

KCL, KVL

09/11/06
EE610

KCL: sum of currents into a closed surface = 0



KVL: algebraic sum of voltages around a closed path = 0



to work with electrical circuits
need these two laws of connections
plus laws of elements connected.

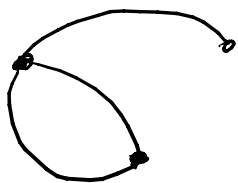
∴ set up graphs for the connections

graph = lines & points in a plane

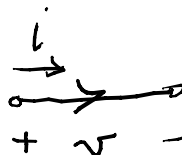
lines will have end points

points = nodes, number of them = n

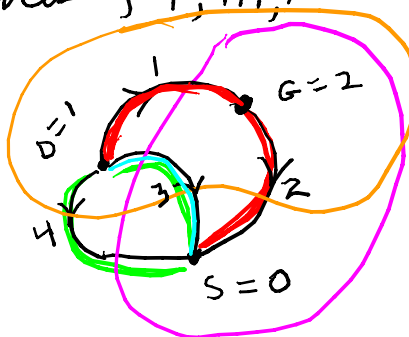
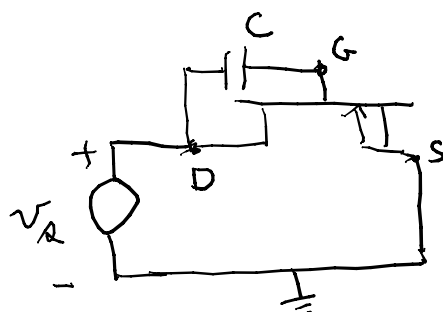
lines = branches, number of them = b



orient by arrows on the lines



give numbers to branches, $1, \dots, b$
" " " nodes, $1, \dots, n$



$n = 3$
 $b = 4$

drain to source

tree = node connections of all nodes = **tree**; tree branches
 cotree = limbs = **cotree**, number of = l number of = t

branches cut for one tree = cut set

$$KCL_1: 0 = i_{b_1} + i_{b_3} + i_{b_4} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_{b_1} \\ i_{b_2} \\ i_{b_3} \\ i_{b_4} \end{bmatrix}$$

$\underbrace{\quad}_{1_2 = 1_t}$

$$KVL_1: 0 = -v_{b_1} - v_{b_2} + v_{b_3}$$

$$KVL_2: 0 = -v_{b_1} - v_{b_2} + v_{b_4}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 1 & 0 \\ -1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_{b_1} \\ v_{b_2} \\ v_{b_3} \\ v_{b_4} \end{bmatrix}$$

$\underbrace{\quad}_{\frac{1}{l} = \frac{1}{2}}$

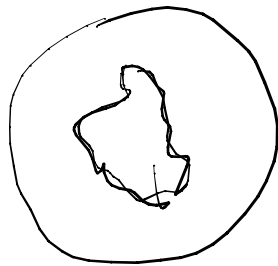
$$KCL: 0_t = C \begin{bmatrix} i_t \\ i_l \end{bmatrix} = \begin{bmatrix} 1_t & K \end{bmatrix} i_b$$

$$KVL: 0_l = \mathcal{T} \begin{bmatrix} v_t \\ v_l \end{bmatrix} = \begin{bmatrix} K_v & 1_l \end{bmatrix} v_b$$

$$t = n - 1$$

$$l = b - t$$

C = cut set matrix
 $\mathcal{T}$ = tie set matrix



for a bounded circuit
 surround by a sphere enclosing
 so no energy enters the
 sphere $\Rightarrow$ power into the
 circuit from outside is 0

$$P_{in}(t) = 0 = v_b^t \cdot i_b \quad ; \quad v^t = \text{transpose}$$

$$= [v_{b_1}, v_{b_2}, \dots, v_{b_b}] \begin{bmatrix} i_{b_1} \\ \vdots \\ i_{b_b} \end{bmatrix} = 0$$

