

08/30/00

ENEE 610

P. 1

R. Newcomb

Electrical Network Theory

Network = connection of
circuits

Circuits = connection of
electrical components
(R, L, C, M, Trans., op-amps,
transistors, ...)

Theory = mathematics
that pertains to circuits

$\Rightarrow$ CAD = ^{spec} math
math cad
mathematics

analysis $\Rightarrow$ matrix
design $\Rightarrow$ synthesis

Now will do a little
synthesis $\Rightarrow$ analysis.

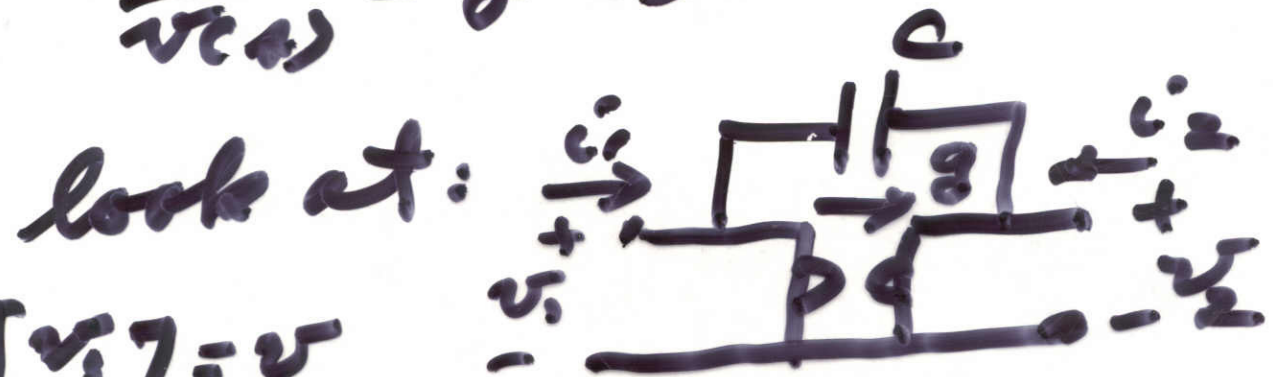
use Richards' function, p. 351

Ex: $g(s) = \frac{s+4}{s+9}$

derive a circuit for this.



$$\frac{i(s)}{v(s)} = g(s)$$



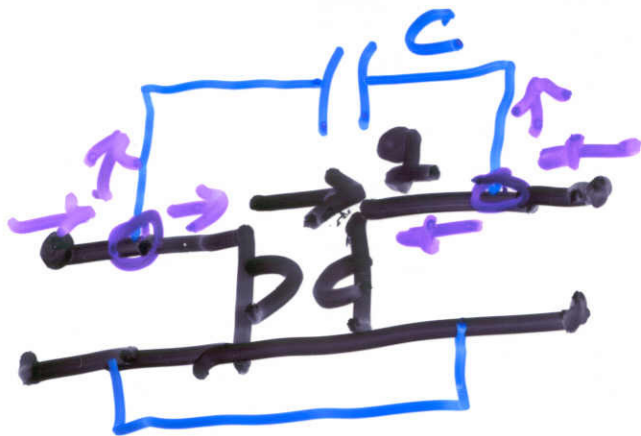
$[v_1] = v$

$$i = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Y(s) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}; i = Yv$$

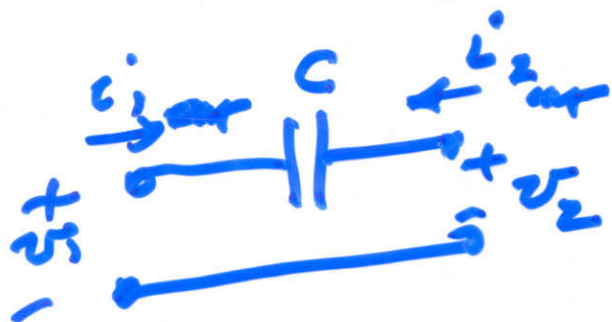
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$$Y = Y_{gpn} + Y_{cap}$$



