UAV Collision Avoidance with Varying Trigger Time

Zijie Lin, Lina Castano, and Huan Xu Member, IEEE

Abstract—This paper examines the effect that the selection of the different collision avoidance trigger time has on UAV safety and avoidance efficiency and finds the optimal trigger time for different cases. The trigger time indicates when a collision avoidance maneuver begins. An earlier trigger time is safer for the UAV to avoid all the obstacles but causes the UAV to deviate away from its intended path, while a delayed avoidance trigger action reduces overall path deviation but entails a higher risk of collision. Thus, the balance between safety and avoidance efficiency is important. Simulations for different mission scenarios show that by selecting specific avoidance trigger times, missed waypoints which are a result of the avoidance maneuver, could be reduced by over 40%. In addition, avoidance time and space required by the avoidance maneuver are also reduced, as compared to always starting the avoidance maneuver when the obstacle is first detected. Hence, selecting the avoidance trigger time can improve the performance of the UAV’s avoidance maneuver, especially for real-time applications.

Index Terms—collision avoidance, trigger time selection, avoidance efficiency, missed waypoints

I. INTRODUCTION

Unmanned Aerial Vehicles (UAVs) play an important role in applications where the environment may be hazardous for humans, such as in search and rescue operations, agricultural monitoring and aerial photography. 1200 UAV incidents and sightings reported to FAA [1]. Near New York City, in September of 2017, a civilian UAV collided with a helicopter and caused serious damages. In August of 2018, a hot air balloon was struck by a UAV while carrying passengers flying over Driggs, Idaho. Fortunately, there were no injuries but the two air vehicles were destroyed. Thus, collision avoidance is one of the key safety issues for UAVs and aviation [2]. Figure 1 shows a simple diagram of a UAV avoiding one dynamic intruder successfully.

Algorithms for UAV collision avoidance have been developed for decades. These depend on the different types of conflicts and available information. For instance, global path planning methods are used when sufficient information on the obstacles and the environment is provided [3]. The A* algorithm [4] and D* algorithm [4][5] are two types of global path planning methods. They use cost functions to calculate conflict-free paths and pick up intermediate waypoints. The A* method checks from the beginning node to the destination node while D* checks from the end and back to the beginning with time variant cost functions. If only local data is available to the UAS sensors, then a local path planning method such as [6] is applied. Sampling-based methods [7], which can be either a global or local algorithm, examine several points around a waypoint, then add the points outside the conflict zone into a valid waypoint set. There are other classifications of collision avoidance approaches, e.g. potential field methods and virtual force field methods, which can be applied when gradients of force and relative variables are easy to compute. The above approaches might have a good obstacle avoidance performance [8][9], but the computational burden is a disadvantage for their real-time application [10][11]. Another algorithm which has been widely applied is the geometric method[12][13], which uses the geometric relationship between the UAV and the obstacles. Variables in this method include: distance, angle, relative speed and the probability of conflicts on the grid between the UAV and obstacles. It then computes the desired direction and speed for the UAV to follow, with less computation time.

![Collision avoidance diagram: UAV avoids a dynamic obstacle](image)

None of the above methods look at the effect that avoidance start and end times have on algorithm outcomes, or whether the efficiency of conflict resolution remains the same or is changed.

In this work, we conduct twenty thousand of simulations and found that even if the same collision avoidance approach is applied to the same scenario with different avoidance action trigger times, the results including the collision avoidance success possibility and the time cost are significantly different.
In [14], the critical avoidance trigger time $t_c$, for the UAV to have last chance to escape from the collision conflicts is calculated, but there isn’t a quantitative analysis of the cost for UAV to operate this action. Also, the different possible variations of successful avoidance when UAV starts its escaping action between $t=0$ and critical time $t_c$ are not explored.

In this work, we use geometry methods as they have a lighter computation burden and a fast calculation time [14], which is an advantage for conducting Monte-Carlo simulations. This paper examines the following parameters under varying avoidance trigger times:

1. $P_{\text{succ}}$, collision avoidance success rate, which means under a large amount of randomly generated scenarios, the percentage that the UAV successfully avoids the collisions with avoidance trigger time $t$. 

2. $\text{Cost}_W$, waypoint-related cost, the number of waypoints that the UAV misses as a consequence of collision avoidance operation.

3. $\text{Cost}_L$, path-related cost, compared to the original path [15], the length of the new path generated due to the collision avoidance.

4. $\text{Cost}_S$, 2D space-related cost, space occupied by collision avoidance operation.

5. $\text{Cost}_T$, the time UAV deviates from its original airway.

