

QUESTION #1

PART A)

$$\frac{\partial g(y)}{\partial y} = f(y) \rightarrow g(y) = \int f(y) dy$$

$$f(y) = [\cosh(y)] - 2 \rightarrow g(y) = \sinh(y) - 2y$$

$$\therefore \epsilon [\cosh(y) - 2] \frac{dy}{dt} = \frac{d}{dt} \{ \epsilon g(y) \} = \frac{d}{dt} [\epsilon (\sinh(y) - 2y)]$$

$$\text{Let } x_1 = y \quad \text{f. } x_2 = \frac{dy}{dt} + \epsilon \sinh(y) - 2y$$

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$$\frac{\partial^2 y}{\partial t^2} + \epsilon f(y) \frac{\partial y}{\partial t} + \omega_0^2 y = 0$$

$$\frac{d}{dt} \left[\frac{dy}{dt} \right] + \frac{d}{dt} [\epsilon \sinh(y) - 2\epsilon y] + \omega_0^2 y = 0$$

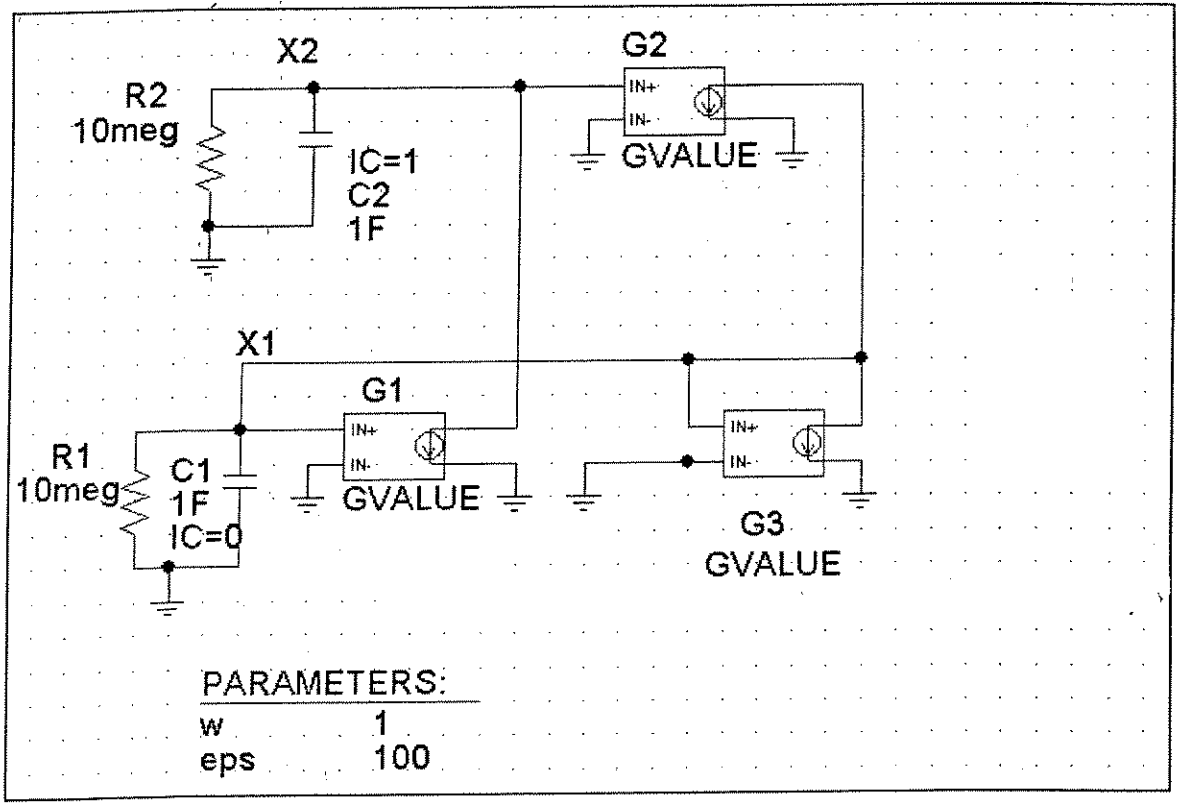
$$\frac{d}{dt} \left(\frac{dy}{dt} + \epsilon (\sinh(y) - 2y) \right) + \omega_0^2 y = 0$$

