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A reliability as an independent variable (RAIV) methodology for optimizing test planning for liquid rocket engines

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Abstract The hot fire test strategy for liquid rocket engines has always been a concern of space industry and agency alike because no recognized standard exists. Previous hot fire test plans focused on the verification of performance requirements but did not explicitly include reliability as a dimensioning variable. The stakeholders are, however, concerned about a hot fire test strategy that balances reliability, schedule, and affordability. A multiple criteria test planning model is presented that provides a framework to optimize the hot fire test strategy with respect to stakeholder concerns. The Staged Combustion Rocket Engine Demonstrator, a program of the European Space Agency, is used as example to provide the quantitative answer to the claim that a reduced thrust scale demonstrator is cost beneficial for a subsequent flight engine development. Scalability aspects of major subsystems are considered in the prior information definition inside the Bayesian framework. The model is also applied to assess the impact of an increase of the demonstrated reliability level on schedule and affordability.

Keywords Multi-criterion decision making · Multi-level Bayesian aggregation · Aspiration equivalent

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Abbreviations

CAD	Computer aided design
CDR	Critical design review
D&D	Design and development
FLPP	Future launcher preparatory programme
FT	Fault tree
HFT	Hot fire test
IOC	Initial operating capability
MCC	Main combustion chamber
MCTPP	Multiple criteria test planning problem
MLC	Main life cycle
MoE	Measure of effectiveness
MRL	Manufacturing readiness level
NAFCOM	NASA/Air Force Cost Model
Ox	Oxidizer
PB	Preburner
PDR	Preliminary design review
QR	Qualification review
R by C	Reliability by confidence
RAIV	Reliability as independent variable
RBD	Reliability block diagram
SC	Staged combustion
SRR	System requirement review
STH	System test hardware
TRL	Technology readiness level
TFU	Theoretical first unit

List of symbols

a	Lower limit for range of area of concern
b	Upper limit for range of area of concern
c	Cycle
C	Credibility bound
CT	Cycle tested
DP	Development period
$D\&D$	Design and development

EQL	Equivalent life
EQM	equivalent mission
f	Objective function
$F(\cdot)$	Cumulative density function
g	Constraint function
HFT	Hot fire test
HFTD	Hot fire test duration
HW	Number of hardware
M	Maintenance
MLC	Main life cycle
M&D	Mounting and dismounting
q	Failure fraction
R	Reliability
t	Life time
TF	Test facility
TO	Test occupation
TR	Test rate
UF	Utility function
w	weight
WY	Work year
x	Decision variable, number of hot fire test
$\hat{x}$	Target value
z	Equivalent number of successes
y	Measure of effectiveness level
α	Shape parameter
α_1, α_2	Weighting factors
β	Shape parameter
γ	Risk aversion coefficient
$\Gamma(\cdot)$	Gamma function
λ	Poisson distribution parameter
π	Probability
Θ	Integration domain

Subscripts

i	Functional node
j	Hot fire test group
m	Index for objective functions
med	Median
MD	Mission duration
MIQ	Mission ignition quantity
M€	Million Euro
k	Index for constraint function
R by C	Reliability by confidence
rel	Reliable
rem	Remaining
tot	Total
u	Upper
0	Indication of prior probability distribution

1 Introduction

The selection of a hot fire test plan for liquid rocket engines is a concern for the space industry and the European Space Agency because there exists no recognized standard that defines quantitatively the scope of hot fire test plans. The current best practice is a blend of art and science that tries to define test plans that will verify performance requirements and demonstrate safety margins against known failure modes. The scope of initial test plans is defined by meeting the stated initial operational capability and the available budget. Updates of test plans are made during the development to adjust the schedule constraints and the remaining budget. The predicted mission success probability is then a result of the executed hot fire test plan. However, the key stakeholders—the space agency, the member states, and launch operators—are concerned about the predicted reliability, the time required for the development including the hot fire testing (“schedule”), and the cost of the development including hot fire testing (“affordability”) in the early program planning stage. The scope definition of a test plan is one of the key drivers for the stakeholder concerns; therefore, the selection of an optimized hot fire test plan becomes a multiple criteria decision-making (MCDM) problem in which the numbers of planned hot fire tests at various system integration levels are the decision variables.

Our proposed approach for the multiple criteria test planning problem (MCTPP) formulates it as an optimization problem and includes elements from utility theory and normative target-based decision making. The number of hot fire tests determines the reliability, defines the schedule, and drives the affordability. The MCTPP formulation seeks to maximize a linear combination of the utilities of these values. We will solve this problem using an evolutionary algorithm that searches for the optimal hot fire test plan.

The MCTPP is demonstrated exemplarily in the context of ESA’s Future Launcher Preparatory Programme (FLPP).[1] Hot fire test plans are found for two scenarios: (1) a reduced thrust scale engine demonstrator precedes the flight engine development and (2) a flight engine development is executed from scratch (without a demonstrator). The results of these two scenarios are used to support the claim that a reduced thrust scale demonstrator is cost beneficial for the subsequent flight engine development. Finally, the impact of an increase in predicted mission success probability on schedule and affordability is addressed.

2 Problem formulation

The decision variables of the MCTPP are the number of hot fire tests. The objective function is a multiattribute utility function that relates the decision variables to the

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[1] http://www.esa.int/esaCP/SEMHBUZO0WF_index_0.html, accessed on 19.02.2011.

stakeholder's areas of concern: reliability, schedule, and affordability, which are all functions of the number of hot fire tests.

2.1 Decision variables

The decision variables of the MCTPP are the number of planned hot fire tests allocated at the different system integration levels, i.e. component, subsystem, and system level. For example, for the LE-7A liquid rocket engine, there are nine types of tests that must be considered (see Table 1). The key component tests are the preburner test and the igniter test. The key subsystem tests are the fuel turbomachinery test, the oxidizer turbomachinery test, and the combustion test. There are also four different system tests (which have different durations). The specific number of tests of each type must be determined, so there are nine decision variables.

For each test type, the specific number of tests is bounded below by the minimum number required to verify the performance requirements, optimize the start-up and shut down sequences, demonstrate margin against known failure modes, and attain an adequate level of demonstrated reliability to assure mission success subject to schedule and budget constraints. In addition, the specific number of tests is bounded above such that the number of required hot fire tests for engine reliability certification is placed on engine system level (Nota Bene: The bounds provided in Table 1 are given only as example). Therefore, various hot fire test strategies can be defined to demonstrate these basic test objectives. However, test facility capabilities and physical hardware degradation phenomena impose constraints to the allocation of the hot fire tests that defines the hot fire test strategy.

Table 1 Hot fire test strategy for LE-7A

Component level	Min. no. of tests	Max. no. of tests	Hot fire test time (s)
Preburner	20	60	10
Igniter	20	80	2
Subsystem level			
Fuel turbomachinery	40	100	60
Ox turbomachinery	40	100	60
Combustion devices	40	100	10
System level			
Test duration 1	5	50	3
Test duration 2	5	50	30
Test duration 3	5	200	150
Test duration 4	5	200	300

The component and subsystem test facilities lack the capability of providing adequate testing boundary conditions that allow the operation of the tested hardware at full rated conditions. At system level, the full rated conditions are achieved but the test facility may lack the capability of providing the required amount of propellants to support the operation of the full mission duration. Both the limitations are superimposed by the fact that start-ups and shut downs are more detrimental than the simple accumulation of hot fire test time. Therefore, a framework is needed to account not only for the various test facility limitations but also for the hardware degradation phenomena.

