DC Bias Equations for Common Source Amplifier

Unlike a BJT, where we make the reasonable approximation the $V_{BE} = 0.7V$ for DC biasing a common emitter amplifier using the Base-Emitter loop, for the MOSFET Common Source Amplifier the DC bias is more complicated because the analogous quantity V_{GS} does not have a simple approximation for its value. Instead, we have to solve device equation and the loop equation simultaneously.

The gate-source loop gives the following:

$$V_{GG} - V_{GS} - I_D R_S = 0 \tag{1}$$

Here we have two unknowns (I_D and V_{GS}) and only one equation. Therefore, we also utilize the drain current versus gate voltage equation for the MOSFET is in satuation region of operation for the second equation.

$$I_D = \gamma(V_{GS} - V_{TH})^2 \tag{2}$$

$$\gamma = \frac{\mu C_{ox} W}{2L} \tag{3}$$

Method 1: We eliminate V_{GS} and obtain the following equation for I_D in terms of known circuit parameters:

$$I_D = \frac{2\gamma R_S(V_{GG} - V_{TH}) + 1}{2\gamma R_S^2} - \frac{[(2\gamma R_S(V_{GG} - V_{TH}) + 1)^2 - 4\gamma^2 R_S^2 (V_{GG} - V_{TH})^2]^{1/2}}{2\gamma R_S^2} \tag{4}$$

Now we can substitute I_D back into the Gate-Source loop equation to obtain V_{GS}.

Method 2: Alternatively, we could have first eliminate I_D from equations (1) and (2), and obtain an equation for V_{GS}. Performing the algebra for this second approach gives the following equation for V_{GS} in terms of known circuit parameters:

$$V_{GS} = \frac{-(1 - 2\gamma V_{TH} R_S)}{2\gamma R_S} + \frac{[(1 - 2\gamma V_{TH} R_S)^2 - 4(\gamma^2 R_S^2 V_{TH}^2 - \gamma R_S V_{GG})]^{1/2}}{2\gamma R_S} \tag{5}$$

Of course, with V_{GS} now known, we can substitute back into the gate-source loop equation (1) to obtain I_D.