$$\frac{d}{dt} (x_2) = -\omega_0^2 y = -\omega_0^2 x_1$$

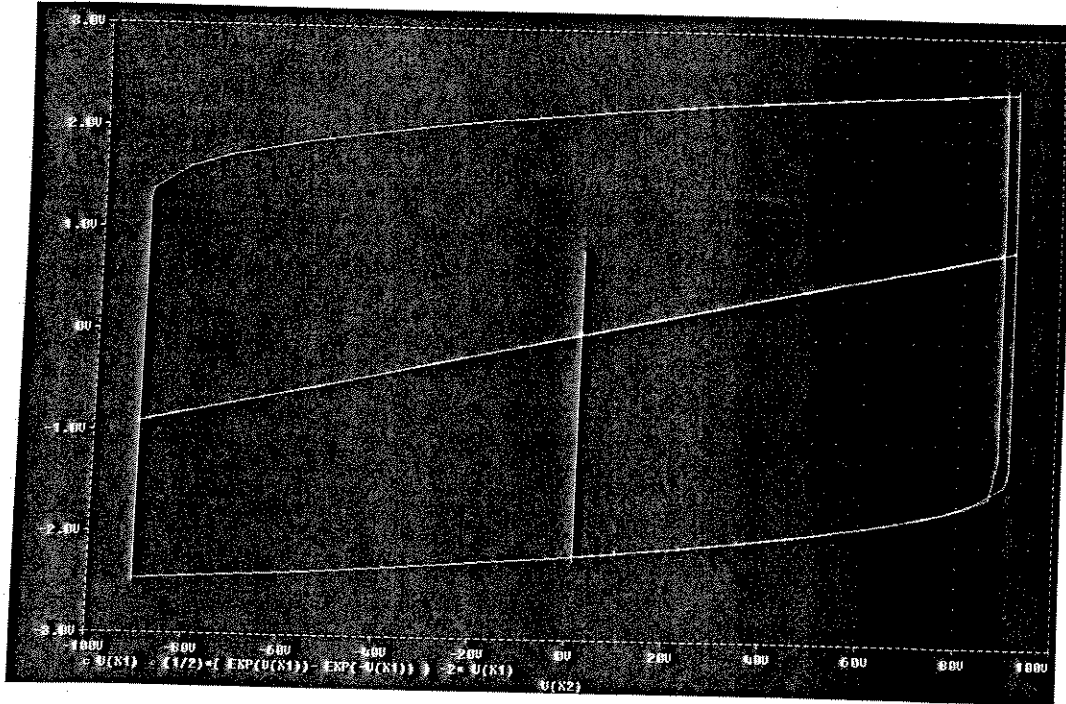
$$\frac{dx_2}{dt} = -\omega_0^2 x_1 \Rightarrow x_2 = \frac{dx_1}{dt} + \epsilon \sinh(x_1) - 2\epsilon x_1$$

