Investigation of the On-Chip MIS Interconnects with the Alternating-Direction-Implicit (ADI) Method

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Abstract — The Alternating Direction Implicit Finite-Difference Time-Domain method is introduced to analyze the Metal-Insulator-Semiconductor interconnects by solving Maxwell’s equations in the time domain. We analyze the quasi-TEM, slow wave and the skin-effect mode, where modern integrated circuit devices are most likely to operate. The silicon substrate loss and the metal line loss are included in the analysis.

Index Terms — ADI-FDTD method, boundary layer problem, MISM interconnects, substrate loss, skin-effect mode.

I. INTRODUCTION

Metal-Insulator-Semiconductor Metal (MISM) structure is a basic interconnect unit in integrated circuits (ICs). Three fundamental mode limits, namely the dielectric quasi-TEM mode, the slow-wave mode, and the skin-effect mode, have been modeled analytically and verified by experiment in early studies of this fundamental IC unit[1]. Subsequent investigators used the conventional FDTD method to analyze on-chip interconnects in two of these modes. However, the analysis was not performed on the skin-effect mode because of the computational overhead required to satisfy the Courant–Friedrichs–Lewy (CFL) stability limit [2]. However, in modern ICs, clock rates are above 3GHz, and harmonics are well into the tens of GHz. At these frequencies, the MISM structure is in the skin-effect mode for typical substrate doping. It is thus important to study wave propagation in this range.

To investigate the skin-effect mode in digital IC’s, we apply the Alternating Direction Implicit Finite-Difference Time-Domain (ADI-FDTD) method, because it is not limited by the CFL condition [3]. The ADI-FDTD is especially powerful in solving electromagnetic problems in digital ICs which have fine physical scales much less than the wavelength of interest in the time domain. Also, to the best of our knowledge, this is the first time the ADI-FDTD method has been applied to analyze on-chip MISM structures.

In this paper, we first generate a quasi-analytical solution of the frequency-doping map, which extends the work presented in [1]. The map gives the attenuation factor (α), and the phase constant (β), as a function of frequency and silicon doping. We then compare the attenuation factor and phase constants, obtained from the map in the various modes, with ones that we obtain from the ADI-FDTD method. In addition to illustrating important physics, this serves to help verify the accuracy of the ADI method. We then use the ADI method to investigate the signal propagation characteristics along 4-layered Metal-Insulator-Semiconductor-Metal (MISM) structures. This work extends the numerical analysis from the slow-wave and the dielectric quasi-TEM modes, to the skin-effect mode. In addition, it provides details of the losses in the semiconductor substrate and the metal interconnect.

II. QUASI-ANALYTICAL AND NUMERICAL ANALYSES: PROCEDURES AND RESULTS

We perform our analyses on the MISM structure. Fig.1 shows the side view of the MISM structure.

![Fig. 1. Side view of the MISM structure. Z is the direction of propagation. h, b1 and b2 is the thickness of the metal layer, the SiO2 layer and the silicon substrate, separately.](image_url)

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A. Quasi-Analytical Analysis and 3-Mode Limits

The MISM structure gives rise to a 4 layered boundary problem. In the quasi-analytical analysis, we neglect the metal thicknesses, and extend the line width to infinity. We use perfectly electric conductor (PEC) boundary conditions for the top and bottom metal layers. The equations for the longitudinal and the transverse propagation constants have been derived previously[1], and are written below:

\[ \gamma_i^2 + \gamma_j^2 = -k_0^2 \mu, \quad i=1,2 \]  \hspace{1cm} (1)

\[ \sum_i \gamma_i \tanh(\gamma_i b_i) = 0, \quad i=1,2 \]  \hspace{1cm} (2)

\[ \gamma_1 \text{ and } \gamma_2 \text{ denote the transverse propagation constants (y direction) in SiO_2 and Si layers, respectively, and } \gamma \text{ is the longitudinal one (z direction). Here, } \gamma = \alpha + j\beta, \text{ where } \alpha \text{ is the attenuation factor, and } \beta \text{ is the phase constant. } k_0 \text{ is the wave number in free space, and } \epsilon_i = \epsilon_0 + \sigma_j/(j\omega \epsilon_0), \text{ i=1,2. } \epsilon_0 \text{ and }
\( \mu_0 \) are the permittivity and permeability in the vacuum, respectively. We solve this set of coupled non-linear equations using the Newton-Raphson method. The resulting attenuation factors and the phase factors on the frequency-doping plane are shown in Fig. 2. In the figure, three bold lines divide the map into 3 fundamental mode regions, and a transition region. The characteristic frequency for the skin-effect in the Si substrate is 
\[ f_\delta = \frac{1}{2\pi} \cdot \frac{2}{\mu_2 \sigma_2 b_2^2} \]
the dielectric relaxation frequency in the Si substrate is 
\[ f_e = \frac{1}{2\pi} \cdot \frac{\sigma_2}{\varepsilon_2 \varepsilon_0} \]
and the characteristic frequency of slow-wave mode is 
\[ f_0 = \left( f_s^{-1} \cdot 2/3 - f_\delta^{-1} \right)^{-1} \]
\[ f_s = \frac{1}{2\pi} \cdot \frac{\sigma_2}{\varepsilon_2 \varepsilon_0} \cdot \frac{b_1}{b_2} \]
the relaxation frequency of the interfacial polarization. The location of these edge lines depends on both the geometrical factors \((b_1, b_2)\), and the electrical factors \((\sigma_2, \varepsilon_2)\). The conductivity and the doping density transform comes from [5].