The proposed framework uses a functional node representation of the physical architecture of a liquid rocket engine and the notion of mission equivalents. The details about these two elements of the framework is described using the LE-7A architecture.

2.1.1 Functional node representation

The functional node representation of a physical architecture of a liquid rocket engine not only describes the structural relation of components known from the fault tree (FT) or reliability block diagram (RBD) techniques but also defines the fundamental hot fire test strategy [1].

The LE-7A liquid rocket engine architecture (see [2]) is used to explain a possible fundamental hot fire test strategy. The main components of the LE-7A, which are most likely pertinent to main failure modes, are the turbomachinery on fuel and oxidizer side, the preburner, the thrust chamber assembly, the two ignition systems, the control valve on the fuel side (MFV), the control valves on the oxidizer side (MOV and POV), the mixture ratio setting device, and the heat exchanger.

Based on the definition of components with pertinent failure modes, the functional node representation can be defined (see Fig. 1). All main functions are in series (if one function fails the system fails). This node representation is node 0 and is used to aggregate all engine level hot fire tests. It should be noticed that not all subassemblies or components of the liquid rocket engine are included in the functional node representation because the reliability levels of the "missing" components are considered to be unity or almost unity and therefore do not affect the reliability analysis. In case a specific subassembly or component is failure mode susceptible, it can be easily included in the node representation.

Once the engine level functional node representation is defined, the fundamental hot fire test strategy at lower system level can be established. Fundamental in that sense means that subsystem level hot fire test configurations at combustion device and turbomachinery level can be defined as shown in Fig. 2.

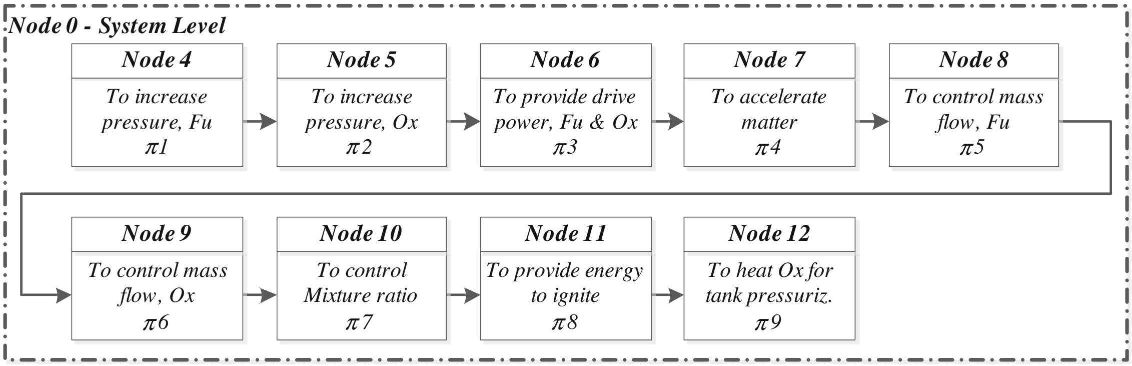


Fig. 1 Node 0: engine level—functional node representation

2.1.2 Mission equivalents

Based on the fundamental hot fire test strategy definition, through the functional node representation, the mission equivalents are needed to relate the planned hot fire tests at the various system integration levels to the mission requirement as well as to capture the two fundamental strength-reduced and stress-increased failure mechanisms into a single metric, the equivalent mission [1].

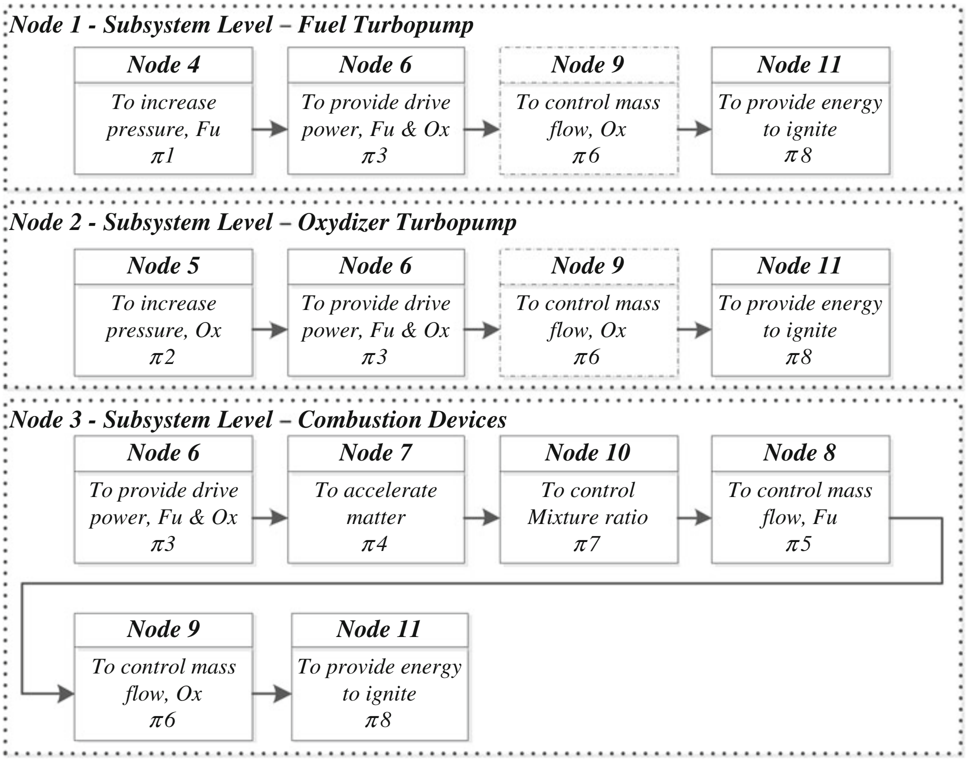
The mission requirement not only includes the actual flight but also any other hot fire tests aggregated throughout the product life cycle. The notion of main life cycle (MLC) is used to normalize the hot fire test events which may consist of

- a single or multiple acceptance hot fire test(s) before the actual flight,
- a possible engine ground start hold-down with launch commit criteria abort, and
- the single flight mission (or several flight missions in case of a reusable main stage engine) or multiple re-ignitions in case of upper stage liquid rocket engines.

During the design maturation and qualification, additional hot fire tests are augmented to the MLC. These tests may be either multiples of a nominal MLC or a fraction of full mission duration.