$$\frac{dx_1}{dt} = x_2 - \epsilon \sinh(x_1) + 2\epsilon x_1$$

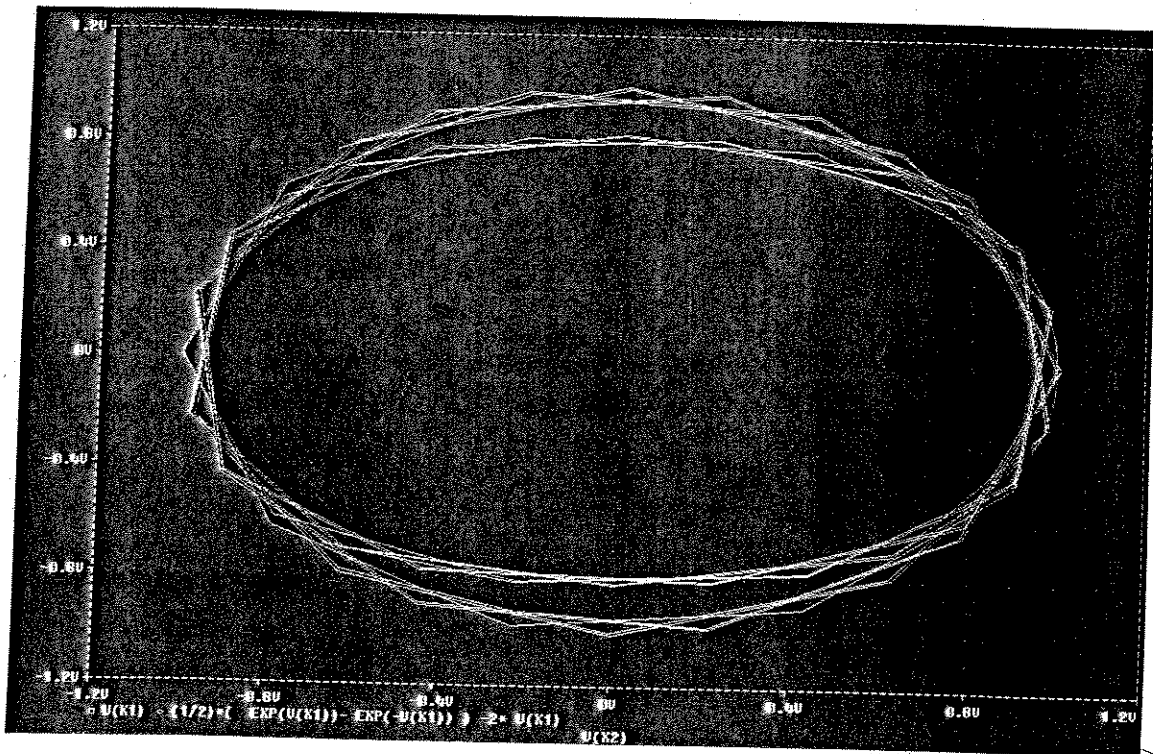
$$\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -\epsilon \sinh(x_1) + 2\epsilon x_1 \\ 0 \end{bmatrix}$$



PSPICE CIRCUIT FOR PART (A)



