ATTRIBUTE-WISE VALUE OF INFORMATION IN ENGINEERING SYSTEMS: A SIMULATION-BASED STUDY

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ABSTRACT
Many engineering decisions involve uncertainty and require tradeoffs between multiple attributes. Often it is possible to reduce or eliminate uncertainty through data collection, or analysis, or both. Although it generally leads to better outcomes, reducing uncertainty may increase cost. This meta-decision of whether to reduce the uncertainty at a given cost is undertaken in the form of Value of Information (VOI) studies. Previous work has considered value of information in single-attribute cases, however, studies of the more challenging multiattribute case are less common. Multiattribute problems can have numerous uncertain variables across alternatives, not all of them easily measurable in money amounts, which makes the VOI analysis more complicated. This paper presents a simulation-based approach for calculating and interpreting the VOI in generic multiattribute decision problems and describes a simulation study that we conducted to explore how the decision problem parameters affect the VOI. The relationship between the VOI and the gain in expected utility (utility-gain) that will result from new information is also discussed. This relationship is not simple and yields some expected as well as counterintuitive results. For example, our results suggest that non-linearity of the utility functions may lead to different VOI calculation for the same utility-gain, depending on the relative position of the alternatives. Finally, this paper discusses the sensitivity of the utility from the decision as a function of decision problem parameters and shows that the magnitude of change in VoI can be predicted from these sensitivities. A vehicle alignment problem in an automotive assembly line is presented to demonstrate the approach. The results show that VoI calculation is a challenging problem that is very much contingent on the parameters of the problem. Our results provide insights into the benefits of different ways to evaluate the value of information in multiattribute problems.

1. INTRODUCTION
In a multiple attribute decision with uncertainty, typically a finite set of alternatives is given, and the alternatives are described by several (possibly uncertain) attributes that are important to the decision-maker. To compare the alternatives and identify the best one, the decision-maker may use a formal method such as modeling their preferences with a multi-attribute utility function. The alternative that maximizes the expectation of this function is then selected.

For example, consider an engineer who needs to select a solder material for use in a printed circuit board (PCB). There may be multiple solder materials available (such as SAC305 and SAC105), with uncertainty about the relevant attributes such as reliability and cost in each case. The engineer may have the option to reduce uncertainty about a particular material’s reliability by developing and running simulation models to evaluate that material for that application, but this requires valuable time and effort. The engineer would like to know in advance the benefit of gathering this information. Another example is that of the many decisions made in a vehicle assembly line. A new vehicle is assembled through a long sequence of steps, where workers or assembly robots perform operations at each work station. Many of these steps involve decisions, such as adjustment decisions to bring certain dimensions that are subject to variability, to within specifications. A plant manager may want to know the value of predicting these dimensions before a vehicle reaches a station, so that best decisions can be made. In both the above examples, while multiple attributes or metrics may be relevant, it may not be possible to reduce uncertainty in all of them.

The analysis of the decision to gather information (or not) is a type of preposterior analysis (Berger, 2010). The economic value of information describes the expected benefit that the
decision-maker would realize from having the information (Lawrence, 1999). This quantity can be used by the decision-maker to determine if and to what extent resources should be spent to get the information. Information is valuable if it leads to a better decision. The net value after subtracting the cost of gathering the information from the expected value of the information is known as the expected net gain of the information.

Generally, the approach described in this paper is related to multi-attribute decision analysis (Keeney and Raiffa, 1976, 1993), statistical decision theory (Raiffa and Schlaifer, 1961; Pratt et al., 1995), optimization under uncertainty (Powell, 2016), and the economic value of information (Lawrence, 1999). The value of new information (e.g., a new sample or measurement) measures the expected gain in the decision-maker’s utility from the next sample (experiment), and this quantity can be measured as a monetary value or as a gain in utility (Raiffa and Schlaifer, 1961; Bernardo, 1979; Lawrence, 1999). Howard (1970) illustrated the key concepts related to analyzing experimentation decisions with a coin-flipping example. Leber and Herrmann (2013, 2014a, b, 2015, 2016) and Herrmann and Mehta (2020) presented and tested one-step lookahead approaches for the problem of how to allocate multiple samples (opportunities for collecting information about uncertain attributes).