Each hot fire test contributes to the degradation of the hardware due to the two fundamental failure mechanisms

Fig. 2 Nodes 1, 2, and: subsystem level—functional node representation



strength-reduced and stress-increased which are present in every liquid rocket engine piece part or subassembly. Equation (1) captures mathematically, the two fundamental failure mechanisms and normalizes them with the hot fire events of the MLC; hence, the notion of equivalent mission (EQM).

$$\mathrm{EQM}_{i,j} = \alpha_{1,i,j}\frac{\mathrm{CT}_{i,j}}{\mathrm{MLC}_{\mathrm{MIQ}}} + \alpha_{2,i,j}\frac{\mathrm{CT}_{i,j}\left(w_{i,j}\cdot\mathrm{HFTD}_{i,j}\right)}{\mathrm{MLC}_{\mathrm{MD}}}. \tag{1}$$

The first term accounts for the strength-reduced and the second for the stress-increased failure mechanism, respectively. The second term includes also the weighing of planned hot fire tests which are shorter than full mission duration.

Therefore, the hot fire tests can be performed with different hot fire test durations which is reflected in the index j. The various system integration levels are defined through the index i, a group of hot fire tests. The number of hot fire tests in each hot fire test group is defined by k. The total number of equivalent missions in each hot fire test group i is given in Eq. (2).

$$\mathrm{EQM}_i = \sum_{j=1}^{k}\mathrm{EQM}_{i,j}. \tag{2}$$

Equation (3) accounts for planned hot fire test failures in each hot fire test group i to reflect the typical design-fail-fix-test cycles present in liquid rocket engine developments. The second term of Eq. (3) is based on Eq. (1) but equated at the failure time.

$$z_i = \mathrm{EQM}_i - \sum_{j=1}^{k}\mathrm{EQM}_{i,j}^{\mathrm{failure}} \tag{3}$$

Equations (2) and (3) are used in Sect. 2.2.1 which describes a methodology to estimate the projected mission success probability based on the number of planned hot fire tests that are allocated at the various system integration levels.

2.2 Measure of effectiveness for the areas of concern

The measure of effectiveness (MoE) for each area of concern is a function of the number of hot fire tests. The MoE for reliability is determined by means of a reliability as independent variable (RAIV) methodology, the MoE for the schedule is effort level driven in terms of work force and test plan scope, and the MoE for the budget is based on cost models that partially depend on the test plan scope, respectively. Section 2.3 uses then the MoE for each area of concern to compute the score value of the utility functions used in the MCTPP formulation.

2.2.1 Reliability

The RAIV methodology estimates the projected mission success probability based on the number of hot fire tests planned. As the number of hot fire tests increases, the reliability MoE and, as a consequence, the reliability utility score increases (see Sect. 2.3). The unique features of RAIV are the multi-level aggregation of hot fire test results (planned or actual), i.e. results may be obtained at component, subsystem, and/or system level using the functional node representation introduced in Subsect. 2.1.1 and the pooling of test results with various hot fire test durations using the notion of mission equivalents as defined in Subsect. 2.1.2.

The fundamental mathematical expression of RAIV is given in Eq. (4). It is based on the Bayesian formulation to estimate parameters (probability of success π_i) given a set of data (the number of hot fire tests) [1].

$$\pi(\underline{\pi}_i|\mathrm{Data}) = \frac{\prod_i \pi_i^{z_i}(1-\pi_i)^{\mathrm{EQM}_i - z_i}\prod_i \pi_{0,i}}{\int\dots\int_\Theta\left(\prod_i \pi_i^{z_i}(1-\pi_i)^{\mathrm{EQM}_i - z_i}\prod_i \pi_{0,i}\right)\mathrm{d}\pi_\Theta} \tag{4}$$

The first product of the numerator expresses the hot fire test strategy defined by the equivalent number of planned hot fire tests (EQM_i) including possible test failures (z_i) at the various functional node levels. The second product defines the prior knowledge of the parameters to be estimated in a Bayesian framework. Each individual function node may feature a different level of prior knowledge due to scalability constraints, e.g. a turbomachinery is limited in terms of scalability from a small to a much larger thrust scale if compared to a thrust chamber.

The solutions for the functional node reliability levels are used to calculate the mean, the variance or any other pth percentile of the projected engine level mission success probability.

2.2.2 Schedule

The MoE for the schedule is effort driven as well as by the time which is needed to perform the hot fire tests to attain the reliability target estimated with the RAIV methodology.

The effort which is needed to design and develop the hardware is estimated with the NASA/Air Force Cost Model (NAFCOM), which is described in Sect. 2.2.3. The test occupation simply depends on the number of hot fire tests allocated to the various integration levels and the number of test facilities, the test cadence per week, a yearly maintenance period, and the mounting and dismounting periods.

Based on empirical evidences given in Koelle [3], a quantile regression equation for the development period in years is defined which relates the cost for the design and development divided by the work force yearly cost [first part of Eq. (5)]. The second part of Eq. (5) is simply the addition of the overall test duration also given in years which is determined by the test occupation model described next.

$$\mathrm{DP} = \left[-6.62 + 1.35\ln\left(\frac{\mathrm{D\&D}}{\mathrm{WY}}\right)\right] + \mathrm{TO} \tag{5}$$

It is the second term of Eq. (5) which links the MoE for the schedule with the decision variable number of hot fire tests. The test operational assumptions such as the number of test facilities, the test cadence per week, a yearly maintenance period in weeks, and the mounting and dismounting periods in weeks define the minimum test occupation. Eq. (6) defines the simple test occupation model used.

$$\mathrm{TO} = \frac{\mathrm{HFT_{tot}}}{(\mathrm{TR})(52 - \mathrm{M} - \mathrm{M\&\ D})(\mathrm{TF_{tot}})} \tag{6}$$

It should also be noted that Eq. (5) is not considering any schedule penalty term due the lack of a proper funding profile. It is assumed that an adequate funding profile exists.

2.2.3 *Budget*

The MoE for the area of concern affordability is based on two cost models: The NAFCOM and the authors' defined effort driven test facility operation cost model. The purchasing power parity principle is used to transfer the US to the EU productivity level in order to obtain an adequate European level for the price estimations obtained from NAFCOM [4].

Available European engine development programmatic evidences were used to anchor/validate the two cost models for a European multi-national environment.

2.2.3.1 Design & development and system test hardware cost model The NAFCOM tool is used to estimate the design and development (D&D) cost as well as the System Test Hardware (STH) cost. The D&D cost includes all the specifications and requirements, engineering drawings as well as program management and configuration control efforts that are required to achieve the built-to baseline for the definition of the STH. It includes also design rework which may become necessary after the hot fire test conductance and evaluation.

The NAFCOM effort-driven input variables for the D&D cost estimate are the development environment, the manufacturing environment, the Manufacturing Readiness Level (MRL), the design scope, and the team experience. However, a correlation exists between the design effort and the team experience as pointed out by Sherman [5], i.e. a high design effort is also linked with a low team experience level and vice versa. In addition, the links between this correlation, the Technology Readiness Level (TRL), the MRL, the Integration Readiness Level (IRL), and project phases exist and are highlighted in Fig. 3 [6–9].