$$i_{2cap} = -i_{1cap}$$

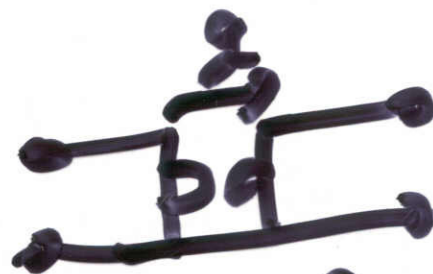
$$i_{cap} = Y_{cap} v$$

$$i_{1cap} = Y_{1cap} v_1 + Y_{12cap} v_2$$

$$= \frac{1}{sC} v_1 - \frac{1}{sC} v_2$$

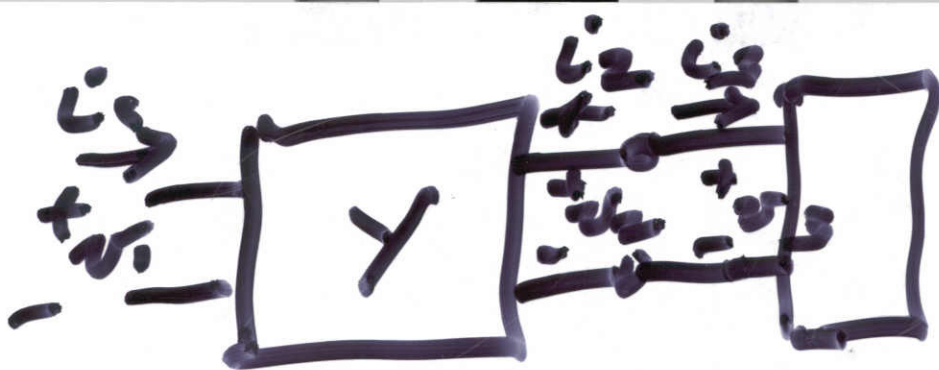
$$Y_{cap} = \begin{bmatrix} \frac{1}{sC} & -\frac{1}{sC} \\ -\frac{1}{sC} & \frac{1}{sC} \end{bmatrix} = sC \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$Y = \begin{bmatrix} sC & g - sC \\ -g - sC & sC \end{bmatrix}$$



$$Y_{gpn} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$$

$$= -Y_{gpn}^T = \text{transpose}$$



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$$v_2 = v_3, \quad i_2 = -i_3$$

$$i_3 = g_1 v_3$$

$$i_2 = -g_1 v_2$$

$$\begin{bmatrix} ac & g-ac \\ -g-ac & ac \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 = -g_1 v_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} ac & g-ac \\ -g-ac & ac+g_1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ 0 \end{bmatrix}$$

add $-\frac{(g-ac)}{ac+g_1} \times \text{row 2 to row 1}$

$$\begin{bmatrix} AC + (g + AC) \left(\frac{g - AC}{AC + y_L} \right) & 0 \\ -(g + AC) & AC + y_L \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$y_{\text{input}} = AC + \frac{(g + AC)(g - AC)}{AC + y_L}$$

$$= \frac{AC y_L^{(a)} + g^2}{AC + y_L^{(a)}} = y^{(a)}$$

solve for y_L :

$$AC y_L + g^2 = AC y + y_L \cdot y$$

$$(AC - y) y_L = AC y - g^2$$

$$y_L^{(a)} = \frac{g^2 - AC \cdot y^{(a)}}{y^{(a)} - AC}$$

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decide to use the
Richards' function:

$$r.36: R(a) = \frac{k_2(a) - a_2(a)}{k_2(a) - a_2(a)}$$

(8.6-1)

use

$$y_1(a) = y(a) \frac{k_2(a) - a_2(a)}{k_2(a) - a_2(a)}$$

$$= \frac{g^2 - a_2(a)}{y(a) - a_2(a)}$$

$$= y(a) \cdot \frac{k_2(a)}{k_2(a)} \cdot \frac{1 - \frac{a_2(a)}{k_2(a)}}{\frac{y(a)}{y(a)} - \frac{a_2(a)}{k_2(a)}}$$

$$= \frac{g^2}{Ck} \left[\frac{1 - \frac{a_2(a)}{g^2/C}}{\frac{y(a)}{Ck} - \frac{a_2(a)}{k_2(a)}} \right]$$

$$y(k) = Ck \quad ; \quad ky(k) = g^2/C$$

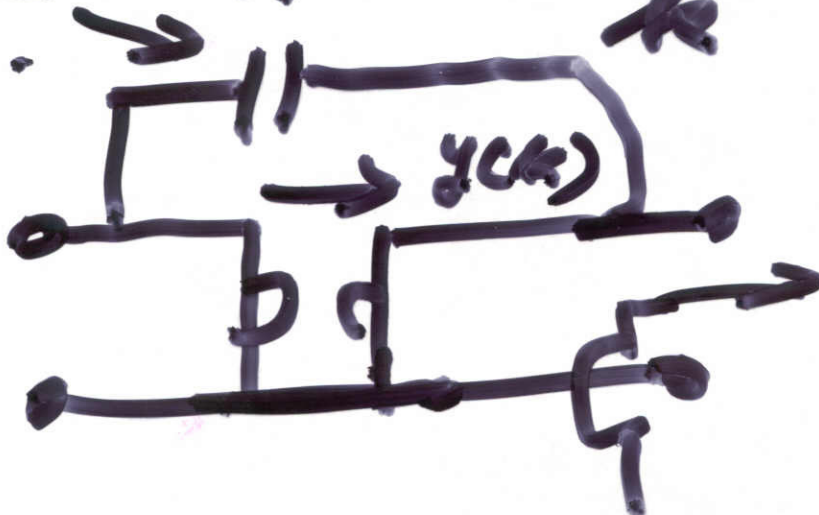
$$\Rightarrow \frac{g^2}{Ck} = y(k) = Ck$$