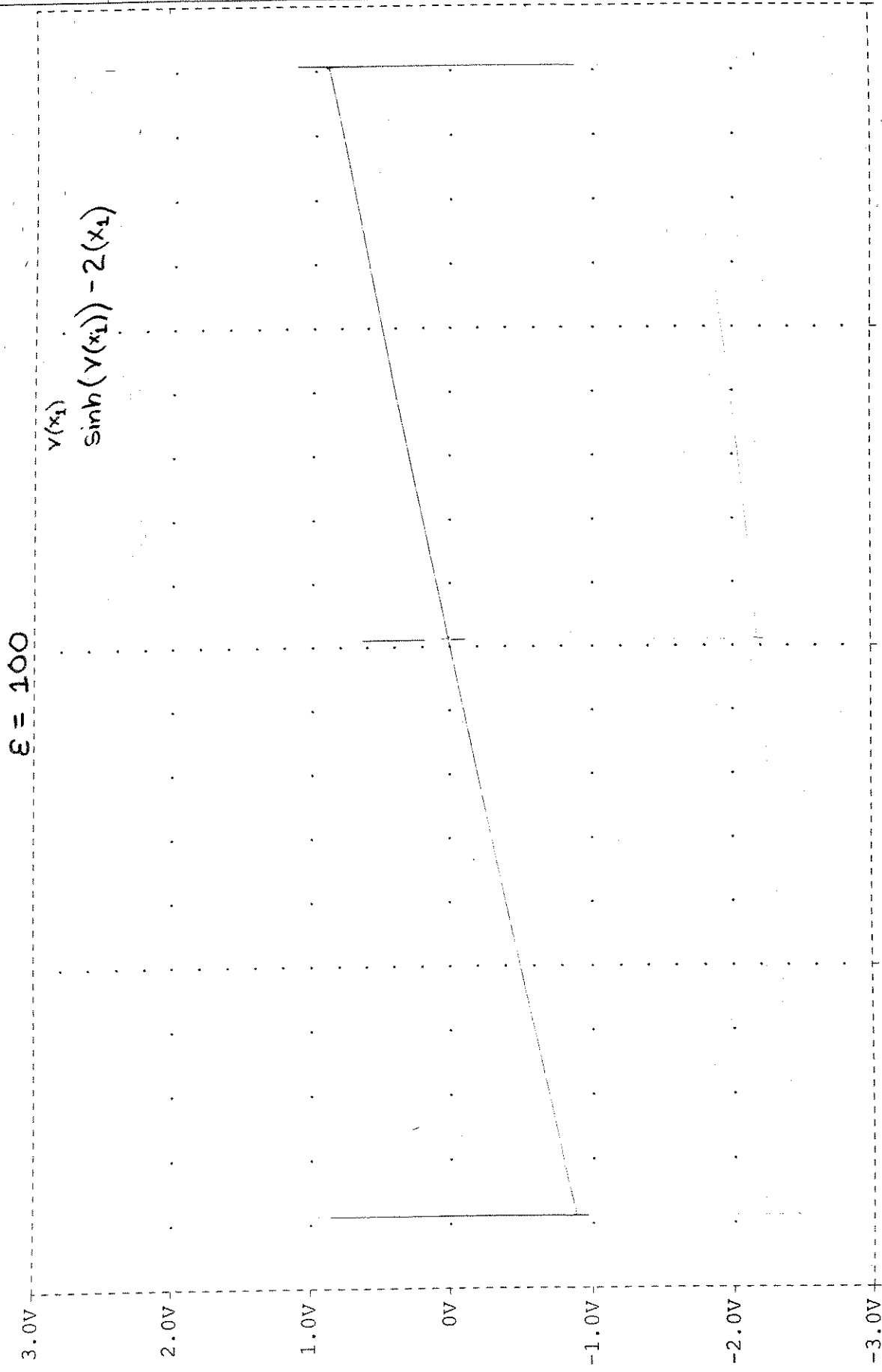
$\epsilon = 100$

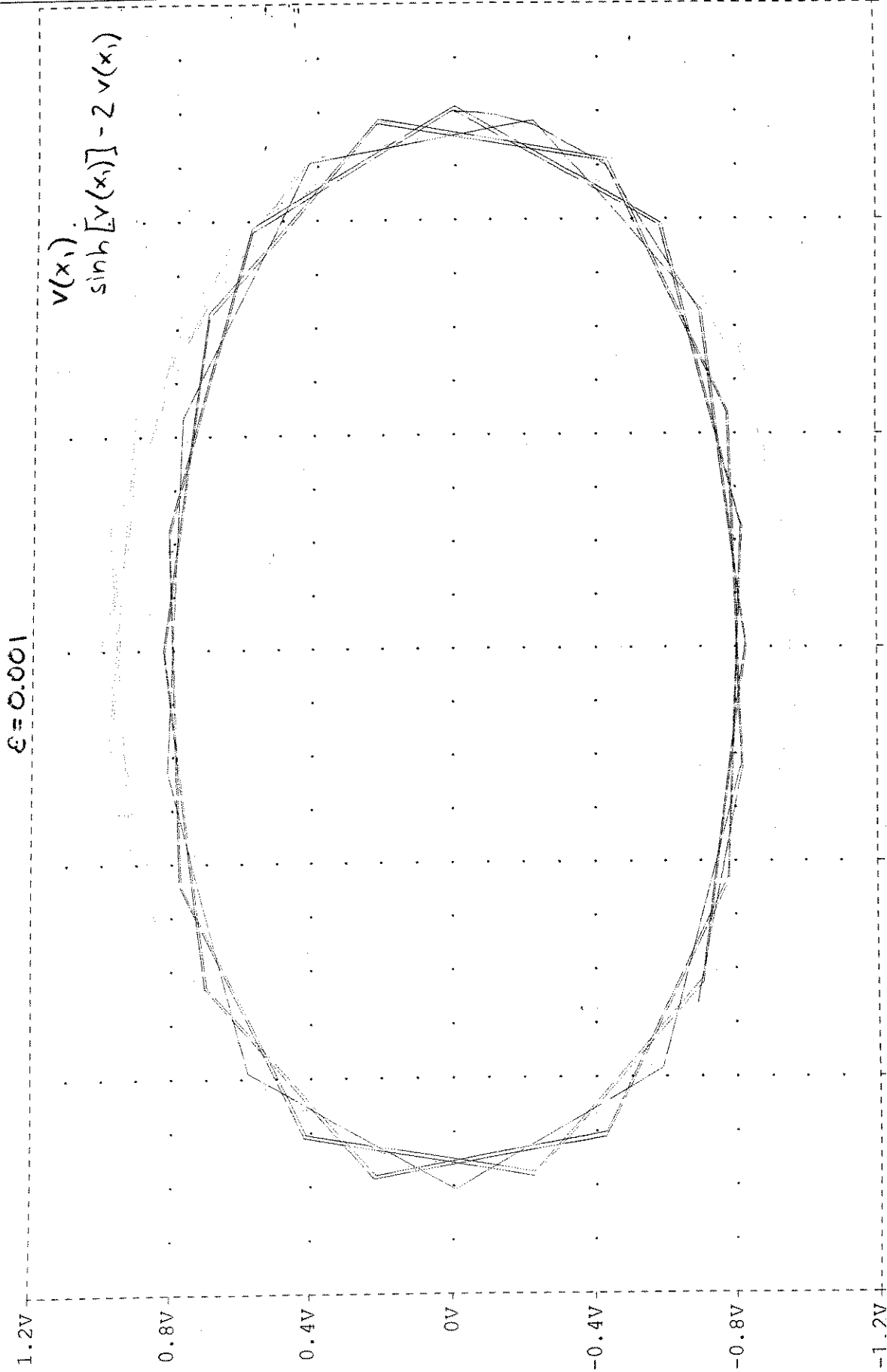


$\epsilon = 0.001$

$$f(y) = \cosh(y) - 2$$

$$g(y) = \sinh(y) - 2y$$





PART B)

$$g(y) = y - |y+1| + |y-1| = \begin{cases} y+2 & y < -1 \\ -y & -1 \leq y < 1 \\ y-2 & 1 \leq y \end{cases}$$

$$f(y) = \frac{\partial g(y)}{\partial t} = \begin{cases} -1 & y < -1 \\ 1 & -1 \leq y < 1 \\ 1 & 1 \leq y \end{cases}$$

$$\frac{d}{dt} \left[\frac{dy}{dt} \right] + \frac{d}{dt} [\epsilon g(y)] + \omega_0^2 y = 0$$