The STH cost is estimated based on the theoretical first unit (TFU) cost but includes a 25% overhead applied to reflect a prototype design approach. No learning curve effect is considered for the STH cost estimation. The total number of STH sets needed to complete the overall hot fire test plan is given in Eq. (7) and is based on elements defined by the RAIV methodology [1].

$$\mathrm{HW_{tot}} = \frac{\mathrm{EQM_{R\ by\ C}}}{\mathrm{EQL_{rel}}} + \frac{\mathrm{EQM_{rem}}}{\mathrm{EQL_{med}}} \tag{7}$$

Equation (7) uses the results obtained from Eq. (4) in terms of total number of equivalent missions required to attain the specified reliability level and relates it to the life capability of the piece parts or subassemblies of the liquid rocket engine components. Two terms are defined: The first one relates the number of equivalent missions without the occurrence of failures to the hardware reliability (reliable

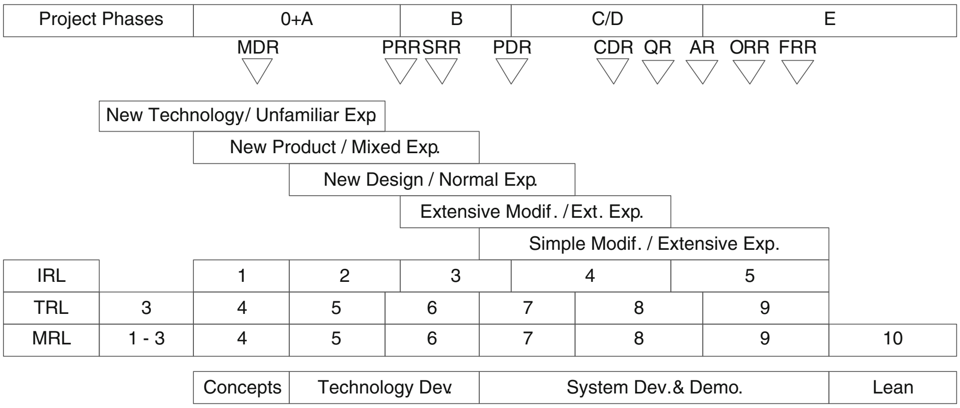


Fig. 3 Effort-driven cost model input variables in relation to TRL, MRL, and IRL

number of cycles and reliable life time). The second term completes the overall test plan by testing the remaining amount of equivalent missions needed to attain the specified level of reliability and relates this number to the medians of the underlying hardware reliability distribution functions describing the two fundamental failure mechanisms, i.e. the Poisson and Weibull distributions.

Equation (8), a Bayesian formulation to estimate the percentile of a binomial distribution, is used to estimate the mission equivalents needed in the reliability by confidence (R by C) success-testing scheme, i.e. the $\mathrm{EQM}_{\mathrm{RbyC}}$.

$$\frac{\mathrm{Beta}_{q_\mathrm{u}}\left(q_\mathrm{u}, \alpha, \beta + \mathrm{EQM}_{\mathrm{R\ by\ C}}\right) \cdot \Gamma\left(\alpha + \beta + \mathrm{EQM}_{\mathrm{R\ by\ C}}\right)}{\Gamma\left(\beta + \mathrm{EQM}_{\mathrm{R\ by\ C}}\right) \cdot \Gamma(\alpha)} = C \tag{8}$$

The percentile or failure fraction q_u is equal to the estimated reliability level given in Sect. 2.2.1. The confidence level C is specified by the customer; typically 60 or 90%. The parameters α and β reflect the prior knowledge about the engine reliability levels either based on the data given in McFadden and Shen [10] or user specific information.

The hardware reliability is defined by specifying the reliable number of cycles c_rel and reliable life time t_rel but is transferred into the EQM notion using Eq. (9). The parameters α_1 and α_2 are used to weigh the two failure mechanisms.

$$\mathrm{EQL}_\mathrm{rel} = \alpha_1 \frac{c_\mathrm{rel}}{\mathrm{MLC}_\mathrm{MIQ}} + \alpha_2 \frac{t_\mathrm{rel}}{\mathrm{MLC}_\mathrm{MD}} \tag{9}$$

The remaining hot fire tests, in terms of equivalent missions needed in Eq. (7), are calculated with Eq. (10).

$$\mathrm{EQM}_\mathrm{rem} = \mathrm{EQM}_\mathrm{tot} - \mathrm{EQM}_\mathrm{RbyC} \tag{10}$$

Similarly to Eqs. (9, 11) is used to transfer the medians of the Poisson and Weibull distribution into the EQM notion which is also needed in Eq. (7).

$$\mathrm{EQL}_\mathrm{med} = \alpha_1 \frac{\lambda}{\mathrm{MLC}_\mathrm{MIQ}} + \alpha_2 \frac{t_\mathrm{med}}{\mathrm{MLC}_\mathrm{MD}} \tag{11}$$

Equations (12, 13) are used to calculate the medians of the Poisson and Weibull distribution, which are required in Eq. (11), based on the assumed reliable number of cycles c_rel and reliable life time t_rel of the piece parts or subassemblies.

$$\Pr(\mathrm{CT} \le c_\mathrm{rel}) = R(c_\mathrm{rel}) = 1 - \frac{\Gamma(1 + \lfloor c_\mathrm{rel} \rfloor, \lambda)}{\Gamma(1 + \lfloor c_\mathrm{rel} \rfloor)} \tag{12}$$

$$t_\mathrm{med} = t_d \cdot \left[\frac{\ln(2)}{-\ln(R)}\right]^{\frac{1}{\beta}}. \tag{13}$$

2.2.3.2 Test operational cost model The test operational cost model is also effort driven, i.e. the test occupation is determined based on assumptions concerning engine mounting, test rate, and test facility operation using empirical data. The test operational cost model estimates the cost based on the values of the decision variables (the number of hot fire tests). Once the test occupation in years is determined using Eq. (6), the yearly cost for a work force year is used to estimate the cost associated to the test conductance. Although minor in magnitude, the propellant cost is also considered which may become more significant in the future if the current tendency of the price increase remains evident for the hydrogen propellant.

2.3 Utility functions and normative decision making

The utility function and normative decision making are used to define the objective function as well as to divide the search space in terms of the decision variable number of hot fire tests into feasible and infeasible regions.

2.3.1 Utility function

For each area of concern, the MoE of a test plan is converted into a utility score. The stakeholder has target values for each MoE, which could be used to define a simple step utility function in which any MoE that meets the target receives a value of one, and any MoE that does not receives a value of zero. However, this type of step function makes optimization difficult because it penalizes all poor performance solutions equally and does not reflect adequately the customer value in case a solution is above the target but is still acceptable with a lower value. Thus, we sought a utility function that would be equivalent in some sense. This will be discussed more in the next subsection.

For reliability, we used the monotonically increasing utility function given in Eq. (14). For schedule and affordability, we use the monotonically decreasing function given in Eq. (15).

$$\mathrm{UF}_\mathrm{increase} = \begin{cases} \frac{1-\mathrm{e}^{-\gamma\,(y-a)}}{1-\mathrm{e}^{-\gamma\,(b-a)}} & \gamma \neq 0 \\ \frac{y-a}{b-a} & \text{otherwise} \end{cases} \tag{14}$$

$$\mathrm{UF}_\mathrm{decrease} = \begin{cases} 1 - \frac{1-\mathrm{e}^{|\gamma|\,(b-y)}}{1-\mathrm{e}^{|\gamma|\,(b-a)}} & \gamma \neq 0 \\ \frac{b-y}{b-a} & \text{otherwise} \end{cases} \tag{15}$$

The range of the MoE y is defined by the stakeholder's least preferred and most preferred values for the particular area of concern. The least preferred value evaluates to a score of zero whereas the most preferred level to a score of one in order to maintain uniformity over the various areas of concern domains [11].