Most value of information studies in the engineering literature have focused on single attribute problems (Nikolaidis et al., 2013; Capser and Nikolaidis, 2017). Value of information in these cases is limited to finding the improvement in utility as a function of reduction in uncertainty in the attribute in question. However, a significant majority of engineering decisions involve multiple attributes, and at the same time, their presence complicates VOI analysis. For example, unless a cost, or cost-type attribute is involved, it is not always possible to quantify the improvement in realistic terms. If a cost attribute is indeed present, one will then project the impact of reduction in uncertainty on the cost attribute. Clearly, in this case one will have to know the tradeoff that the decision maker allows between cost and other attributes. Kassoumeh (2018) investigated VOI in multiattribute problems, provided an approach to evaluate it efficiently in problems involving Gaussian uncertainties, and investigated the effect of statistical correlation between various attributes within and across alternatives. However, their work is limited mostly to two-alternative problems involving the multilinear utility function, and does not provide a general framework for analyzing VOI in multiattribute problems, which is the objective here.

This paper presents a simulation-based approach for calculating and interpreting the VOI in generic multiattribute decision problems and describes a simulation study that we conducted to explore how the decision problem parameters affect the VOI. This paper also discusses the relationship between the VOI and the gain in expected utility that will result from new information. A heuristic based on sensitivity of decision utility to problem parameters is also presented to approximate the value of information.

Although it presents an approach that is closely related to the broad area of multiobjective optimization under uncertainty, this paper does not consider new optimization methods or techniques for multiobjective decision making. Such techniques are useful in selecting the best alternative under uncertainty. Instead, as mentioned in the preceding paragraphs, VOI studies investigate the potential for an improved decision if uncertainty were reduced. Since different individuals have different risk attitudes and preferences for tradeoffs, calculating VOI for a particular individual requires a formal model of their preferences, which is captured by the decision-maker’s multiattribute utility function.

This paper is organized as follows. Section 2 of this paper defines Attribute-specific Value of Information (AVOI) and presents a general approach for determining AVOI. Section 3 discusses the expected gain in utility. Section 4 demonstrates the approach in a case study of a vehicle alignment problem in an automotive assembly line. Section 5 discusses the insights that our work has yielded, and Section 6 concludes the paper.

2. Attribute-specific Value of Information

In this work, we use the multilinear, multiattribute utility function (MAUF) to measure the decision-maker’s utility over multiple attributes (Keeney, 1977; Keeney and Raiffa, 1993). This is done for demonstration purposes and the treatment in this paper does not depend on the type of utility function chosen. This functional choice is common in engineering practice because of its flexibility in modeling a wide range of preference structures. For $n$ attributes denoted by the vector $\mathbf{x} = (x_1, ..., x_n)^T$, the multilinear utility functional form is given by:

$$U(x_1, ..., x_n) = \frac{1}{K} \left[ \prod_{i=1}^{n} (Kk_iU_i(x_i) + 1) - 1 \right]$$

This functional form directly utilizes single attribute utility functions, $U_i(x_i)$’s, assessed over the attributes individually, simplifying the assessment process. The construction of the MAUF guarantees that, if an alternative has attained the best level on all attributes simultaneously (all $U_i(x_i) = 1$), then $U(x_1, ..., x_n) = 1$, and, if the alternative has attained the worst values on all attributes simultaneously (all $U_i(x_i) = 0$), $U(x_1, ..., x_n) = 0$. The scaling constants, $k_i$’s, quantify the relative disinclination of the decision-maker (DM) to accept the worsening of the $i$-th attribute in exchange for improving the other attributes. The single attribute utility functions and the corresponding scaling constants are assessed using lottery techniques described in the literature (Clemen and Reilly, 1997; Nikolaidis et al., 2011). The normalizing parameter, $K$,
is calculated by substituting the values of the scaling constants, setting every single attribute utility and the multi-attribute utility equal to 1, and then solving the resulting polynomial for \( K \).