$$g^2 = (Ck)^2$$

$$g = \pm Ck, \text{ choose } +$$

$$y(k) = g$$

$$C = g/k = \frac{y(k)}{k}$$



$$y_k = y(1) \left| \frac{k y(1) - 1 y(1)}{k y(1) - 0 y(1)} \right|$$

at $k=1$, the numerator & denominator have a common

zero $\Rightarrow$ it cancels

$$dy(x) = k y(x)$$

look at $k \rightarrow -k$

$\Rightarrow -k y(-k)$ on the right

$$= k y(k)$$

$$\text{if } -y(-k) = y(k)$$

$\therefore$ would get a second
cancellation at $x = -k$

i.e. if

$$y(k) + y(-k) = 0$$

$\therefore$ choose ~~to be~~ k to be a

zero of $y(x) = \frac{y(x) + y(-x)}{2}$

$$y(x) = \frac{x+4}{x+9} \quad \begin{array}{c|c} \omega & \text{E610} \\ \hline & \text{P. 9} \\ & 05/30/0 \end{array}$$

$$2\text{Ev } y(x) = \frac{x+4}{x+9} + \frac{-x+4}{-x+9}$$

$$= \frac{(x+4)(-x+9) + (-x+4)(x+9)}{(x+9)(-x+9)}$$

$$= \frac{-2x^2 + 2x + 4x + 36}{\dots} = \frac{-2(x^2 - 4x + 18)}{\dots}$$

zeros at $x = \pm 2 \times 3 = \pm 6$

choose $x = +6$

$$y(x) = \frac{6+4}{6+9} = \frac{10}{15} = \frac{2}{3}$$

$$y_L(x) = \frac{2}{3} \left[\frac{6 \cdot \frac{2}{3} - x \left(\frac{x+4}{x+9} \right)}{6 \cdot \left(\frac{x+4}{x+9} \right) - x \cdot \frac{2}{3}} \right]$$

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$$= K \cdot \frac{(a-6)(a+6)}{(a-6)(a+6)}$$

know this from the theory

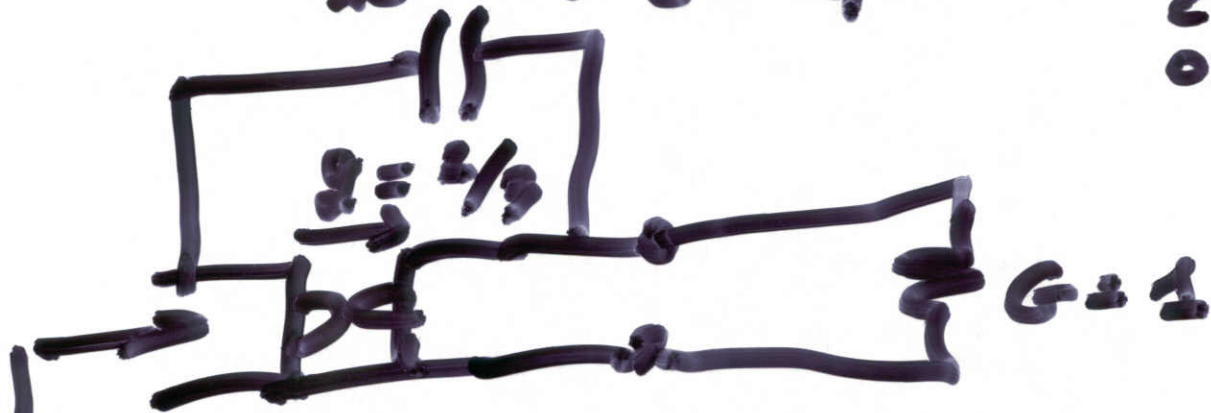
$$= \frac{2}{3} \left[\frac{4(a+9) - a^2 - 4a}{6(a+4) - \frac{2}{3}(a+9)a} \right]$$

$$= -\frac{2}{3} \left[\frac{a^2 - 4 \times 9}{6a + 6 \times 4 - \frac{2}{3}a^2 - \frac{2}{3} \times 9a} \right]$$

$$= \frac{-\frac{2}{3}}{-\frac{2}{3}} \left[\frac{a^2 - 4 \times 9}{a^2 - 6 \times 4 \times \frac{3}{2}} \right] = 1 = \text{degree zero}$$

$$C = \frac{y(x)}{x} = \frac{3}{3} \cdot \frac{1}{6} = \frac{1}{6}$$

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$$y(x) = \frac{x+4}{x+9}$$