The utility assigned to an intermediate value of the MoE is determined by the utility function. The shape of the utility function is determined by the risk aversion coefficient γ. For each area of concern, this parameter is set so that the utility function has an aspiration equivalent equal to the stakeholder's target for that MoE.

Based on the three individual exponential utility functions, the objective function of the MCTPP is the weighted linear combination of the three exponential utility functions. The weights are provided by the stakeholder based on his preferences about the tradeoffs between the three areas of concerns.

2.3.2 Normative target-based decision making

The selection of an adequate value for the risk aversion coefficient γ is based on the normative target-based decision making framework because stakeholders are usually not in a position to directly express a value. Instead, stakeholders define their preferences in terms of a target for each area of concern, e.g. the reliability level should be at least 0.95, the development duration (schedule) should be at most 8 years, and the budget (affordability) should be no more than 1.00 (normalized cost), respectively.

We wish to define a utility function for each area of concern that reflects the customer target and is equal to the expected value of the utility function.

From normative target-based decision making theory, we know that there exists a unique effective risk aversion coefficient γ for any stated aspiration-equivalent (target) and probability distribution (likelihood) that results in the same expected utility and aspiration-equivalent of a particular utility function [12, 13]. That is, we can find the appropriate value of the risk aversion coefficient γ by finding the value that satisfies the equality of Eq. (16).

$$F(\hat{x}) = \int_{a}^{b} \frac{\gamma \mathrm{e}^{-\gamma y}}{\mathrm{e}^{-\gamma a} - \mathrm{e}^{-\gamma b}} F(y)\,\mathrm{d}y \tag{16}$$

The cumulative density function $F(\hat{x})$, which expresses the uncertainty of the degree of attainment of the target for each area of concern, is evaluated at the target value $\hat{x}$ (aspiration-equivalent) and set equal to the product of the derivative of the utility function and the cumulative density function (expected utility). The integration limits are defined through the range of the particular area of concern.

In particular, for each area of concern, the stakeholder can provide a probability distribution $F(y)$ for the MoE that captures the general uncertainty associated with that MoE. This distribution (over the range $[a, b]$ for this MoE) may be based on the performance of previous development programs or expert opinion. Among the various distributions, the general Beta, the Uniform or a truncated Lognormal are the preferred ones.

The first two moments, mean and variance, are used to find the general Beta distribution parameters given the range $[a, b]$. The parameters for the truncated lognormal are found using the bounds of the range $[a, b]$ as the 5th and the 95th percentile, respectively.

In case for the exponential utility function and the use of the general Beta distribution to reflect the uncertainty about the MoE, the solution for the risk aversion coefficient γ is found by applying first the integration by parts technique to simplify the integral such that a closed form solution is obtained. In a second step, Brent method is used to solve finally for the risk aversion coefficient γ. Note that this has to be performed appropriately for each area of concern (see Subsect. 3.2 for the application in the context of the MCTPP).

3 Application of the MCTPP

The hot fire test strategies are determined for two scenarios of interest in the context of FLPP: (1) a flight engine development after a successful completion of a demonstrator project at reduced thrust scale and (2) a flight engine development without a prior execution of a demonstrator project. The reason for these two scenarios is linked to the claim that the execution of a prior demonstrator project is cost beneficial for the subsequent flight engine development especially in case of considerable involvement of new technology maturation. The results of the two scenarios are used to support that claim. In addition, the impact on the schedule and affordability of an increase in the demonstrated reliability level prior to the maiden flight is addressed.

The MCTPPs were solved with a genetic algorithm. Each run took about 3 h on an Intel Duo Core CPU with 2.40 GHz with an optimization run time setting of 0.01% change of the fitness function within the last 100 trials.

The pertinent parameter which drives the overall run time is linked to the solution of the reliability MoE which requires a Markov chain Monte Carlo (MCMC). In order to optimize the MCMC sampling from the posterior, a one-variable-at-a-time with independent candidate density Metropolis–Hastings algorithm was selected which uses already the burn-in samples to tune the independent candidate density properties such that the required acceptance rate of 35% is obtained. The time required to run a single MCMC for nine parameters takes about 2 min with 500 burn-in samples and chain lengths of 5,000 samples.

3.1 Key liquid rocket engine requirements

The key liquid rocket engine requirements are determined in early design trade-off studies performed at launch vehicle level. The launch vehicle optimizations vary the thrust level, the nozzle area ratio, and the combustion chamber pressure level to obtain optimum solutions at given payload weight into a particular orbit. An optimum exists between the gross lift off weight of the vehicle and

Table 2 Key performance requirements

Performance characteristics	Values
Combustion chamber pressure (bar)	150
Vacuum thrust (kN)	2,300
Main life cycle	
Acceptance test (s)	150
Acceptance test (s)	150
Hold-down, launch commit (s)	10
Mission duration (s)	300
Number of ignitions	4
Reliable cycle @ 0.98 reliability	12
Reliable life @ 0.98 reliability (s)	2,600

thrust level of the propulsive system. This optimum should correlate with minimum launcher affordability. Geometric constraints of the launch vehicle limit the nozzle area ratio and higher levels of the combustion chamber pressure increase the sea-level performance. The mission profile defines the mission durations of the propulsion system(s).

The launch vehicle optimizations are not finalized within the FLPP but the following key liquid rocket engine assumptions were made to perform the study (see Table 2). The reduced thrust scale is set to 1,400 kN for the demonstrator. In addition, the liquid rocket engine architecture is similar to LE-7A which allows the reuse of the fundamental hot fire test strategy as already defined in Fig. 1 for the engine system level and Fig. 2 for the subsystem level.

3.2 Stakeholder preference

The stakeholder preferences about the three areas of concern reliability, schedule, and budget were elicited. The main outcomes are listed in Table 3. The budget figures are proprietary data and are given only as normalized values. In both scenarios, an IOC in 2025 is required.

Based on the customer responses, the three aspiration-equivalent exponential utility functions, as shown in Fig. 4, were determined using the techniques presented in Sects. 2.3.1 and 2.3.2.

The stakeholder preferences for the three area of concern influence the search for an optimal test plan because the fitness function used in the objective function of the MCTPP is a weighted linear combination of the three utility functions which are influenced by the stakeholder preferences through the shapes (risk aversion).

