# Corrigendum to Risk Sensitive Control of Markov Processes in Countable State Space

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Formula (3.10) in [3] is incorrect. As noted in [1], for the conclusion of Theorem 3.1 to hold the risk factor $\gamma^{-1}$ must be small enough. The upper bound for $W_\beta(x) - W_\beta(e)$ is obtained as follows. Firstly, we note that (3.2) in [3] implies that $P_x^f[\tau \geq k] \leq \rho^{k-C}$, with $\rho = (\frac{C}{1+C})^{\frac{1}{1+C}} < 1$. This follows from the Markov property and an induction argument [1]. Therefore, if $\gamma \geq \frac{2\|c\|}{-\log\rho}$, and using (3.7), (3.9) in [3],

$$\begin{aligned}
P_x^{f^*,\xi^*}[\tau = k] &= \int_{S\setminus\{e\}} \cdots \int_{S\setminus\{e\}} \int_{\{e\}} \frac{\mathrm{e}^{\beta W_\beta(x_1)}}{P(\mathrm{e}^{\beta W_\beta})(x, f^*(x))} \cdots \frac{\mathrm{e}^{\beta W_\beta(e)}}{P(\mathrm{e}^{\beta W_\beta})(x_{k-1}, f^*(x_{k-1}))} \cdot \\
&\quad \cdot P(dx_1|x, f^*(x)) \cdots P(dx_k|x_{k-1}, f^*(x_{k-1})) \\
&= \int_{S\setminus\{e\}} \cdots \int_{S\setminus\{e\}} \int_{\{e\}} \exp\{\frac{1}{\gamma}\sum_{t=0}^{k-1} c(x_t, f^*(x_t)) - (1-\beta)\sum_{t=0}^{k-1} W_\beta(x_t) + \\
&\quad +\beta(W_\beta(e) - W_\beta(x))\} P(dx_1|x, f^*(x)) \cdots P(dx_k|x_{k-1}, f^*(x_{k-1})) \\
&\leq \exp\{W_\beta(e) - W_\beta(x) + \frac{(k+1)\|c\|}{\gamma}\} P_x^{f^*}[\tau = k] \\
&\leq \exp\{W_\beta(e) - W_\beta(x) + \frac{\|c\|}{\gamma}\} \rho^{-\frac{k}{2}} P_x^{f^*}[\tau = k] \\
&\leq \rho^{-C} \exp\{W_\beta(e) - W_\beta(x) + \frac{\|c\|}{\gamma}\} \rho^{\frac{k}{2}}.
\end{aligned}$$

Then,

$$P_x^{f^*,\xi^*}[\tau \geq k] \leq \rho^{-C} \exp\{W_\beta(e) - W_\beta(x) + \frac{\|c\|}{\gamma}\} \rho^{\frac{k}{2}} \cdot \frac{1}{1-\rho^{\frac{1}{2}}},$$

and

$$1 = P_x^{f^*,\xi^*}[\tau < \infty] \leq \rho^{-C} \exp\{W_\beta(e) - W_\beta(x) + \frac{\|c\|}{\gamma}\} \cdot \frac{1}{1-\rho^{\frac{1}{2}}}.$$

Thus,

$$W_\beta(x) - W_\beta(e) \leq \log(\rho^{-C} \mathrm{e}^{\frac{\|c\|}{\gamma}} \frac{1}{1-\rho^{\frac{1}{2}}}),$$

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which, together with (3.12) in [3], implies the conclusion of Theorem 3.1, under the additional hypothesis that $\gamma \geq \frac{2\|c\|}{-\log \rho}$. Additional results on risk sensitive control of countable state Markov processes with small risk factor appear in [1], which uses probabilistic and analysis methods, and in [2] which also uses the stochastic games approach.

[1] R. Cavazos-Cadena, E. Fernández-Gaucherand, Controlled Markov chains with risk-sensitive criteria: Average cost, optimality equations, and optimal solutions. Preprint, March 1998.

[2] R. Cavazos-Cadena, E. Fernández-Gaucherand, Markov decision processes with risk-sensitive average cost criterion: the discounted stochastic games approach. Preprint, March 1998.

[3] D. Hernández-Hernández, S.I. Marcus, Risk sensitive control of Markov processes in countable state space, Sys. Control Letters 29 (1996), pp. 147-155.

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