

$$\frac{1}{1(\sigma + j\omega)} + \frac{1}{1(\sigma - j\omega)} = Y + Y^*$$

$$= \frac{1}{1} \left[\frac{2\sigma}{|\sigma + j\omega|^2} \right] > 0 \quad \text{in } \sigma > 0$$

$$\therefore Y(\sigma) = \frac{1}{2\sigma} \text{ is positive real}$$

$$2V^1 = V + I, \quad 2V^2 = V - I$$

$$= \left(1 + \frac{1}{2\sigma}\right)V = \left(1 - \frac{1}{2\sigma}\right)V$$

$$\frac{2V^1}{2V^2} = S(\sigma) = \frac{1 - \frac{1}{2\sigma}}{1 + \frac{1}{2\sigma}} = \frac{2\sigma - 1}{2\sigma + 1}$$

$$S(-\sigma) = \frac{-2\sigma - 1}{-2\sigma + 1} = \frac{1 + \sigma}{1 - \sigma} = \frac{1}{S(\sigma)}$$

}

lossless: $\mathbf{I}_n - S^T[-j\omega] S[j\omega] = 0$

set $s = \omega/j$ get $\mathbf{I}_n = S^T[-s] S[s]$

$\Rightarrow S^T[s] = S^T[-s]$

Ex: $\begin{bmatrix} 1 \\ C > 0 \end{bmatrix}$

$S[s] = \frac{-Cs}{1 + Cs}$

$S[-s] = \frac{1 + Cs}{1 - Cs} = \frac{1}{S[s]}$

lossless: $Y^H[j\omega] + Y[j\omega] = 0_n$

$\Rightarrow Y^T[-s] + Y[s] = 0_n$

or $Y[s] = -Y^T[-s]$

Ex: $\begin{bmatrix} 1 \\ L > 0 \end{bmatrix}$

$Y[s] = \frac{1}{Ls}$

$Y[-s] = \frac{1}{L(-s)} = -\frac{1}{Ls}$

or check

$Y[s] + Y[-s] = 0$

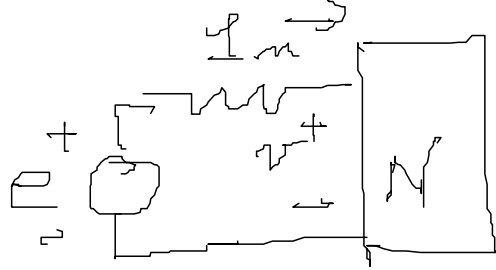
look at $\operatorname{Re} Y[s]$ is $0 \geq 0$

$2\operatorname{Re} Y = Y(s + j\omega) + Y^*(s + j\omega)$ is $0 \geq 0$

$$\text{If } E(\infty) = 0$$

all energy

sent into R



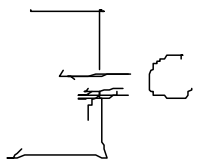
came back out and gets dissipated

in the terminating $R = 1 \Omega$, call

$$\text{this loss } \Rightarrow 1 = \|S_{21}(j\omega)\|^2$$

$$1_n = S^{T*}(j\omega) \cdot S(j\omega) = S^T(-j\omega) S(j\omega)$$

for real components



$$Y(s) = sC$$

look at s or $j\omega$ axis

$$S(s) = \frac{1 - sC}{1 + sC} \quad \text{no pole on } j\omega \text{ axis}$$

positive if $C > 0$, $S(s)$ has a pole at $-\frac{1}{C}$

$$S(j\omega) = \frac{1 - j\omega C}{1 + j\omega C} ; S^{*}(j\omega) \cdot S(j\omega)$$

$$= 1 = \|S(j\omega)\|^2$$

$$\frac{Y(j\omega) + Y^{*}(j\omega)}{2} = \text{Re } Y(j\omega)$$

$$\Rightarrow \frac{j\omega C - j\omega C}{2} = 0$$

$$\|S\|_F^2 = (1 - \|S\|_F^2) \cdot \|V\|_F^2 \Rightarrow \|S\|_F \text{ bounded by 1}$$