3.3 Measure of effectiveness settings

3.3.1 *Reliability*

The required inputs for calculating the reliability MoE are the MLC, the weights for the two failure mechanisms (α_1 and α_2), the weights for hot fire test durations which are

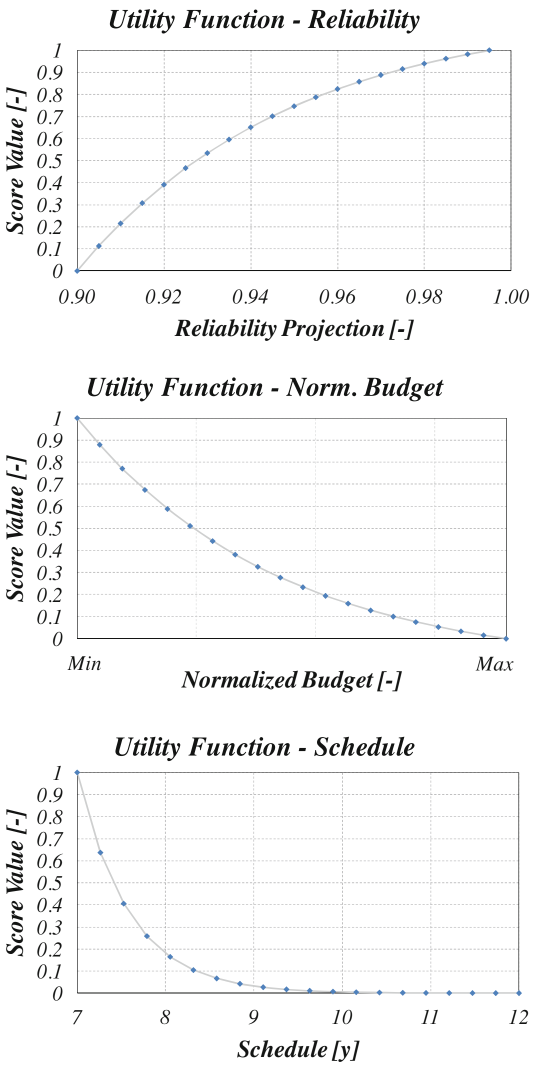


Fig. 4 Utility functions reflecting customer preference

Table 3 Customer preferences

Trade space	Min	Target	Max	Mode	Weights (%)	Remarks
Reliability	0.90	0.95	0.995	0.98	50	The higher the better
Budget	0.67	1.00	1.42	1.17	35	Defined by the authors
Schedule (year)	7	8	12	10	15	Mode defined by the authors

shorter than full mission duration (w_i), the number of anticipated hot fire test failures, and prior information about the component reliabilities. The following paragraphs provide details for these input parameters. All remaining model parameters are calculated internally by the model setup using the mathematical expressions given in Sect. 2.2.1.

The MLC is already defined in Table 2. The weights α_1 and α_2 for the two failure mechanism depend on the planned hot fire test durations and are based on previous European engine development programs [1]. The weights w_i for planned hot fire tests which are shorter than full mission duration are based on a quantile regression using data from previous cryogenic liquid rocket engine programs [1]. The numbers of anticipated hot fire test failures are set to zero in all scenarios. The prior information about the component reliabilities depend on the scenarios. Two cases are discussed next.

No prior information is available because Europe has never demonstrated the mastery of a cryogenic staged combustion liquid rocket engine. Therefore, a non-informative (uniform) prior distribution is assumed for the reduced thrust scale demonstrator engine in scenario I as well as for the flight engine development in scenario II.

Prior information is, however, available for the flight engine development after an assumed successful execution of the demonstrator project in scenario I. The data given in McFadden and Shen [10] is used to estimate the prior distribution parameters [1].

3.3.2 *Schedule*

The required inputs for calculating the schedule MoE are limited to the assumptions concerning the number of available test facilities, weekly test cadence, maintenance periods, and mounting and dismounting activities. All remaining model parameters are calculated internally by the model setup using the mathematical expressions given in Sect. 2.2.2. It should be recalled that the presented model setup does not include any schedule penalty due to the lack of an adequate funding profile.

There are two engine test facilities available in Europe. Both were assumed to be operational for the flight engine development. The demonstrator engine is tested only on one test facility. The component and subsystem test facilities are limited to one for turbomachinery tests and one for combustion devices hot fire tests. The weekly test cadence is set to 0.6 which may seem to be low but was set to that level to account for possible testing interferences with other hot fire test facilities. The non-testing periods due to maintenance and mounting/dismounting activities were set to 4 months per year for engine level test facilities. No impact was considered for component and subsystem test facilities.

3.3.3 *Affordability*

The required inputs for calculating the affordability MoE are linked to the settings for the design and development cost and the test facility operation cost. All remaining model parameters are calculated internally by the model setup using the mathematical expressions given in Sect. 2.2.3.

Table 4 lists the input parameters for the design and development as well as the TFU cost needed to estimate a single STH cost. The total STH cost is a multiple of the single STH based on the number of required hardware sets defined by the technique discussed in Sect. 2.2.3. The inputs for the test facility operation cost were already defined in the discussion above.

3.4 Scenario assessment

3.4.1 *Scenario I: demonstrator and flight engine development*

3.4.1.1 Reduced thrust scale demonstrator engine No customer preference consideration is needed because the programmatic elements are defined by means of the requirements with regards to an IOC for the subsequent flight engine in 2025 and a limited testing scope of 30 hot

Table 4 NAFCOM Settings used to assess the Scenarios

Model parameter	Scenario 1: demonstrator	Scenario 1: flight engine	Scenario 2: flight engine
Development environment	CAD	CAD	CAD
Manufacturing environment	Semi automated	Semi automated	Semi automated
MRL	Similar/modified	New	New
Design scope	New technology	New design	New technology
Team experience	Unfamiliar	Normal	Unfamiliar
Engine cycle	SC-Single PB	SC-Single PB	SC-Single PB
MCC pressure (bar)	150	150	150
Thrust, vac (kN)	1,400	2,300	2,300

Table 5 Hot fire test plan defining characteristics for demonstrator

Component level	Number of tests	HTF time (s)	Accumulated test time (s)
Preburner	20	10	200
Igniter	35	2	70
Subsystem level			
Fuel turbomachinery	20	60	1,200
Ox turbomachinery	20	60	1,200
Combustion devices	20	10	200
System level			
Test duration 1	5	3	15
Test duration 2	5	30	150
Test duration 3	10	150	1,500
Test duration 4	10	300	3,000

Key programmatic elements and test plan characteristics: Total number of hot fire tests (system level): 30, Number of hardware sets: 3, Reliability projection level: 62.8%, Reliability projection level at 90% confidence: 50.0%, Total duration (schedule): to be finished in 2017/2018, Total budget: 0.770

fire tests spread over two engine hardware sets. On engine level, four hot fire test groups were defined, i.e. the 3-s tests are used as start-up verification tests, the 30-s tests as ramp-up tests, the 150-s tests as an intermediate test step, and the 300-s tests as full duration tests. The component and subsystem testing scope were defined by systems engineering best practices. An additional hardware was assumed for the component and subsystem level tests. The results in terms of key programmatic elements and test plan characteristics are listed in Table 5.

3.4.1.2 Subsequent flight engine development The customer preferences were considered when solving the MCTPP for the flight engine development after the successful completion of the reduced thrust scale demonstrator project. The same numbers of hot fire test groups as defined for the demonstrator were kept on engine level for the flight engine. The lower bounds for the number of tests for each hot fire test group is set by the minimum number of hardware sets and the associated MLC, i.e. five in this scenario. The upper bounds are set to 300 for each hot fire test group. The results in terms of key programmatic elements and test plan characteristics are listed in Table 6. The customer targets in terms of development duration and development budget were met. The demonstrated reliability target is marginally not met.