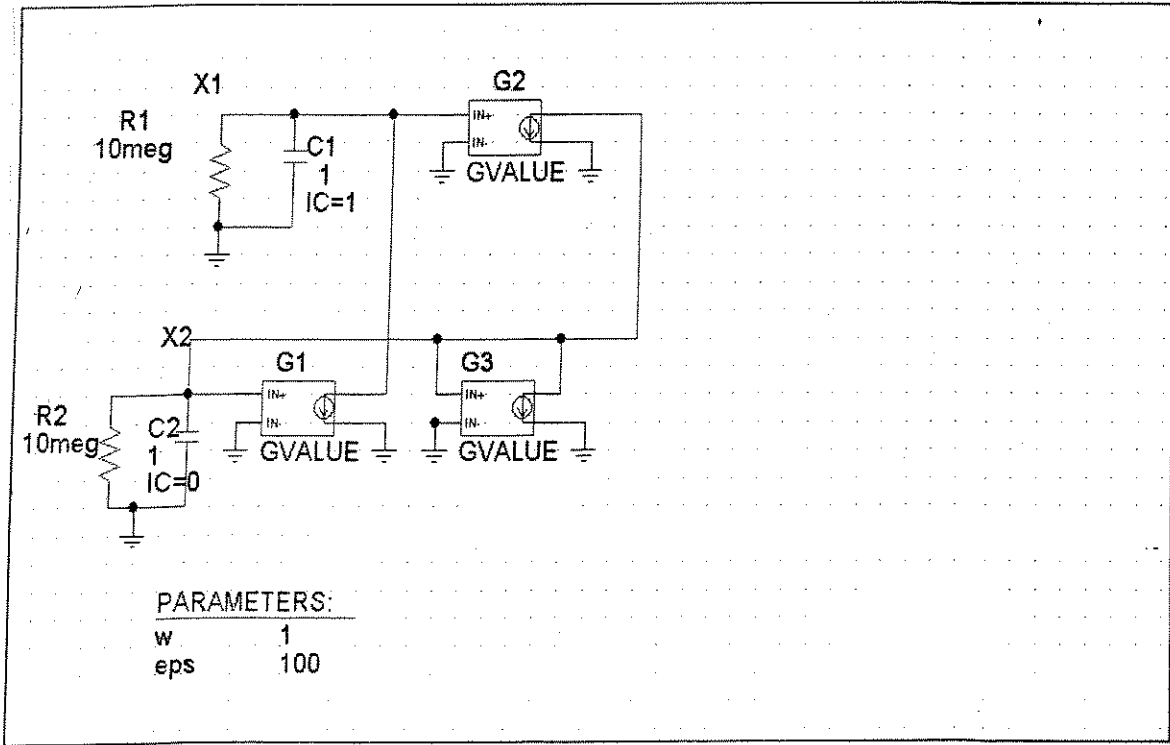
$$\frac{d}{dt} \left[\frac{dy}{dt} + \epsilon g(y) \right] + \omega_0^2 y = 0$$

$$\text{letting } x_1 = y \quad ; \quad x_2 = \frac{dx_1}{dt} + \epsilon g(x_1)$$