**General approach to determine AVOI**

Let \( \mathcal{A}_j \), \( j \in \{1, \ldots, m\} \) be the alternatives that the DM is considering, \( \mathbf{X}^j = (X^j_1, \ldots, X^j_d)^T \) be the vector of attributes corresponding to \( \mathcal{A}_j \), and \( \mathbf{X} = (X^1_T, \ldots, X^m_T)^T = (X^1_1, \ldots, X^1_n, \ldots, X^m_1, \ldots, X^m_m)^T \) be the concatenated vector of all the attributes in the decision problem across all the alternatives. We assume, without any loss of generality, that \( x^j_n \) is the cost of \( \mathcal{A}_j \). Let the attributes or a subset of them be uncertain with a joint pdf \( f_X(x) \) defined over a domain \( D_X \). To highlight the \( k \)-th attribute when needed, we write the vector of attributes for \( \mathcal{A}_j \) as \( (\mathbf{X}^j_{-k}, x^j_k) \), where \( \mathbf{X}^j_{-k} \) is the vector of all attributes except the \( k \)-th attribute. Similarly, the pdf \( f_{X^j_{-k}}(x^j_{-k} | x^j_k) \) is the conditional joint-pdf of the attributes when the \( k \)-th attribute of \( \mathcal{A}_j \) is fixed. Let \( V \) be the VOI associated with uncertain attributes. In the case where all the uncertainties are resolved before making the decision, we can use the following relationship to determine \( V \):

\[
\int_{D_X} \left\{ \max_{x_n} U(x_{-n}^j, x_n^j + V) \right\} f_X(x) dx = \max \left\{ \int_{D_X} U(x_n^j) f_X(x) dx \right\} \tag{2}
\]

If the uncertainty over only one attribute in one of the alternatives is resolved, namely \( x^j_k \), we can write:

\[
\int_{D_{X^j_k}} \left\{ \max_{x_{-k}} U(x_{-k}^j, x_k^j + V) f_{X_{-k}^j}(x_{-k}^j | x_k^j) dx_{-k}^j \right\} f_{X^j_k}(x_k^j) dx_k = \max \left\{ \int_{D_{X^j_k}} U(x_k^j) f_{X^j_k}(x_k) dx \right\} \tag{3}
\]

The above expression can similarly be generalized to multiple attributes and a joint pdf which we collect information. If uncertainty over only one attribute is resolved, we call it the attribute specific value of information or AVOI.

### 2.1 One uncertain attribute in a two-alternative problem

Let \( \mathcal{A}_1 \) and \( \mathcal{A}_2 \) be the two alternatives each involving two attributes, \( X_1 \) and \( X_2 \). As noted before, we assume that \( X_2 \) is the cost of the alternative, and \( x_1^2 \) and \( x_2^2 \) are the known costs of alternatives \( \mathcal{A}_1 \) and \( \mathcal{A}_2 \). For alternative \( \mathcal{A}_1 \), attribute \( X_1^1 \) is uncertain and has pdf \( f_{X_1^1}(x_1^1) \) over domain \( D(X_1^1) \); for alternative \( \mathcal{A}_2 \), attribute \( X_1^2 \) is known and equals \( x_1^2 \).

Let \( V \) be the AVOI for uncertain attribute \( X_1^1 \). We can use the following relationship to determine \( V \):

\[
\int_{D(X_1)} f_{X_1^1}(x_1^1) \max \{ U(x_1^1, x_2^1 + V), U(x_1^2, x_2^2 + V) \} dx_1^1 = \max \left\{ \int_{D(X_1)} f_{X_1^1}(x_1^1) U(x_1^1, x_2^1) dx_1^1, U(x_1^2, x_2^2) \right\} \tag{4}
\]