Note: $0 = C \cdot i_b = [1_t \mid K_i] \begin{bmatrix} i_t \\ i_l \end{bmatrix} \Rightarrow -i_t = K_i i_l$

$$\therefore i_b = \begin{bmatrix} i_t \\ i_l \end{bmatrix} = \begin{bmatrix} -K_i i_l \\ 1_l i_l \end{bmatrix} = \begin{bmatrix} -K_i \\ 1_l \end{bmatrix} i_l = \begin{bmatrix} -1 & -1 \\ -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_{b3} \\ i_{b4} \end{bmatrix}$$

$$0 = \mathcal{V} v_b = [K_v \mid 1_l] \begin{bmatrix} v_t \\ v_l \end{bmatrix} \Rightarrow -v_l = K_v v_t$$

$$\text{or } v_b = \begin{bmatrix} v_t \\ v_l \end{bmatrix} = \begin{bmatrix} 1_t \cdot v_t \\ -K_v v_t \end{bmatrix} = \begin{bmatrix} 1_t \\ -K_v \end{bmatrix} v_t = v_b$$

$$v_b^t = v_t^t \cdot [1_t \mid -K_v^t]$$

$$p_{in}(t) = v_b^t i_b = v_t^t [1_t \mid -K_v^t] \begin{bmatrix} -K_i \\ 1_l \end{bmatrix} i_l = 0$$

here v_t & i_l can be freely chosen as this will hold for arbitrary voltages in the tree and arbitrary current sources in the links $\Rightarrow$ can cancel v_t & i_l
 scalar zero $\Rightarrow t \times l$ matrix 0

$$0_{t \times l} = [1_t \mid -K_v^t] \begin{bmatrix} -K_i \\ 1_l \end{bmatrix} = -K_i - K_v^t \Rightarrow K_v^t = -K_i$$

i.e KVL follows from KCL

$$C = \begin{bmatrix} 1_t & K_i \end{bmatrix}, \quad \mathcal{T} = \begin{bmatrix} K_v & 1_x \end{bmatrix} = \begin{bmatrix} -K_i^T & 1_x \end{bmatrix}$$

$$i_b = \begin{bmatrix} -K_i \\ 1_x \end{bmatrix} i_x = \mathcal{T}^t i_x \quad ; \quad v_b = \begin{bmatrix} 1_t \\ -K_v \end{bmatrix} v_t = \begin{bmatrix} 1_t \\ K_i^t \end{bmatrix} v_t = C^t v_t$$

$$i_b = \mathcal{T}^t i_x, \quad v_b = C^t v_t \quad \left. \vphantom{i_b = \mathcal{T}^t i_x} \right\} \text{ laws for a finite graph}$$

Elements

$$\begin{array}{c} \rightarrow \\ + \\ v \\ \downarrow \\ - \end{array} \quad \left. \vphantom{\begin{array}{c} \rightarrow \\ + \\ v \\ \downarrow \\ - \end{array}} \right\} L$$

$$v = L \frac{di}{dt} = L \Delta i$$

time invariant

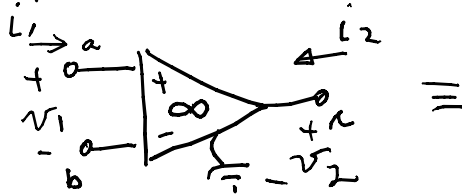
inductor

$$\begin{array}{c} i \\ \rightarrow \\ + \\ v \\ \downarrow \\ - \end{array} \quad \left. \vphantom{\begin{array}{c} i \\ \rightarrow \\ + \\ v \\ \downarrow \\ - \end{array}} \right\} C$$

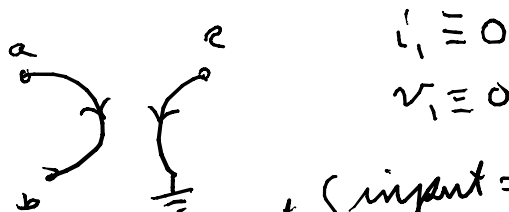
$$i = C \frac{dv}{dt} = C \Delta v$$

TIV capacitor

op-amp



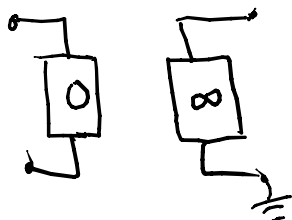
|||



$$i_1 \equiv 0$$

$$v_1 \equiv 0$$

nullator { input = open & short at once
norator { output = any voltage
any current

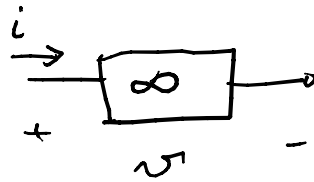


the nullator



$$i = 0$$
$$v = 0$$

the norator



$$i = \text{arbitrary}$$
$$v = \text{arbitrary independent of } i$$