

09/13/06
ΣΣ610

New classroom starting M 09/18

EGR 1108

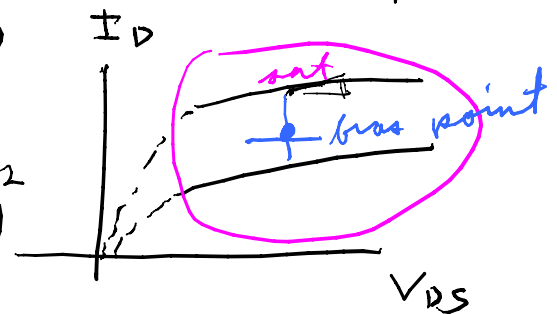
hw extended

NMOS, in saturation $V_{DS} \geq V_{GS} - V_{th}$ threshold voltage

VTO if $a=b$

$$I_D = \frac{K_P}{2} \frac{W}{L} (V_{GS} - V_{th})^2 (1 + \lambda V_{DS})$$

$$g_o = \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{V_{GS} = \text{const.}} = \lambda \left(\frac{K_P}{2} \frac{W}{L} \right) (V_{GS} - V_{th})^2 \approx \lambda I_D$$



$$g_m = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V_{DS} = \text{const.}} = \lambda \left(\frac{K_P}{2} \frac{W}{L} \right) (V_{GS} - V_{th}) (1 + \lambda V_{DS}) = 2 \frac{I_D}{V_{GS} - V_{th}}$$

$$i_D = I_D + i_d \approx \text{total} = \text{bias} + \text{small signal}$$

$$= f(v_{GS}, v_{DS}) = f(V_{GS}, V_{DS}) + \left. \frac{\partial f}{\partial v_{GS}} \right|_{v_{DS} = V_{DS}} (v_{GS} - V_{GS})$$

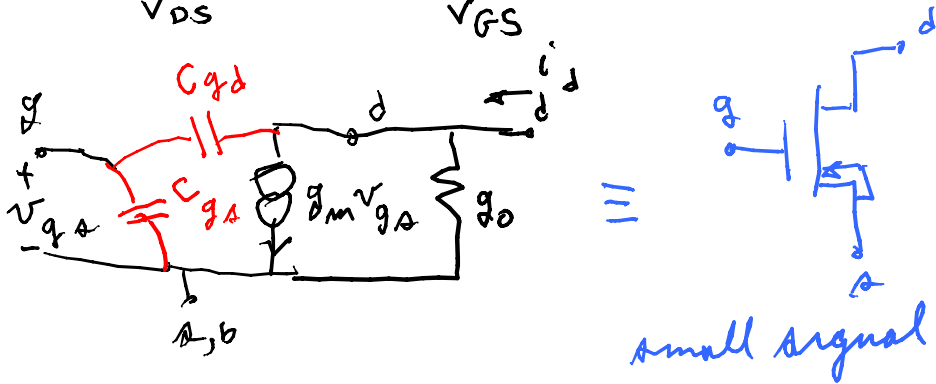
$$+ \left. \frac{\partial f}{\partial v_{DS}} \right|_{v_{GS} = V_{GS}} (v_{DS} - V_{DS}) + \frac{\partial^2}{\partial^2}, \dots$$

$$i_d = i_D - I_D$$

$$v_{gs} = v_{GS} - V_{GS}$$

$$v_{ds} = v_{DS} - V_{DS}$$

$$i_d = \frac{\partial i_D}{\partial v_{GS}} \bigg|_{v_{DS}} v_{GS} + \frac{\partial i_D}{\partial v_{DS}} \bigg|_{v_{GS}} v_{DS} = g_m v_{GS} + g_o v_{DS}$$