6. $\text{Cost}_{\text{total}}$, summary of the above cost, $\text{Cost}_W$, $\text{Cost}_L$, $\text{Cost}_S$ and $\text{Cost}_T$.

A selection of the collision avoidance trigger time can (1) save over 40% path length and 40% missed waypoints, (2) save on collision avoidance operation space, (3) spend less time deviating from its initial path, which can have less effect on the route of other aircraft, and (4) achieve a relatively high collision avoidance success rate.

This paper is organized as follows: Section II presents the problem formulation and model set up. Section III describes the the selection of the different trigger times. Section IV presents the simulation results and analysis. Section V presents the conclusions and discusses future work.

## II. PROBLEM FORMULATION

Cost functions are frequently applied to UAV collision avoidance and path planning algorithms to determine an appropriate trajectory for UAV. Research in [16] uses the function of UAV deviated angle to calculate the cost, aiming at choosing the collision-free path with minimum direction variation. [17] uses the function of the deviation distance to compute the cost, selecting the collision-free path with minimum variation from the initial path. Work in [18] chooses the function of visibility range, and its goal is to determine the collision avoidance solution for UAV with maximum visibility of the environment. In our work, the reward function $\text{Reward}$ is composed of two items, one is avoidance success rate $P_{\text{succ}}$ and another is cost function $\text{Cost}_{\text{total}}$, where the cost function $\text{Cost}_{\text{total}}$ is related to the waypoint missed, path length, space occupied and the time spent while deviating from its original path.

The definition of the variables used in this work are found in Table I. The characteristics being compared in this work are collision avoidance success rate $P_{\text{succ}}$, waypoint-related cost $\text{Cost}_W$, path-related cost $\text{Cost}_L$, space-related cost $\text{Cost}_S$, environment-related cost $\text{Cost}_T$ and total cost $\text{Cost}_{\text{total}}$ with corresponding reward $\text{Reward}$. These characteristics are explained in details as follows.

### TABLE I

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U = (u_x, u_y, u_z)$</td>
<td>UAV and the location of UAV</td>
</tr>
<tr>
<td>$A_i = (A_{ix}, A_{iy}, A_{iz})$</td>
<td>$i$th obstacle and its location</td>
</tr>
<tr>
<td>$V_o = (V_{ox}, V_{oy}, V_{oz})$</td>
<td>Velocity of the UAV</td>
</tr>
<tr>
<td>$V_{ai} = (V_{axi}, V_{ayi}, V_{azi})$</td>
<td>Velocity of the $i$th obstacle</td>
</tr>
<tr>
<td>$\text{CPA}$</td>
<td>Point of closest approach</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Minimum turning radius</td>
</tr>
<tr>
<td>$d_{min}$</td>
<td>Minimum separation radius</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Distance between the UAV and the obstacle</td>
</tr>
<tr>
<td>$V_e$</td>
<td>Relative speed between the UAV and the obstacle</td>
</tr>
<tr>
<td>$t_c$</td>
<td>Critical avoidance trigger time</td>
</tr>
<tr>
<td>$t_t$</td>
<td>Avoidance trigger time</td>
</tr>
</tbody>
</table>

#### A. Conflicts avoidance success rate

If the UAV triggers avoidance action at time $t_t$ after detecting the obstacles, the possibility of avoiding the collision successfully is defined as $P_{\text{succ}}$, the success rate. 5000 cases of UAV and obstacles with randomly generated speed, direction, distance and altitude are used to calculate the success rate every time.

#### B. Waypoints missed

Figure 2 (a) shows that due to collision avoidance operations, the waypoints that are inside the collision zone will be missed since the area is considered unsafe. In Figure 2 (a) at $t = 0$, the UAV (located at U) detects the obstacles. If it starts the avoidance action immediately (gray-blue line), the 1st, 2nd 3rd and 4th waypoints $W_1$, $W_2$, $W_3$ and $W_4$ will not be followed. In contrast, starting later at a particular time $t_c$ (green line), the 3rd and 4th waypoints $W_3$ and $W_4$ are missed by the UAV while 1st, 2nd and 5th waypoints are reached. The portion of missed waypoints from the total waypoints is defined as waypoint cost, $\text{Cost}_w$.

In general, assume the total number of waypoints in the UAV’s initial path is $N$. The location when the UAV detects the obstacles is $U$, the conflict location is $C$, and the number of waypoints between $U$ and $C$ is $M$, Figure 2 (a).

Suppose the time at which the UAV collides with the obstacles is $t_c$, the time that detects the obstacles is $t_0$, the time that starts the avoidance is $t_t$. Thus, the approximated number of missed waypoints $M_w$ is

$$M_w = M \frac{t_c - t_t}{t_c - t_0}$$  \hspace{1cm} (1)

Where $t_c > t_t > t_0$. As $t_t$ increases from $t_0$ to $t_c$, the waypoint missed $M_w$ decreases.