Table 6 Optimized hot fire test plan defining characteristics—flight engine after demonstrator

Component level	Number of tests	HTF time (s)	Accumulated test time (s)
Preburner	40	10	400
Igniter	35	2	70
Subsystem level			
Fuel turbomachinery	160	60	9,600
Ox turbomachinery	160	60	9,600
Combustion devices	210	10	2,100
System level			
Test duration 1	5	3	15
Test duration 2	30	30	900
Test duration 3	33	150	4,950
Test duration 4	90	300	27,000

Key programmatic elements and test plan characteristics: Total number of hot fire tests (system level): 158, Number of hardware sets: 5, Reliability projection level: 94.4%, Reliability projection level at 90% confidence: 92.2%, Total duration (schedule): 7.5 years, Total budget: 0.737

Table 7 Optimized hot fire test plan defining characteristics—flight engine only

Component level	Number of tests	HTF time (s)	Accumulated test time (s)
Preburner	50	10	500
Igniter	45	2	90
Subsystem level			
Fuel turbomachinery	165	60	9,900
Ox turbomachinery	165	60	9,900
Combustion devices	162	10	1,620
System level			
Test duration 1	11	3	33
Test duration 2	90	30	2,700
Test duration 3	30	75	2,250
Test duration 4	30	150	4,500
Test duration 5	30	240	7,200
Test duration 6	90	300	27,000

Key programmatic elements and test plan characteristics: Total number of hot fire tests (system level): 281, Number of hardware sets: 11, Reliability projection level: 94.3%, Reliability projection level at 90% confidence: 91.9%, Total duration (schedule): 11.1 years, Total budget: 1.781

3.4.2 Scenario II: flight engine development without demonstrator

The customer preferences were also considered when solving the MCTPP for the flight engine development without a prior execution of a demonstrator project. The results in terms of key programmatic elements and test plan characteristics are listed in Table 7. The numbers of hot fire test groups were increased to six in this scenario to provide an additional degree of freedom for the optimal hot fire test allocation. The lower and upper bounds for the number of tests for each hot fire test group is set in a similar way as it was done for scenario I. The minimum number of hot fire tests is, however, set to 11 which corresponds to the number of hardware sets needed in scenario II. The customer targets in terms of demonstrated reliability, development duration, and development budget were not met. However, the demonstrated reliability level is only marginally not met as it was the cased in the flight development of scenario I. Both demonstrated reliability levels obtained in scenario I and II are at about the same level which allows an easy comparison.

3.4.3 *Comparison of results with previous liquid rocket engine programs*

Before the two scenarios are compared, the results of the MCTPP are reflected against previous liquid rocket engine key programmatic elements and test plan characteristics.

Koelle [3] provides a figure about the empirical relation of engine reliability versus the number of development and qualification hot firings (see Fig. 5). The figure was expanded with additional flight engines, results of a study performed by Strunz and Herrmann [1], and the results obtained from the two scenario assessments. The model results follow rather well the empirically determined relation.

The number of hardware sets required in the two scenarios, five and 11, correspond also well with previous experiences if one considers the planned number of tests and the assumed hardware reliability level. Evidences of similar hardware set utilizations for developments are given in Emdee [14]. Strunz and Herrmann [1] further highlight the impact on too stringent hardware reliability requirements on the overall hot fire test plan credibility. Based on this information, the assumed hardware reliability levels as given in Table 2 are reasonable.

The development duration (schedule) results for the two scenarios fit also well with previous evidences given in Emdee [14], e.g. LE-7 with 282 hot fire tests required 11 years and Vulcain 1 with 278 hot fire tests 10 years, respectively.

Therefore, the results obtained for the two scenarios by solving the MCTPP can be seen as credible based on the comparison of the key programmatic and test plan characteristics with evidences from previous liquid rocket engine programs.

3.5 Cost advantages of a demonstrator project

The claim that a prior demonstrator project is cost beneficial for the flight engine development can be confirmed by assessing the results obtained from the two scenarios as summarized in Fig. 6. The customer targets for the reliability level and the development duration are also included for ease of comparison.

By looking at Fig. 6, the longer development duration for scenario I should not raise any concern by the stakeholders because the budget for a demonstrator project is limited and as a consequence the work force level allocated to such a project which directly impacts the development duration. In addition, the IOC in 2025 is met even with this longer development duration.

3.6 The cost of reliability

The cost of reliability is also a long lasting question in the space industry and by the European Space Agency. The MCTPP setup provides the proper framework for answering this question with quantitative facts. Figure 7 shows the impact of an increase in the demonstrated reliability level

Fig. 5 Engine reliability versus number of development and qualification hot firings