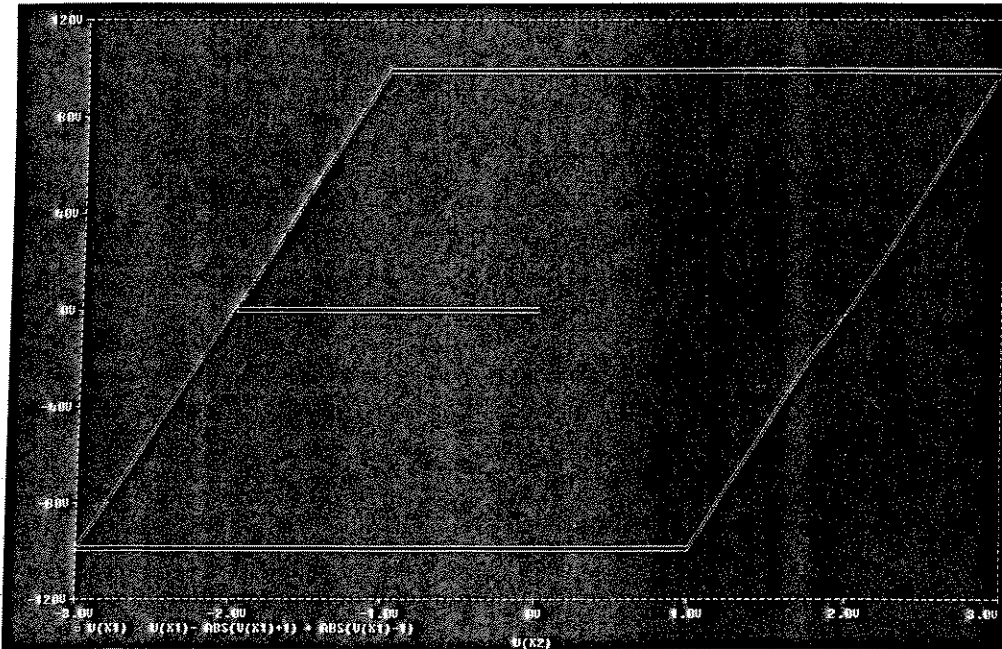
$$\text{Therefore, } \frac{dx_2}{dt} = -\omega_0^2 x_1$$

$$\frac{dx_1}{dt} = x_2 - \epsilon (x_1 - |x_1+1| + |x_1-1|)$$

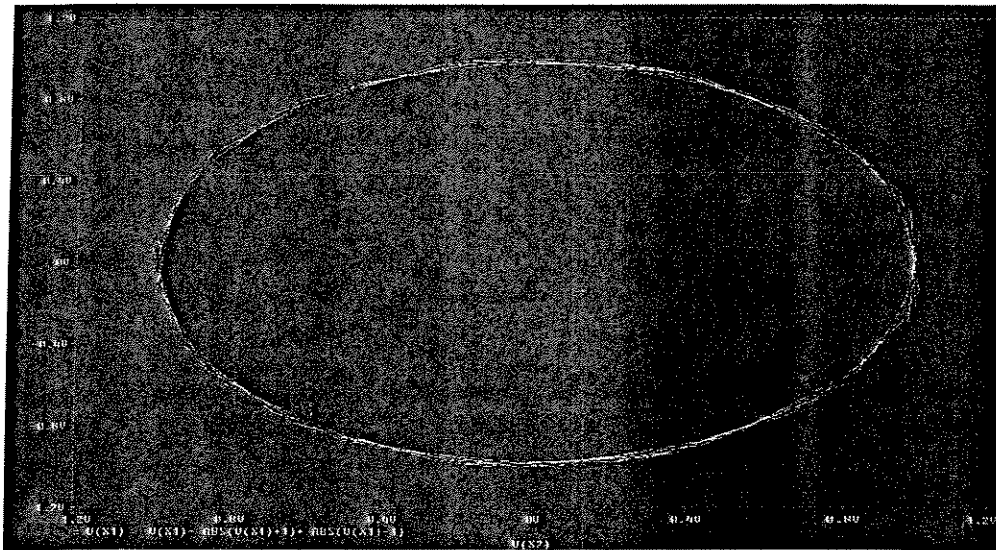
$$\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -\epsilon (x_1 - |x_1+1| + |x_1-1|) \\ 0 \end{bmatrix}$$



PSPICE CIRCUIT FOR PART (B)