Figure 2 (a) shows that the waypoints are located in a straight line while Figure 2 (b) shows that the waypoints are located randomly. The conclusions are similar.
The cost relative to the waypoint missed is defined as,
\[ \text{Cost}_w = \frac{M_w}{N} = \frac{M}{N} \frac{t_c - t_l}{t_c - t_0} \] (2)

**C. Additional path length**

In Figure 2 (a), different trigger times results in a different path length. The green line represents the UAV’s trajectory where the avoidance operation starts at \( t = t_c \), and the blue line is the UAV’s trajectory where the avoidance operation starts at \( t = 0 \). From the work in [14], it’s concluded that the green line is shorter than the blue line in almost all situations. The ratio of the additional trajectory’s length over the UAV original trajectory’s length is defined as the cost of path, \( \text{Cost}_L \).

The comparison of the length of green and blue line (length of path saved) as shown in Figure 2 is proved at [14].

Suppose the original length of path from beginning to the destination is \( L_0 \), the additional path due to collision avoidance action is \( L \). Then the cost of the path length is defined as,
\[ \text{Cost}_L = \frac{L}{L_0} \] (3)

**D. Avoidance operation space**

The areas under the effect by the UAV collision avoidance action is claimed as avoidance operation space \( S \). The avoidance action is in 3D space with yaw angle, pitch angle and speed variation. For easy calculation, the space is defined as the surface instead of the volume, which is calculated as when UAV starts its obstacle avoidance operation, the planar space surrounded by the original path, collision point (CPA, the closet point of approach) and the new path. Figure 3 (a) shows a example of it. In Figure 3 (a), if UAV starts avoidance at the time \( t_0 \) (the time UAV first detects the obstacles), the corresponding operation space is \( S_1 \), the pink area plus blue area. In contrast, if the UAV starts the avoidance action at \( t_c \), the operation space is \( S_2 \), the pink area only. \( S_1 \) and \( S_2 \) have different size. In Figure 3 (b), where the waypoints are located randomly, the above definition remains similarly.

The length of the original path from start to the destination is \( L_0 \). Since the UAV should have separation distance \( d_m \) to keep safe, the space used during the flight task is,
\[ S_0 = 2d_m * L_0 \] (4)

The avoidance operation space is \( S \). Thus, the cost relative to the space occupied is defined as,
\[ \text{Cost}_S = \frac{S}{S_0} = \frac{S}{2d_m L_0} \] (5)

The quantitative comparison of the avoidance space size due to different trigger times is provided in the simulation section.

**E. Effects on the environment**

The different collision avoidance trigger times will have the different effects on the environment (other air vehicles’ airways). As shown in Figure 4, the UAV travels in Airway\(_1\) initially, with yellow line. The width of the Airway\(_1\) is \( 2d_m \). To avoid obstacles located in its future path, the UAV deviates from its original path. The light blue line means the UAV deviates at time \( t_0 \) while the green line means it deviates at time \( t_c \). The time during which the UAV travels outside the Airway\(_1\) (green line or blue line beyond the Airway\(_1\)) is defined as the deviation time \( T_{eff} \), which has the possibility to affect another aircraft nearby.

Then the percentage of the deviation time over the original time consumed to finished flight task is \( \text{Cost}_T \).

![Fig. 3. Space occupied by collision avoidance operation](image3)

![Fig. 4. Collision avoidance operation deviates the UAV from its original airway and might have effects on the air corridors of other aircraft (schematic is not drawn to scale, especially in airspace)](image4)
If $T$ is the total time taken by the UA V to complete its mission without any obstacles on its way, we can define the time cost as the following ratio

$$Cost_T = \frac{T_{eff}}{T} \quad (6)$$

F. Total cost and reward function

The total cost function for a UAV’s flight mission and conflict resolution can be defined in many ways, as previously mentioned [16][17] and [18], depending on flight plan goals and overall scenario. In this paper, the final cost function and reward function are defined as,

$$Cost_{total} = Cost_w \ast Cost_L \ast Cost_S \ast Cost_T \quad (7)$$

$$Reward = P_{suc} / Cost_{total} \quad (8)$$

G. Model setup

The model applied in this paper is based on a small size, fixed-wing UAVs, to be consistent with future hardware tests. The speed of commanded UAV ranges from 5 m/s to 15 m/s. The minimum separation radius $d_{m}$ is set to be 30 meters. This work assumes that obstacles’ sensor data, e.g. ADSB data is available to the UAV.

