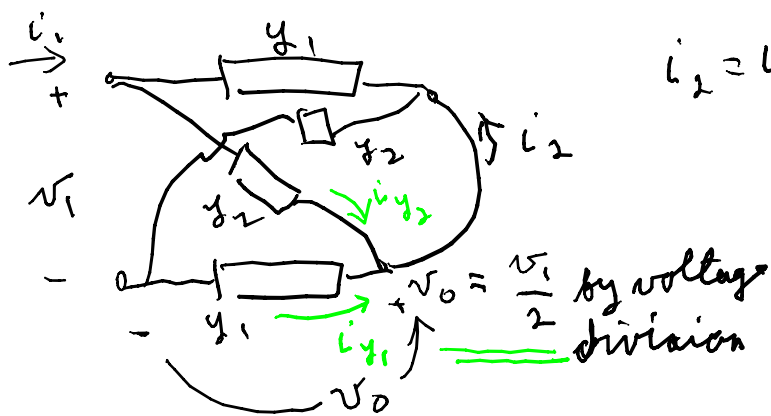


$$Z_o = \frac{1}{y_1 + y_2}$$

$$\frac{v_1}{i_1} \Big|_{v_2=0} = 2Z_o = \frac{2}{y_1 + y_2} = Z_{in} \Big|_{\text{port 2 shorted}}$$

$$y_{21} = y_{11} = \frac{y_1 + y_2}{2} \equiv y_{22} \text{ by symmetry}$$

$$i_2 = y_{21}v_1 + y_{22}v_2 \Big|_{v_2=0} \Rightarrow y_{21} = \frac{i_2}{v_1} \Big|_{v_2=0}$$



$$i_2 = i_{g_1} + i_{g_2}$$

on g_2 the voltage is

$$v_1 - v_0$$

$$i_{g_2} = g_2 (v_1 - v_0)$$

$$i_{g_1} = -g_1 v_0$$

$$\begin{aligned} \therefore i_2 &= i_{g_1} + i_{g_2} = -g_1 v_0 + g_2 (v_1 - v_0) = -\frac{1}{2} g_1 v_1 + g_2 \left(v_1 - \frac{v_1}{2} \right) \\ &= \frac{1}{2} (g_2 - g_1) v_1 \quad \text{when } v_2 = 0 \Rightarrow y_{21} = \left. \frac{i_2}{v_1} \right|_{v_2=0} = \frac{g_2 - g_1}{2} \end{aligned}$$

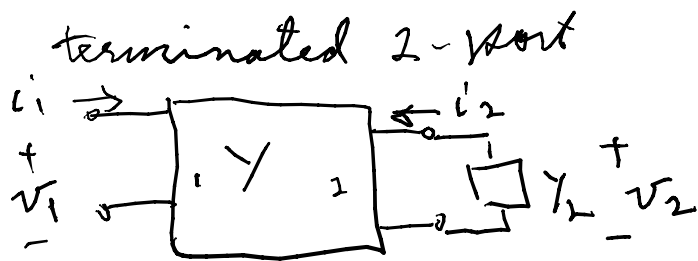
$$Y = \frac{1}{2} \begin{bmatrix} g_2 + g_1 & g_2 - g_1 \\ g_2 - g_1 & g_2 + g_1 \end{bmatrix} = \text{admittance matrix for the symmetric lattice} = y_{12} \text{ by symmetry}$$

$$\begin{aligned} Z &= Y^{-1}; \quad \text{need } \Delta = \det Y = \frac{1}{4} \left((g_2 + g_1)^2 - (g_2 - g_1)^2 \right) \\ &= \frac{1}{4} (g_2^2 + 2g_1g_2 + g_1^2 - [g_2^2 - 2g_1g_2 + g_1^2]) \\ &= g_1g_2 \end{aligned}$$

$$Z = \frac{1}{\Delta} \begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix} = \frac{1}{g_1g_2} \begin{bmatrix} g_2 + g_1 & g_1 - g_2 \\ g_2 - g_1 & g_2 + g_1 \end{bmatrix} = \begin{bmatrix} z_1 + z_2 & z_2 - z_1 \\ z_2 - z_1 & z_1 + z_2 \end{bmatrix}$$

$$z_i = 1/g_i \quad i=1,2$$

$$\begin{aligned} v &= zi \\ i &= yv \Rightarrow z = 1/y \end{aligned}$$



find $i_1 = y_{in} v_1$ with this termination

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ -\frac{1}{y_L} i_2 \end{bmatrix}$$