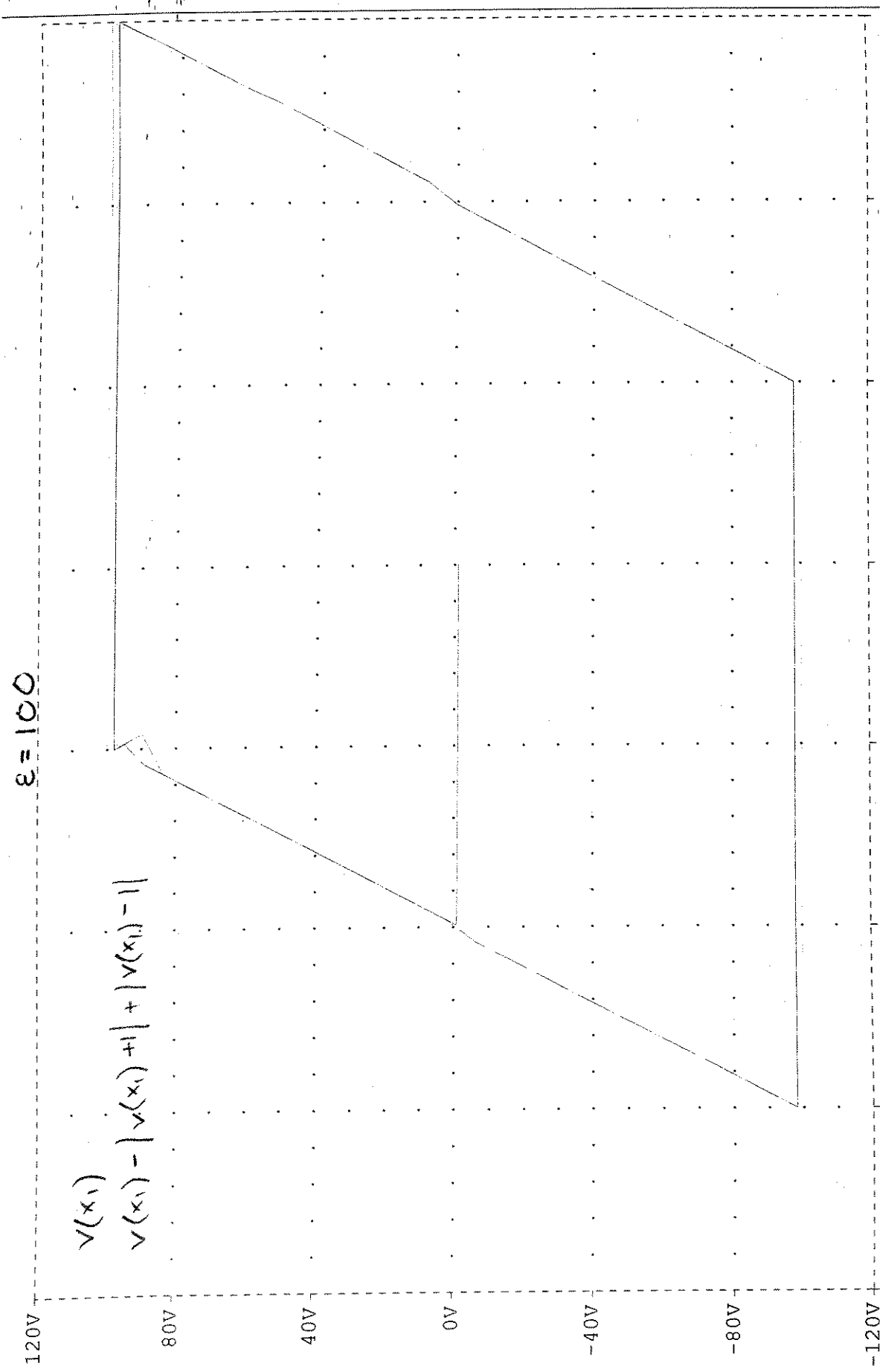
$\epsilon = 100$



$\epsilon = 0.001$

$$f(y) = \begin{cases} 1 & y < -1 \text{ or } y > 1 \\ -1 & -1 < y < 1 \end{cases}$$

$$g(y) = y - |y+1| + |y-1|$$



$g = 100$

$$v(x_1)$$
$$v(x_1) - |v(x_1) + 1| + |v(x_1) - 1|$$

120V
80V
40V
0V
-40V
-80V
-120V