During collision avoidance maneuvers, the UAV changes its yaw angle, pitch angle, bank angle and speed to avoid the obstacles in 3D space. To reach the desired value, PID control is applied.

$$\dot{\psi} = K_p(\psi_e - \psi) + K_i \int (\psi_e - \psi) + K_d \frac{d}{dt}(\psi_e - \psi) \quad (9)$$

$$\dot{\theta} = K_p(\theta_e - \theta) + K_i \int (\theta_e - \theta) + K_d \frac{d}{dt}(\theta_e - \theta) \quad (10)$$

Where $\psi$ and $\theta$ are the yaw and pitch angle. Then at $\Delta t$,

$$\psi(\Delta t) = \psi + \psi \Delta t \quad (11)$$

$$\theta(\Delta t) = \theta + \theta \Delta t \quad (12)$$

since the relationship between the yaw and bank angle $\psi$ is,

$$\omega = \psi = \frac{g tan \phi}{V_u} \quad (13)$$

The bank angle at $\Delta t$ is updated to:

$$\phi(\Delta t) = tan^{-1}(\frac{\dot{\psi}V_u}{g}) \quad (14)$$

For less computation time in the Monte-Carlo simulations, the standard kinematic model [2] of UAV is used. Hence, the velocity of the UAV in the x, y and z direction is,

$$V_{ux} = V_u cos \theta cos \psi \quad (15)$$

$$V_{uy} = V_u cos \theta sin \psi \quad (16)$$

$$V_{ux} = V_u sin \theta \quad (17)$$

The position of the UAV is calculated as,

$$U_x(\Delta t) = U_x + U_x \Delta t = U_x + V_{ux} \Delta t \quad (18)$$

$$U_y(\Delta t) = U_y + U_y \Delta t = U_y + V_{uy} \Delta t \quad (19)$$

$$U_z(\Delta t) = U_z + U_z \Delta t = U_z + V_{uz} \Delta t \quad (20)$$

III. COLLISION AVOIDANCE OPERATION TRIGGER TIME

The key issue is the set up of the different avoidance trigger times. The first step is to find the critical avoidance time $t_c$, which is also the final possible time for the UAV to successfully escape from the conflicts. In simulation, the trigger times are set between 0 and $t_c$.

A. Find the critical avoidance operation trigger time $t_c$

From our previous research [14], the critical avoidance trigger time $t_c$ is defined as the last possible time by which the UAV could escape from a collision conflict.

Suppose $\rho$ is the UAV minimum turning radius. $d_{m}$ is the safe separation radius between the UAV and obstacle, Figure 5.

As shown in Table I, $\vec{U}$ is the location vector of the UAV and $\vec{A}$ is the location vector of the obstacle. Where

$$\vec{U} = (U_x, U_y, U_z) \quad (21)$$

$$\vec{A} = (A_x, A_y, A_z) \quad (22)$$

Also as claimed before, $\vec{V_u}$ is the velocity vector of the controlled UAV and $\vec{V_o}$ is the velocity vector of the obstacle ($||\vec{V_o}|| = 0$ for the static obstacle).

$$\vec{V_u} = (V_{ux}, V_{uy}, V_{uz}) \quad (23)$$

$$\vec{V_o} = (V_{ox}, V_{oy}, V_{oz}) \quad (24)$$

Therefore, the relative velocity vector and the relative location vector are:

$$\vec{V_r} = \vec{V_u} - \vec{V_o} \quad (25)$$

$$\vec{r} = \vec{A} - \vec{U} \quad (26)$$
Use $\vec{V}_r^0$ and $\vec{r}_0^0$ to represent the $\vec{V}_r$ and $\vec{r}$ at the time that UAV detects the obstacle (set $t = 0$), then the angle between $\vec{V}_r^0$ and $\vec{r}_0^0$ is $\alpha_0$. Thus,

$$\alpha_0 = \cos^{-1}\left(\frac{\vec{r}_0^0 \cdot \vec{V}_r^0}{||\vec{r}_0^0|| ||\vec{V}_r^0||}\right) \tag{27}$$

From Figure 5 [14], the ideal critical avoidance trigger time is calculated by the following equation:

$$t_c(\text{ideal}) = \frac{r_0^0 \cos \alpha_0 - \sqrt{(p + d_m)^2 - (r_0^0 \sin \alpha_0)^2}}{||V_r||} \tag{28}$$

where

$$\rho = \frac{||V_r||^2}{gtan \phi} \tag{29}$$

$g$ is the acceleration of gravity and $\phi$ is the maximum banking angle of each UAV ($\phi$ is a fixed value for each UAV).