For a given value of \( V \), the domain \( D(X_1^j) \) can be partitioned into two subdomains, \( D_1 \) and \( D_2 \), such that the following hold:

\[
\begin{align*}
U(x_1^1, x_2^1 + V) & \geq U(x_1^2, x_2^2 + V) \quad \forall x_1^1 \in D_1 \\
U(x_1^1, x_2^1 + V) & < U(x_1^2, x_2^2 + V) \quad \forall x_1^1 \in D_2
\end{align*}
\]

The subdomain \( D_1 \) includes only the realizations of \( X_1^1 \) that make alternative \( \mathcal{A}_1 \) more (or equally) desirable compared to alternative \( \mathcal{A}_2 \) when the additional cost equals \( V \), and subdomain \( D_2 \) includes the realizations that make alternative \( \mathcal{A}_1 \) less desirable than alternative \( \mathcal{A}_2 \). Then, the left-hand side of Equation 4 can be expressed as follows:

\[
\begin{align*}
\int_{D_1} f_{X_1^1}(x_1^1) & \max \{ U(x_1^1, x_2^1 + V), U(x_1^2, x_2^2 + V) \} dx_1^1 = \\
\int_{D_1} f_{X_1^1}(x_1^1) & U(x_1^1, x_2^1 + V) dx_1^1 + P(\{ X_1^1 \in D_2 \}) U(x_1^2, x_2^2 + V) \tag{5}
\end{align*}
\]

### 2.2 Multiple uncertain attributes

Let \( \mathcal{A}_1 \) and \( \mathcal{A}_2 \) be the two alternatives each involving three attributes, \( X_1, X_2 \), and \( X_3 \). Here, \( X_3 \) is the cost of the alternative, and \( x_1^3 \) and \( x_2^3 \) are the known costs of alternatives \( \mathcal{A}_1 \) and \( \mathcal{A}_2 \). For alternative \( \mathcal{A}_1 \), both attributes \( X_1^1 \) and \( X_2^1 \) are uncertain. Their joint pdf over domain \( D \) is \( f(x_1^1, x_2^1) \). The first one has marginal pdf \( f_1(x_1^1) \) and conditional pdf \( f_{2|1}(x_2^1 | x_1^1) \) over domain \( D^1 \); the second one has marginal pdf \( f_2(x_2^2) \) and conditional pdf \( f_{2|1}(x_2^1 | x_1^1) \) over domain \( D^2 \). For alternative \( \mathcal{A}_2 \), the first two attributes are known: \( x_1^2 = x_1^2 \) and \( x_2^2 = x_2^2 \).

Let \( V_i \) be the AVOI for uncertain attribute \( X_i^j \). (Let \( X_i^j \) be the other uncertain attribute.) We can use the following relationship to determine \( V_i \):

\[
\int_{D} f_i(x_1^j) \max \left\{ \int_{D_1} f_{j|1}(x_j^i | x_1^1) U(x_1^1, x_2^1, x_3^i + V_j^i) dx_1^1, \right. \\
\left. U(x_1^2, x_2^2, x_3^i + V_j^i) \right\} dx_j^i = \\
\max \left\{ \int_{D} f_i(x_1^j, x_2^1) U(x_1^1, x_2^1, x_3^i) dx_1^1, U(x_1^2, x_2^2, x_3^i) \right\} \tag{6}
\]

Due to the complexity of Equation 6, it will generally be impossible to solve directly for \( V_i \), so determining \( V_i \) to a desired level of precision will require a search over the possible values. The obvious lower bound for the range of \( V_i \) is 0; at this point, the left-hand side of Equation 1 will be greater than or equal to its right-hand side. The upper bound can be set to a value \( V_i^UB \) such that \( U(x_1^1, x_2^1, x_3^i + V_i^UB) \leq U(x_1^2, x_2^2, x_3^i) \) for all values of \( x_1^1 \in D^1 \) and \( x_2^1 \in D^2 \). At this point, the left-hand side of Equation 1 will be less than its right-hand side.