$\Rightarrow$ passive $S(s)$ are bounded real

by definition $S(s)$ is bounded real if

1) $S(s)$ has $\sigma(s)$ real in $\sigma > 0$

2) $S(s)$ is analytic in $\sigma > 0$ (real element stability)

3) $I_m - S^*(s) \cdot S(s)$ is positive semidef in $\sigma > 0$

if rational 1) real coefficients

2) $\rightarrow$ stable $\rightarrow$ no poles in $\sigma > 0$

3) $\rightarrow$ no poles on $j\omega$ axis $\rightarrow L^2 \rightarrow L^2$ for

$$\text{Ex: } 2V^1 = V + U, \quad 2V^1 = V - U$$

$$2V^2 = V(s) + I(s), \quad 2V^1(s) = V(s) - I(s)$$

$$\text{representation: } I(s) = C \cdot s \cdot V(s)$$

$$2V^1(s) = (1 + C s) V(s), \quad 2V^1(s) = (1 - C s) \cdot V$$

$$\frac{2V^1}{2V^1} = S(s) = \frac{1 - C s}{1 + C s} \Rightarrow \frac{1}{1 + C s}$$

$$\int_{-\infty}^{\infty} v^n v^i dx = \int_{-\infty}^{\infty} v^i - v^n dx$$

$$E(\infty) = \langle v^i, v^i \rangle - \langle v^n, v^n \rangle = \|v^i\|^2 - \|v^n\|^2$$

≥ 0 if passive

Now $V^n = S(a) V^i$, $S(a)$ = scatter matrix

Parseval's theorem:

$$\int_{-\infty}^{\infty} g^*(t) h(t) dt = \int_{-\infty}^{\infty} G^*(j\omega) H(j\omega) \frac{d\omega}{2\pi}$$

$$= \int_{-\infty}^{\infty} G^*(j2\pi f) H(j2\pi f) df$$

if g^* = complex conjugate into

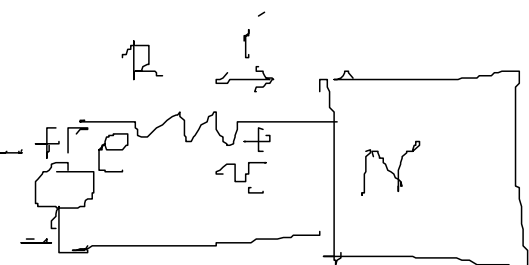
$$\langle x, y \rangle = \int_{-\infty}^{\infty} x^T y dt \quad \text{then}$$

$$\langle v^i, v^i \rangle = \langle V^i, V^i \rangle, \quad \langle v^n, v^n \rangle = \langle V^n, V^n \rangle$$

$$E(\infty) = \|v^i\|^2 - \|v^n\|^2 = \|V^i\|^2 - \|V^n\|^2$$

$$= \|V^i\|^2 - \|S \cdot V^i\|^2 = \|V^i\|^2 - \|S\|^2 \|V^i\|^2$$

Passive circuits - n-ports



$$[v, i] \in N; \quad \Sigma(t) = \int_{-\infty}^t v(\tau) i(\tau) d\tau$$

$$\Sigma(t) = \int_{-\infty}^t v^T(\tau) i(\tau) d\tau = \int_{-\infty}^t p_{in}(\tau) d\tau$$

passive if $\Sigma(t) \geq 0$ for all t & all $[v, i] \in N$

$$t \rightarrow \infty, \quad \Sigma(\infty) = \langle v^i, i^i \rangle$$

$$v = v^i + v^r = 2v^i \quad \text{choose } R = 1 \Omega$$

$$2v^r = v - v^i = v - Ri^i \Rightarrow \text{reflected voltage}$$

$$2v^i = v + Ri^i \Rightarrow \text{incident voltage}$$

$$\text{add: } v = v^i + v^r, \quad Ri^i = v^i - v^r = i^i, \quad R = 1 \Omega$$

$$\Sigma(\infty) = \langle v, i \rangle = \langle v^i + v^r, v^i + v^r \rangle$$

$$= \langle v^i, v^i \rangle - \langle v^i, v^r \rangle + \langle v^r, v^i \rangle - \langle v^r, v^r \rangle$$

$$\text{but } \langle v^i, v^r \rangle = \langle v^r, v^i \rangle$$