If we consider the time delay $t_{res}$ associated to the response of the UAV, the critical time becomes:

$$t_c = t_c(\text{ideal}) - t_{res} \tag{30}$$

If there are $n$ multiple obstacles, first calculate the critical avoidance trigger time $t_{ci}$ for each obstacle, and then the overall trigger time is computed as follows:

$$t_c = t_c(\text{ideal}) - t_{res} = \min(t_{ci}) - t_{res} \tag{31}$$

where $i = 1, 2, \ldots n$.

**B. Set the sequence of avoidance operation trigger times**

After calculating the critical avoidance time $t_c$, the next step is to set the trigger time. The UAV could choose to trigger the collision avoidance anytime between 0 and $t_c$. In our experiments, the test times will be chosen in a uniform distribution from 0 to $t_c$, as presented in Table II.

**TABLE II**

<table>
<thead>
<tr>
<th>Trigger time $t/t_c$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
</table>

For easier visualization of results, results of $t_c = 0$, $t_c = 0.2t_c$, $t_c = 0.5t_c$, $t_c = 0.8t_c$, $t_c = t_c$ will be presented in the tables and figures in section IV, including missed waypoints, path length, space occupation, total cost and the total reward. The Algorithm 1 explains the processes involved in the trigger time selection and reward calculation.

**IV. SIMULATION RESULTS AND ANALYSIS**

Table III displays two scenarios of collision avoidance, case 1 for three static obstacles and case 2 for two dynamic obstacles.

For case 1, the UAV detects static obstacles from 70-100 meters away and calculates the critical avoidance time $t_c = 3s$.

(1) If the UAV starts the avoidance operation right away, that is $t_t = 0$, it can avoid the obstacles successfully with 30% missed waypoints. The path length changed by 37m, the planar air space affected by the collision avoidance operation was 432 $m^2$ and the UAV flew beyond its airway for 9.49 seconds.

(2) If the UAV starts the avoidance operation at $t_t = 0.2t_c$, $t_t = 0.5t_c$, $t_t = 0.8t_c$, $t_t = t_c$, percentage of missed waypoints are 20%, 20%, 10%, 10%. The path lengths changed by 33m, 29m, 26m and 23m. The planar airspace affected was 405$m^2$, 370$m^2$, 319$m^2$ and 290$m^2$, respectively. And the UAV traveled beyond its air corridor by 8.23 seconds, 5.95 seconds, 5.84 seconds and 5.12 seconds. All of the variables have smaller values (smaller cost) as compared to when the collision avoidance is triggered at time $t = 0$. The result of collision avoidance remains successful.

For case 2, the UAV detects dynamic obstacles from 70-100 meters away and calculates the critical avoidance time $t_c = 1.5s$. (1) If the UAV starts the avoidance operation right away, that is $t_t = 0$, it can avoid the obstacles successfully with 30% missed waypoints. The path length changed by 35m, the planar air space affected by the collision avoidance operation was 450 $m^2$ and the UAV flew beyond its airway for 11.05 seconds. (2) If the UAV starts the avoidance operation at $t_t = 0.2t_c$, $t_t = 0.5t_c$, $t_t = 0.8t_c$, $t_t = t_c$, percentage of missed waypoints are 20%, 10%, 10%, 10%. The path lengths changed by 32m, 27m, 25m and 20m. The planar airspace affected was 337$m^2$, 325$m^2$, 287$m^2$ and 256$m^2$, respectively. And the UAV traveled beyond its air corridor by 8.87 seconds, 7.49 seconds, 7.47 seconds and 5.95 seconds. All of the variables have smaller values (smaller cost) as compared to when the collision avoidance is triggered at time $t = 0$. The result of collision avoidance remains successful.

It is clear that as the trigger time $t_t$ gets closer to the critical avoidance time $t_c$, the overall cost, including waypoints missed, additional path, occupied space and the time flying outside the original lane is decreased. This is further indicated by the Monte-Carlo simulation results in Table IV.