2nd set

$$i_2 = y_{21} v_1 - y_{22} (y_L^{-1}) i_2 \Rightarrow (1 + y_{22}/y_L) i_2 = y_{21} v_1$$

$$\text{or } i_2 = \frac{y_{21}}{1 + \frac{y_{22}}{y_L}} v_1 \Rightarrow v_2 = -\frac{1}{y_L} i_2 = -\frac{y_{21}/y_L}{1 + y_{22}/y_L} v_1$$

from 1st eq.

$$\therefore i_1 = y_{11} v_1 + y_{12} v_2 = \left(y_{11} - \frac{y_{12} y_{21}/y_L}{1 + y_{22}/y_L} \right) v_1 = y_{in} v_1 \quad \left| \begin{array}{l} \text{when} \\ \text{load} \\ \text{port 2} \\ \text{with } y_L \end{array} \right.$$

$$y_{in} = y_{11} - \frac{y_{12} y_{21}}{y_L + y_{22}} = \frac{y_{11} y_L + (y_{11} y_{22} - y_{12} y_{21})}{y_{22} + y_L}$$

$$\text{or } y_{in} = \frac{y_{11} y_L + \Delta_y}{y_{22} + y_L}$$

if $y_L = G$ (a constant), if $y_{11} = y_{22}$ then $y_{in} = \frac{y_{11} G + \Delta_y}{y_{11} + G}$

then $y_{in} = G$ if $G = \frac{y_{11} G + \Delta_y}{y_{11} + G} \Rightarrow y_{11}/G + G^2 = y_{11}/G + \Delta_y$

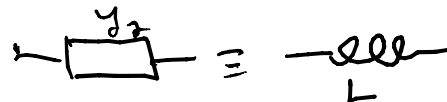
$\therefore$ if $\Delta_y = G^2$ then $y_{in} = y_L = G$ if $y_{11} = y_{22}$

For our lattice $Y_{11} = Y_{22} = \frac{1}{2}(Y_2 + Y_1)$

$$\Delta Y = Y_1 Y_2$$

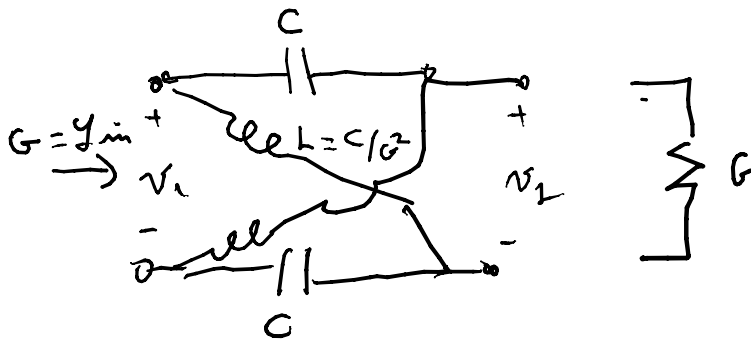
$\therefore$ if $Y_1 Y_2 = G^2$ then $Y_{in} = G$ when load is G

Let $Y_1 = sC$, $Y_2 = \frac{1}{sL}$



$$Y_1 Y_2 = sC \cdot \frac{1}{sL} = \frac{C}{L}$$

$$\text{if } = G^2 \Rightarrow G^2 = \frac{C}{L} \Rightarrow L = C/G^2$$



$$V_2 = \frac{Y_{21} V_1}{1 + Y_{22}/Y_L}$$

$$\frac{V_2}{V_1} = \frac{-Y_{21}}{G + Y_{22}} = \frac{-(Y_2 - Y_1)}{2(G + [Y_2 + Y_1])}$$

$$\frac{V_2}{V_1} = \frac{Y_1 - Y_2}{2G + Y_2 + Y_1} = \frac{sC - \frac{1}{sL}}{2G + \frac{1}{sL} + sC}$$