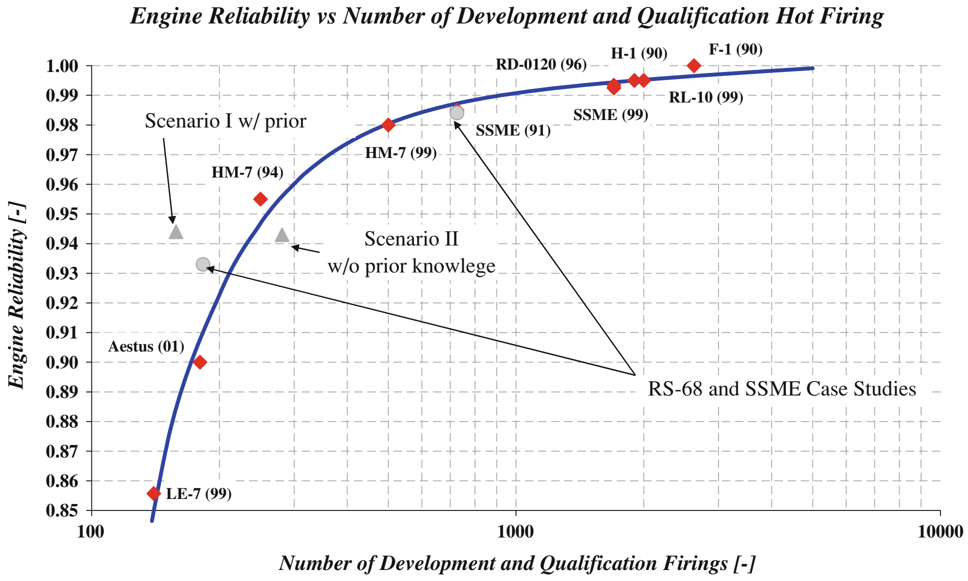


Fig. 6 Comparison of scenarios and customer targets

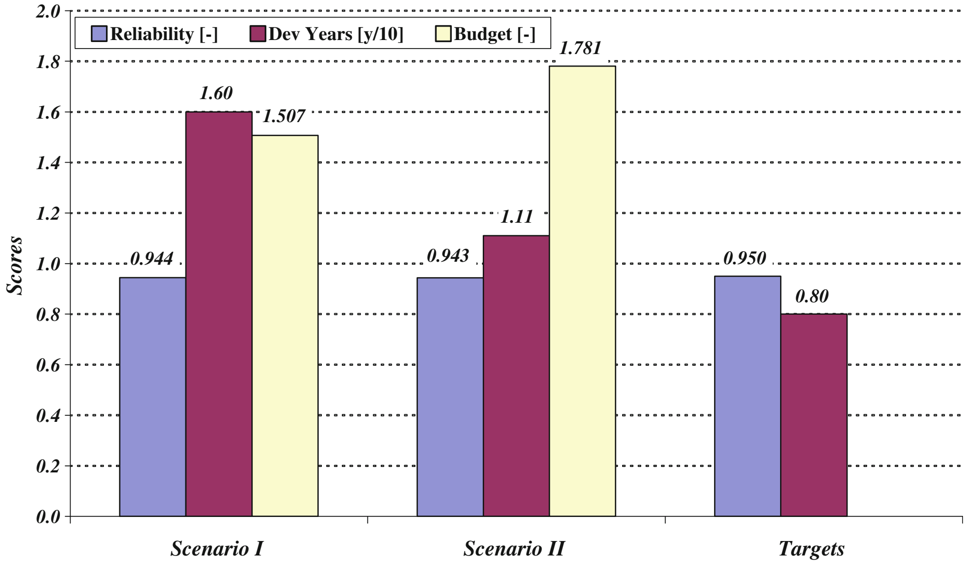
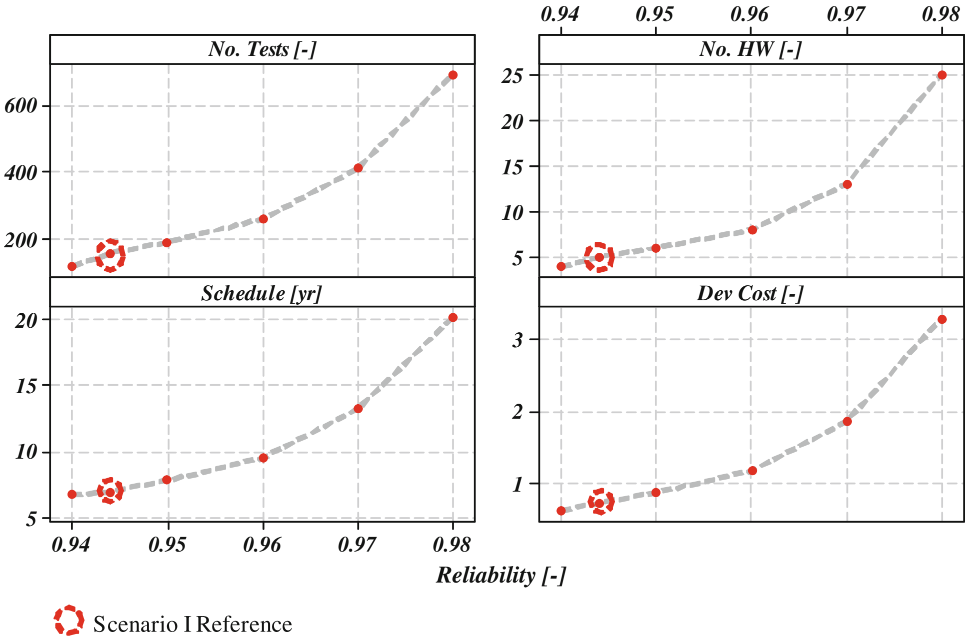


Fig. 7 Impact of reliability level on development schedule and cost for flight engines



on the schedule and affordability. The flight engine development of scenario I is included as reference.

By looking at Fig. 7, the effect on the number of hot fire tests on engine level, the number of engine hardware sets, development duration (schedule), and affordability (development cost) of an increase of the reliability from 0.95 to 0.98 (roughly 3%) can be assessed. The number of hot fire tests on engine level is increased by 260%, the number of hardware sets by 320%, the development duration by 150%, and the affordability by 270%, respectively. The number of hardware sets needed can be significantly reduced in case of an enhanced life capability of the piece parts and subassemblies is given but at the expense of an increase in the production cost for later flight utilization. The development duration may be significantly reduced by erecting additional test facilities for engine level tests but at the expense of an increase in development cost.

4 Conclusion

The MCTPP presented here supports early design trade-off studies by providing quantitative relationships between the hot fire test plan and reliability, schedule, and affordability performance measures. Moreover, the model allows one to find the best hot fire test strategy that meets customer targets for these performance measures. (The best test strategy has the smallest number of tests and hardware sets.)

In addition, the study substantiated the claim that a prior test bed or demonstrator project reduces the development cost of the actual flight engine in case there is a substantial technology maturation need. Scalability aspects for the technology maturation at lower scale are adequately accounted for the different components and subsystems through the prior in the Bayesian framework.

The sensitivity of the development schedule and development cost to an increased level of reliability is quantitatively confirmed as well.

Of course, optimal plans increase the likelihood of success but do not guarantee it. The actual flight mission success is still subject to good workmanship, brilliant engineers, and luck.

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References

1. Strunz, R., Herrmann, J.W.: Reliability as independent variable: applied to liquid rocket engine hot fire test plans. AIAA J. Propuls. Power (submitted for publication)
2. Fukushima, Y.: Development status of LE-7A and LE-5B engines for H-IIA Family. Acta Astronautica. **50**(5), 275–284 (2002)
3. Koelle, D.E.: Handbook of Cost Engineering for Space Transportation System, TCS-TR-190 (2010)
4. Krugman, P.R., Obstfeld, M.: International Economics. Addison Wesley, Boston (2002)
5. Shermon, D.: Systems Cost Engineering. Gower Publishing, Surrey (2009)
6. Schankman, M.S., Reynolds, J.W.: Advancing the Art of Technology Cost Estimating—A Collaboration between NASA and Boeing, ISPA/SCEA International Conference, San Diego (2010)
7. OSD: Manufacturing Readiness Level Deskbook, Department of Defense (2010)
8. ECSS Space Project Management: Project Planning and Implementation, ECSS-M-ST-10C Rev. 1 (2009)
9. Macret, J.L.: Maitrise des Risques des Développements Technologiques: Méthodologie TRL et IRL en Ingénierie Système, Ingénierie Grands Projets et Systèmes Complexes III, Arcachon, 23–24 Juin (2008)
10. McFadden, R.H., Shen, Y.: An Analysis of the historical Reliability of US liquid-fuel Propulsion Systems, AIAA 90-2713, 26th AIAA/ASME/SAE/ASEE Joint Propulsion, Orlando (1990)
11. Kirkwood, C.W.: Strategic Decision Making: Multiobjective Decision Analysis with Spreadsheets. Duxbury Press, California (1995)
12. Abbas, A.E., Matheson, J.E.: Normative target-based decision making. Manag. Decis. Econ. **26**, 373–385 (2005)
13. Abbas, A.E., Matheson, J.E.: Utility—Probability Duality. http://ideas.repec.org/ (2011)
14. Emdee, J.L.: A survey of development test programs for hydrogen oxygen rocket engines. In: AIAA 2001-0749, 39th AIAA Aerospace Sciences Meeting & Exhibit, Reno, NV, 8–11 January 2001