Table IV summarizes Monte Carlo simulation results for the success rate of avoiding static and dynamic obstacles and their corresponding cost. From the results, similar to Table III, as the trigger time $t_t$ gets closer to the critical avoidance time $t_c$, the overall cost, including waypoints missed, length of...
TABLE III
AVOIDANCE OPERATION COST BY DIFFERENT TRIGGER TIMES

<table>
<thead>
<tr>
<th>Case</th>
<th>Number and type of obstacles</th>
<th>Initial distance</th>
<th>Initial altitude</th>
<th>Speed m/s</th>
<th>Total number of waypoints</th>
<th>Avoidance success?</th>
<th>Waypoints missed num</th>
<th>Waypoint missed percentage</th>
<th>Additional path length /m</th>
<th>Space occupied /m²</th>
<th>Time outside airway /s</th>
<th>Collision avoidance starts at t = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3 static</td>
<td>70-100</td>
<td>550-650</td>
<td>0</td>
<td>10</td>
<td>yes</td>
<td>3</td>
<td>30%</td>
<td>37</td>
<td>432</td>
<td>9.49</td>
<td>10.23, 9.39, 9.08</td>
</tr>
<tr>
<td>2</td>
<td>2 dynamic</td>
<td>70-100</td>
<td>550-650</td>
<td>0</td>
<td>10</td>
<td>yes</td>
<td>3</td>
<td>30%</td>
<td>35</td>
<td>350</td>
<td>11.05</td>
<td>9.39, 10.23, 9.08</td>
</tr>
<tr>
<td>3</td>
<td>3 static</td>
<td>256</td>
<td>370</td>
<td>5</td>
<td>2</td>
<td>yes</td>
<td>2</td>
<td>20%</td>
<td>29</td>
<td>370</td>
<td>7.95</td>
<td>7.95, 7.84, 7.37</td>
</tr>
<tr>
<td>4</td>
<td>2 dynamic</td>
<td>256</td>
<td>370</td>
<td>5</td>
<td>1</td>
<td>yes</td>
<td>1</td>
<td>10%</td>
<td>26</td>
<td>319</td>
<td>5.34</td>
<td>5.34, 5.24, 4.74</td>
</tr>
<tr>
<td>5</td>
<td>3 static</td>
<td>15-20</td>
<td>370</td>
<td>20</td>
<td>1</td>
<td>yes</td>
<td>1</td>
<td>10%</td>
<td>25</td>
<td>250</td>
<td>5.12</td>
<td>5.12, 5.12, 4.74</td>
</tr>
</tbody>
</table>

additional path, occupied space and the time outside its airway decrease.

In Table IV, the second column depicts the UAV avoiding a single static obstacle. Single static obstacle avoidance is a simple scenario, thus under the proper initial detection distance between the UAV and the obstacle, avoidance success rate remains approximately 100%. But the costs vary. As the trigger time changes from 0, 0.2tₖ, 0.5tₖ to 0.8tₖ, tₖ, the cost, including Costₖ, Costₕ, Costₗ and Costₚ, decreases monotonously. For example, the cost of missed waypoints Costₖ decreases from 0.710, 0.633, 0.570 to 0.493 and 0.427. As a result, the total reward calculated from Equation 8 as Pₕ(cost_total), increases from 769, 1123, 1198 to 2127 and 2160, which means choosing a later collision avoidance trigger time would be better for the UAV.

Now let’s take a look at the fourth column of Table IV, the UAV avoiding a single dynamic obstacle. The single obstacle avoidance is not a tough task if the initial detection distance between the UAV and the obstacle is not a very small margin. And the success rate remains approximately the same as the trigger time varies. But the costs vary. As the trigger time tₖ changes from 0, 0.2tₖ, 0.5tₖ to 0.8tₖ, tₖ, the cost, including Costₖ, Costₕ, Costₗ and Costₚ decreases monotonously. For example, the cost of missed waypoints Costₖ decreases from 0.316, 0.298, 0.288 to 0.255 and 0.231. As a result, the total reward calculated from Equation 8 as Pₕ(cost_total), increases oppositely from 270, 279, 667 to 1000 and 1754, which means choosing a later collision avoidance trigger time is better for the UAV.

Next, the analysis of situations for multiple static and dynamic obstacles, which will be more complex. From Table IV third column, it is clear that the safety level (avoidance success rate) of multiple static obstacles decreases as tₖ increases, which means the failure rate increases. The total cost Cost_total, composed of missed waypoint Costₖ, additional path cost Costₕ, space occupied cost Costₗ and deviation time cost Costₚ decreases. The trajectory of the above parameter is also shown in Figure 6. The final reward Reward = Pₕ(cost_total), the changing trend of the reward is dependent on the decreasing rate of the Pₕ and the Cost_total. For multiple static obstacles, the cost decreasing speed is greater than success rate decreasing speed. Thus, the final reward still increases as action trigger time tₖ approaches the critical time tₖ, as Figure 7 presents.