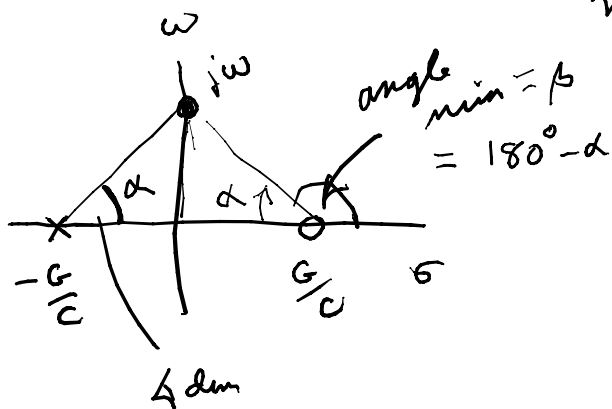
$$= \frac{s^2 C^2 - 1}{s^2 C^2 + 2\frac{C}{G}s + 1} = \frac{s^2 - \frac{G^2}{C^2}}{s^2 + \frac{2G}{C}s + \frac{G^2}{C^2}} = \frac{(s - \frac{G}{C})(s + \frac{G}{C})}{(s + \frac{G}{C})^2}$$

$$= \frac{s - G/C}{s + G/C} \text{ an all pass function}$$

$$\frac{v_2}{v_1}(1) = \frac{1 - G/c}{1 + G/c}, \quad y_{in}(s) = G \quad \text{loaded in } G$$

$$\left| \frac{v_2}{v_1}(j\omega) \right| = \left| \frac{j\omega - G/c}{j\omega + G/c} \right| = \frac{|j\omega - G/c|}{|j\omega + G/c|} = \frac{\sqrt{(-G/c)^2 + \omega^2}}{\sqrt{(G/c)^2 + \omega^2}} = 1$$

phase of $\frac{v_2}{v_1}(j\omega)$; $\frac{v_2}{v_1}(j\omega) = \left| \frac{v_2}{v_1}(j\omega) \right| e^{j\angle(\frac{v_2}{v_1}(j\omega))}$



$$\begin{aligned} &= 1 \cdot e^{j\delta_{num}} \cdot e^{-j\delta_{den}} \\ &= e^{j(\beta - \alpha)} = e^{j(\neq 0)} \\ &= e^{j(180 - 2\alpha)} \end{aligned}$$

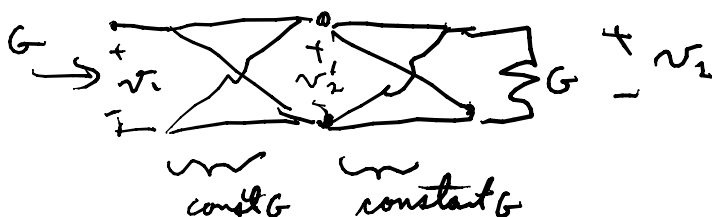
$$\tan \alpha = \frac{\omega}{G/c} \Rightarrow \alpha = \arctan\left(\frac{\omega}{G/c}\right) ; \quad \angle\left(\frac{v_2}{v_1}(j\omega)\right) = \pi - 2\arctan\left(\frac{\omega}{G/c}\right)$$

∴ the constant R lattice, where $y_1 y_2 = G^2$

can be used to change the phase of a sinusoidal signal without changing magnitude

$$\text{if } y_1 = sC, \quad y_2 = \frac{1}{sC/G^2} = \frac{1}{sA}$$

if cascade two of these:



$$T_1 = \frac{v_2'}{v_1} = \left(\frac{1-a}{1+a} \right) \quad a = \frac{G}{C}$$

$$T_2 = \frac{v_2}{v_2'} = \left(\frac{1-b}{1+b} \right) \quad b = \frac{G}{C'}$$

$$\text{then } \frac{v_2}{v_1} = \frac{v_2}{v_2'} \cdot \frac{v_2'}{v_1} = T_2 \cdot T_1 = \frac{(a-a)(a-b)}{(a+a)(a+b)} = T$$

↑
same load
for T_2 & T_1

as when
calculate $\frac{v_2}{v_1}$ as loads stay as G