

ENEE 610
Homework Problems for Grading, Set 2 (100 points)
Due at class ~~M-09/26/05~~ **W 09/28/05**
Van der Pol type equations and matrix functions

1.(50 points)

For the following Van der Pol type equation

$$\frac{d^2 y}{dt^2} + \epsilon f(y) \frac{dy}{dt} + \omega_0^2 y = 0$$

set up state variable equations for each of the following two cases using (**note following changes**) $x_1=y$ and $x_2=(dy/dt) + \epsilon g(y)$ as state variables, with $dg(y)/dy=f(y)$.

- a) $f(y)=(\cosh(y)) - 2$
- b) $g(y)= y - |y+1| + |y-1|$

In each case set up a PSpice circuit and simulate. For grading submit in each case [a) and b)] your equations, a sketch of $f(y)$ and $g(y)$, along with the PSpice schematics and **a+two** plots of x_2 vs x_1 (along with $g(y)$ on the same plot) **for $\omega_0=1$ and the two values of $\epsilon=100$ and $\epsilon=0.001$.**

2. (50) points

a) Find the range of values of the parameters a and b such that the following two matrices commute. (**note B(3,3) changed to -2**)

$$A = \begin{bmatrix} 4 & a & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 0 & b \\ 0 & -2 & 2 \\ 0 & 0 & -2 \end{bmatrix}$$

- b) Choose $a=1$ and the b for which A and B commute. Find the eigenvalues of A and B.
- c) Under the assumption of b) use the eigenvalues to calculate $\exp(A)$ and $\exp(B)$.
- d) Continue c) and calculate $[\exp(A)][\exp(B)]$ and compare with $\exp(A+B)$.
- e) Choose $a=1$ and $b=1$ [for which A and B should not commute] calculate $[\exp(A)][\exp(B)]$ and show this is not $\exp(A+B)$. For these values ($a=b=1$) find C such that $[\exp(A)][\exp(B)]=\exp(C)$.

Hint: find the eigenvalues of $\{[\exp(A)][\exp(B)]\}$ and work with their ln's.