In Table IV, the fifth column describes the collision avoidance experiment for multiple dynamic obstacles. The safety level (avoidance success rate) of multiple dynamic obstacles decreases a lot as tₖ increases, which means the failure rate increases. The total cost Cost_total, composed of missed waypoint Costₖ, additional path cost Costₕ, space occupied cost Costₗ and deviation time cost Costₚ decreases. The trajectory of the above parameter is also shown in Figure 8. The final reward Reward = Pₕ(cost_total), the changing trend of the reward is dependent on the decreasing rate of the Pₕ and the Cost_total. For multiple dynamic obstacles, the situation is totally different from the above three. In the beginning, the speed at which cost decreases is greater than the rate at which avoidance success decreases, which increases the reward, but after the 0.5tₖ, the success rate decreases fast, which decreases the reward. Thus, the shape of the reward here is like a parabola with a vertex as the trigger time tₖ approaches the critical time tₖ, as Figure 9 shows.

Figure 6 presents the Monte-Carlo simulation results of Table IV third column for UAV avoiding multiple static obstacles. As the trigger time varies from tₖ = 0, tₖ = 0.2tₖ, tₖ = 0.5tₖ, tₖ = 0.8tₖ, tₖ = tₖ, the average waypoints missed are 0.519, 0.444, 0.353, 0.269, 0.226, reducing around 60% of missed waypoints. The average path increasing proportion varies from 0.810 to 0.427. The space proportion decreases from 0.227 to 0.086 while the deviation time proportion decreases from 0.633 to 0.427. Time, path, space saved are around 30% to 70%.

In summary, the overall cost decreases, from 0.06 to 0.0034 while the collision avoidance success rate decreases from 99.90% to 92.30% (failure rate increases from 0.1% to 7.7%). As cost decreases, the safety level also decreases while in-
TABLE IV
MONTE CARLO SIMULATION STATISTIC RESULTS OF UAV AVOIDING OBSTACLES BY DIFFERENT TRIGGER TIMES

<table>
<thead>
<tr>
<th>Num of cases</th>
<th>5000</th>
<th>5000</th>
<th>5000</th>
<th>5000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Num of obs each case</td>
<td>single</td>
<td>multi</td>
<td>single</td>
<td>multi</td>
</tr>
<tr>
<td>Obstacle type</td>
<td>static</td>
<td>static</td>
<td>dyn</td>
<td>dyn</td>
</tr>
<tr>
<td>Initial distance /m</td>
<td>100-200</td>
<td>100-200</td>
<td>200-300</td>
<td>200-300</td>
</tr>
<tr>
<td>Initial altitude /m</td>
<td>550-650</td>
<td>550-650</td>
<td>550-650</td>
<td>550-650</td>
</tr>
<tr>
<td>Average speed m/s</td>
<td>0</td>
<td>0</td>
<td>20-25</td>
<td>20-25</td>
</tr>
<tr>
<td>Avoidance starts at t=0</td>
<td>0</td>
<td>0</td>
<td>20-25</td>
<td>20-25</td>
</tr>
<tr>
<td>Avoid suc rate $P_{succ}$</td>
<td>100%</td>
<td>99.90%</td>
<td>100%</td>
<td>90.00%</td>
</tr>
<tr>
<td>Waypoint missed Cost$_w$</td>
<td>0.511</td>
<td>0.319</td>
<td>0.316</td>
<td>0.395</td>
</tr>
<tr>
<td>Additional path Cost$_L$</td>
<td>0.710</td>
<td>0.810</td>
<td>0.440</td>
<td>0.540</td>
</tr>
<tr>
<td>Space occupied Cost$_S$</td>
<td>0.149</td>
<td>0.227</td>
<td>0.093</td>
<td>0.308</td>
</tr>
<tr>
<td>Deviation time Cost$_T$</td>
<td>0.025</td>
<td>0.633</td>
<td>0.293</td>
<td>0.733</td>
</tr>
<tr>
<td>Total cost Cost$_{total}$</td>
<td>0.0013</td>
<td>0.0080</td>
<td>0.0037</td>
<td>0.0040</td>
</tr>
<tr>
<td>Total reward Reward</td>
<td>769.2</td>
<td>16.6</td>
<td>270.3</td>
<td>19.2</td>
</tr>
</tbody>
</table>

| Avoidance starts at 0.2$t_c$ | 100%   | 99.05% | 100%   | 86.17% |
| Waypoint missed Cost$_w$ | 0.452  | 0.444  | 0.298  | 0.363  |
| Additional path Cost$_L$ | 0.633  | 0.707  | 0.443  | 0.510  |
| Space occupied Cost$_S$ | 0.110  | 0.187  | 0.110  | 0.296  |
| Deviation time Cost$_T$ | 0.024  | 0.349  | 0.271  | 0.592  |
| Total cost Cost$_{total}$ | 0.0009 | 0.0331 | 0.0056 | 0.0237 |
| Total reward Reward | 1123.6 | 29.9   | 277.8  | 36.8   |