analysis: graph theory

KCL: $\underline{0} = \underline{C} i_b$

KVL: $\underline{0} = \underline{\sigma}^T v_b$

$v_b = \underline{C}^T v_t$

$i_b = \underline{\sigma}^T i_l$

$\underline{C}$ = cutset matrix

$= [1_t | K_i]$ if $i_b = \begin{bmatrix} i_t \\ i_l \end{bmatrix}$

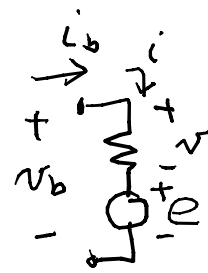
$\underline{\sigma}$ = tie set matrix

$= [-K_i^T | 1_l]$ if use the same tree

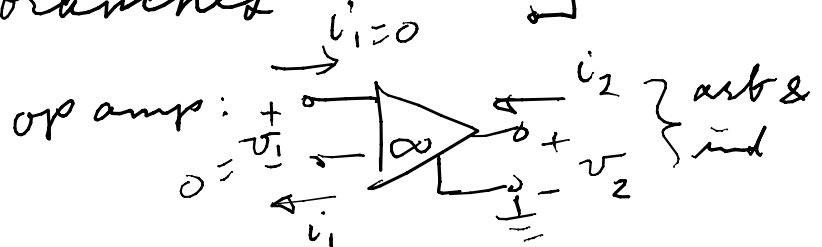
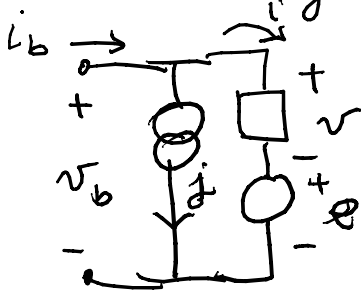
these are for the graph connections

For linear components

$$A v = B i$$



Take a generic branches



$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}; A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

here A & B are singular, no Y matrix or Z matrix

$$\begin{aligned} v_b &= v + e & v &= v_b - e, \quad i = i_b - j \\ i_b &= i + j & Av &= Av_b - Ae = Bi = Bi_b - Bj \end{aligned}$$

$$\begin{aligned} \text{or } Av_b - Ae &= A e^+ v_t - Ae \\ Bi_b - Bj &= B \sigma^+ i_\ell - Bj \end{aligned} \quad \Leftrightarrow$$

$$A e^+ v_t - B \sigma^+ i_\ell = \underbrace{Ae - Bj}_{\text{sources}}$$

$$\underbrace{b}_{\text{known}} \left\{ \underbrace{\begin{bmatrix} A e^+ & -B \sigma^+ \end{bmatrix}}_{\substack{\text{known} \\ t \quad \ell \\ b}} \underbrace{\begin{bmatrix} v_t \\ i_\ell \end{bmatrix}}_{\substack{\text{unknowns} \\ \# = b \\ b\text{-vectors}}} \right\} = \underbrace{Ae - Bj}_{\substack{\text{known sources} \\ a \quad b\text{-vectors}}}$$

eqs. in b unknowns

to solve, invert the coefficient matrix

$$\begin{bmatrix} v_t \\ i_\ell \end{bmatrix} = \begin{bmatrix} A e^+ & -B \sigma^+ \end{bmatrix}^{-1} \begin{bmatrix} Ae - Bj \end{bmatrix}$$

gives all voltages and all currents

$$v_b = e^+ v_t, \quad i_b = \sigma^+ i_\ell$$

If have admittance matrix for components

$$\underline{I} = \underline{Y}_{b \times b} \cdot \underline{V} = \underline{I}_b - \underline{J}$$

$$\times \underline{e} : \underline{e} \underline{I}_b = \underline{0}_t \Rightarrow \underline{e} \underline{Y}_{b \times b} \underline{V} = \underline{e} \underline{I}_b - \underline{e} \underline{J} = \underline{0} - \underline{e} \underline{J}$$

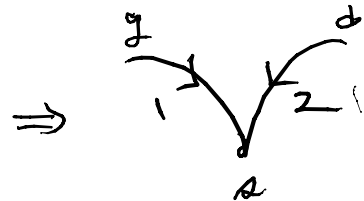
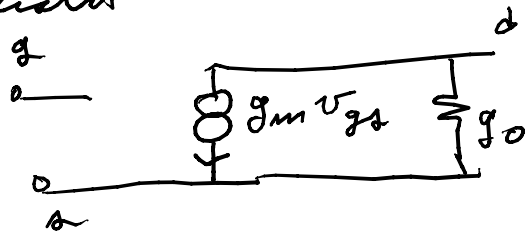
$$\underline{V} = \underline{V}_b - \underline{Q} \quad \underline{e} \underline{Y}_{b \times b} (\underline{V}_b - \underline{Q})$$

$$\underline{V}_b = \underline{e}^t \underline{V}_t \quad + \quad \left\{ \underline{e} \underline{Y}_{b \times b} \underline{e}^t \cdot \underline{V}_t = \underline{e} \underline{Y}_{b \times b} \underline{e} - \underline{e} \underline{J} \right.$$

t eqs & t unknowns in $\underline{V}_t$

or as $b = t + r$ have fewer equations if $\underline{Y}_{b \times b}$ is known

for the transistor



$$\underline{Y}_{b \times b} = \begin{bmatrix} 0 & 0 \\ g_m & g_o \end{bmatrix}$$