B. Numerical Analysis: Extracting Propagation Modes and Constants

We next use our ADI-FDTD simulator to analyze the structure in Fig. 1. In contrast with the quasi-analytical approach, here we account for the thickness of the metal interconnect. The metal line is excited at one end; Mur’s 1st absorbing boundary condition is applied on the open region, and the PEC boundary condition is added on the ground plane. We take the MISM structure to have 
\( h=1.8\mu m, b_1=2\mu m, b_2=200\mu m, \) and the metal conductivity to be \( \sigma_{Al} = \frac{7}{10^3} \times mS \). A sharp Gaussian pulse with time constant \( \tau = 8.83 \) ps (bandwidth is 50GHz) is used as the excitation. 82 uninformed grids are laid out along the y direction with the minimum grid size of 0.1µm inside the oxide and the metal layers; 29 grid points in the y direction are laid out in the upper free space region; 240 uniform grid points are laid out along the z direction with \( \Delta z = 150\mu m \). For the traditional FDTD, \( \Delta t_{FDTD} \) must be less than \( 3.3 \times 10^{-16} \) sec to satisfy the CFL stability limits. In our simulation, the time step is \( \Delta t_{ADI-FDTD} = 2 \times 10^{-13} \) sec, which is an acceptable choice according to [4]. \( \Delta t_{ADI-FDTD}/\Delta t_{FDTD} = 600 \). This helps to calculate the field distribution in the very thin silicon skin depth region in the skin-effect mode, as well as in the other two modes of propagation.

From the ADI solution we extract the attenuation factor and the phase factor, as a function of frequency and semiconductor doping. This is achieved by applying the Gaussian pulse in the time domain as described above, and then taking the Fourier transform. The propagation constants extracted from ADI-FDTD electromagnetic field time domain solutions are shown in Fig.3. The frequency range varies from 1GHz to 50GHz, and the substrate doping of \( 8.9 \times 10^{19} \) cm\(^{-3} \). Under the frequencies considered, the curve corresponding to a doping of \( 6.9 \times 10^{19} \) cm\(^{-3} \) represents operation in the skin-depth mode. The curve corresponding to a doping of \( 8.9 \times 10^{17} \) cm\(^{-3} \) reflects propagation ranging from the slow-wave mode to the transition mode, and then to the skin-depth mode as the frequency increases. In the skin-depth mode, the loss is higher, and the phase velocity is relatively large, while in the dielectric quasi-TEM mode, they are both small and relatively constant. The numerical results in each case also match the quasi-analytical calculations.

C. Numerical Analysis: Calculating Field Distributions in Mixed Dimensional Structures

To further understand the energy flow and distribution in each mode, a comparison of the field distributions obtained from the ADI-FDTD full-wave results is made. In Fig.4 (a)-(c), the sine wave with the frequency of 50GHz is excited on the MISM with doping = \( 5 \times 10^{19} \) cm\(^{-3} \), and the field is taken at \( t=40\)ps. The skin-depth mode field distribution is observed as we expected. In Fig.4 (d)-(f), a sine wave with frequency of 1GHz is excited on the MISM structure with \( 8.9 \times 10^{18} \) cm\(^{-3} \).Fig. 2. For the MISM structure, \( b_1=2\mu m, b_2=200\mu m \). The three solid lines divide the map into 3 regions of fundamental modes as marked: Top figure: contour of attenuation factor \( \alpha \) (along propagation direction z) vs. substrate doping and wave frequency. Bottom figure: contour of normalized phase constant \( \beta/(\omega/c) \) (along propagation direction Z) vs. substrate doping and wave frequency.
the field is taken at \( t=1\text{ns} \), and the slow-wave mode field distribution is shown. In Fig. 4 (g)-(i), a sine wave with frequency of 4GHz is excited on the MISM with doping \( = 1.8 \times 10^{13} \text{cm}^{-3} \), and the field is taken at \( t=0.6 \text{ns} \), and the dielectric quasi-TEM field distribution is shown. The field is normalized to the field in the oxide layer in each of the 3 cases. In the last two modes, in order to show at least one period of the wave pattern, we elongated the interconnects to 144 mm, which is larger than their actual on-chip length. In the skin-depth mode, the field in the Si substrate is concentrated close to the SiO2-Si interface, with an equivalent skin-depth of 10 to 20 \( \mu \text{m} \); while in the dielectric quasi-TEM mode, the field penetrates through the Si substrate. In the slow-wave mode, although the field line goes all the way down to the substrate, its magnitude is orders less than the field in the SiO2 layer. This implies that very little energy not penetrates into the Si substrate. In our numerical simulation, we are also able to show that in the skin-effect mode, the field in the metal layer is concentrated close to the Al-SiO2 interface with a skin depth of less than one micron. In the other two modes, the field is distributed more evenly in the metal layer, the skin depth is larger.

VII. CONCLUSION

In this paper, we used the ADI-FDTD method to extend the investigation from the slow-wave and the dielectric quasi-TEM modes into the skin-effect mode. The propagation loss, the dispersion, and the skin-depth is examined over different substrate dopings, in the frequency range from 1GHz to 50 GHz. The instantaneous field distribution is also analyzed in this paper. The work in this paper can help pave the way for accurate numerical analysis of more complicated on-chip interconnect and doping structures.

REFERENCES