| Avoidance starts at 0.5$t_c$ | 100%   | 97.79% | 100%   | 86.53% |
| Waypoint missed Cost$_w$ | 0.485  | 0.357  | 0.288  | 0.319  |
| Additional path Cost$_L$ | 0.570  | 0.591  | 0.447  | 0.480  |
| Space occupied Cost$_S$ | 0.108  | 0.148  | 0.098  | 0.256  |
| Deviation time Cost$_T$ | 0.004  | 0.397  | 0.122  | 0.399  |
| Total cost Cost$_{total}$ | 0.0009 | 0.0122 | 0.0018 | 0.0202 |
| Total reward Reward | 1198.9 | 80.2   | 666.7  | 42.8   |

| Avoidance starts at 0.8$t_c$ | 100%   | 95.65% | 100%   | 81.11% |
| Waypoint missed Cost$_w$ | 0.271  | 0.269  | 0.255  | 0.299  |
| Additional path Cost$_L$ | 0.495  | 0.487  | 0.443  | 0.487  |
| Space occupied Cost$_S$ | 0.084  | 0.105  | 0.096  | 0.266  |
| Deviation time Cost$_T$ | 0.004  | 0.389  | 0.098  | 0.498  |
| Total cost Cost$_{total}$ | 0.0005 | 0.0053 | 0.0010 | 0.0193 |
| Total reward Reward | 2122.7 | 178.8  | 1000   | 41.6   |

| Avoidance starts at $t_c$ | 100%   | 92.30% | 100%   | 70.15% |
| Waypoint missed Cost$_w$ | 0.225  | 0.226  | 0.231  | 0.252  |
| Additional path Cost$_L$ | 0.427  | 0.430  | 0.443  | 0.487  |
| Space occupied Cost$_S$ | 0.012  | 0.088  | 0.094  | 0.229  |
| Deviation time Cost$_T$ | 0.007  | 0.342  | 0.085  | 0.461  |
| Total cost Cost$_{total}$ | 0.0005 | 0.0054 | 0.0006 | 0.0195 |
| Total reward Reward | 2160.8 | 274.4  | 1754   | 35.9   |

As the trigger time gets from $t_t = 0$, $t_t = 0.2t_c$, $t_t = 0.5t_c$, $t_t = 0.8t_c$ to $t_t = t_c$, the proportion of missed waypoints varies from 0.395 to 0.252, reducing about 40%. The average proportion of additional path decreases from 0.541 to 0.482. The proportion of the space affected by the avoidance operation drops from 0.309 to 0.229 while the deviation time drops from 0.736 to 0.462.

The overall cost decreases from 0.0470 to 0.0195. Path, space and overall cost saved are around 40% to 60%. The success rate also decreases from 90% to 70.15%, meaning that the failure rate increases from 10% to 29.85%. After applying the reward function $Reward = P_{succ}/Cost_{total}$, we got Figure 9. However, different from the Figure 7, as shown in Figure 9 for the dynamic obstacles, the shape of reward changes. The total reward, composed of the collision avoidance success rate and the costs, are not a monotonous function, it is similar to a parabola, which goes up from 19.2 to 36.8, reaches its peak at 42.8, then decreases to 41.6 and 35.9. That is because both collision avoidance success rate and the cost decreases as the $t_t$ gets closer to $t_c$. After $t_t = 0.5t_c$,
Fig. 8. Different cost of UAV avoiding multiple dynamic obstacles with different avoidance trigger times

the success rate has a higher decreasing speed than at which the cost decreases. So the reward function reaches its peak when avoidance action starts at time $t_s = 0.5t_c$. Thus, $0.5t_c$ is the optimal time point for the UAV to start avoidance action for dynamic obstacles if 85% to 90% collision avoidance success rate is acceptable.

In addition, the selection of the different trigger times not only could schedule the tasks for multiple UAVs [19], but also could let the UAV classify the hazard level of the obstacles, breaking the large amount of obstacles into several smaller groups according to the critical avoidance time, and then avoid them in sequence, which could solve the difficult simultaneous collision avoidance problem.

V. CONCLUSION AND FUTURE WORK

In summary, this paper provides the analysis of cost and success rate for UAV collision avoidance performance, as well as determination of the proper time at which the UAV should start the avoidance maneuver to get the most efficient results. Around 40% to 60% time and other relative cost is reduced while keeping the collision avoidance success rate, which is promising for the real time applications. In addition, the proper trigger time point determined from large amount of Monte-carlo simulation results can be used for UAV future machine learning training.

REFERENCES

[1] “Reported us sightings by faa (July 2019–September 2019).”