### 3. ESTIMATING VALUE OF INFORMATION

If collecting perfect information about an uncertain attribute were free, then collecting that information should increase the expected utility of the decision situation (at worst,
there will be no increase in expected utility). To avoid searching for the precise value of AVOI, this section focuses on evaluating the expected gain in utility and how it is correlated with AVOI. We also present a heuristic method based on the sensitivity analysis similar to the first-order second-moment method used in reliability engineering.

**Expected gain in utility**

Let $E[L]$ be the expected utility of the decision situation with no information:

$$E[L] = \max_i \left\{ \int_{\mathcal{X}_i} U(x_i) f(x_i')dx_i \right\}$$

(7)

Let $E[LI(k, l)]$ be the expected utility of the decision situation with perfect information about uncertain attribute $X_k^l$, the $k$-th attribute for $\mathcal{A}_l$:

$$E[LI(k, l)] = \int_{\mathcal{X}_k^l} \left\{ \max_j \int_{\mathcal{X}_{-k}} U(x_{-n}, x_k^l) f_{X_{-k}}^{-1}(x_k^l) dx_{-k} \right\} f_k(x_k^l) dx_k^l$$

(8)

Let $G(k, l)$ be the expected gain in the utility of the decision situation by collecting perfect information about attribute $X_k^l$:

$$G(k, l) = E[LI(k, l)] - E[L]$$

(9)

Naturally, we should expect attributes with greater AVOI to also have greater expected gain, but this is not always true when the attribute is the cost. For example, let us consider a generic decision example with three alternatives and three attributes (the third attribute is cost). All of the attributes for the first two alternatives are uncertain and normally distributed, but none of the attributes for the third alternative are uncertain. We consider two cases: in case 1 the uncertain attributes are independent; in case 2 they are correlated. The correlation values are given in Table 2.

**Table 1.** Probability distributions for both cases 1 and 2 in the generic decision example.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Distribution for attribute</th>
</tr>
</thead>
<tbody>
<tr>
<td>$j$</td>
<td>$X_1^j$</td>
</tr>
<tr>
<td>1</td>
<td>N(5.5, 1.65)</td>
</tr>
<tr>
<td>2</td>
<td>N(7, 2.1)</td>
</tr>
<tr>
<td>3</td>
<td>7.5</td>
</tr>
</tbody>
</table>

**Table 2.** Attribute correlations for cases 1 and 2

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{X_1X_2}$</td>
<td>0</td>
<td>-0.5</td>
</tr>
<tr>
<td>$\rho_{X_1X_3}$</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho_{X_2X_3}$</td>
<td>0</td>
<td>-0.5</td>
</tr>
<tr>
<td>$\rho_{X_1X_3}$</td>
<td>0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Smaller values of the attributes are more desirable, and the attribute utility functions are identical:

$$U_i(x_i) = \left( 1 - e^{-(12-x_i)/5} \right) / \left( 1 - e^{-12/5} \right)$$

(10)

For the multi-attribute utility function, all three scaling constants $k_i = 0.4$, and $K = -0.4428$.

In case 1, we find that the expected utility of alternative 1 is 0.7399. The expected utility of alternative 2 is 0.7025. The utility of alternative 3 is 0.6814. Thus, with no information, the expected utility $E[L] = 0.7399$. With perfect information about every uncertain alternative, the expected utility equals 0.7791, a gain of 0.0392. The value of that perfect information is 1.4737.

In case 2, we find that the expected utility of alternative 1, which is still the best alternative, is 0.7405. The expected utility of alternative 2 is 0.7034. The utility of alternative 3 is 0.6814. Thus, with no information, the expected utility $E[L] = 0.7405$. With perfect information about every uncertain alternative, the expected utility equals 0.7660, a gain of 0.0255. The value of that perfect information is 1.0297.

We notice that there is a decrease in the value of information and in the utility gain from case 1 to case 2. This is because there is dependence between the attributes, both within and across alternatives, in case 2 (see Table 2). Therefore, finding information about an alternative or its attributes reduces the overall uncertainty present in the decision problem since corresponding attributes across alternatives are positively correlated.

**Table 3.** AVOI and Expected Gains for Generic Example

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1^k$</td>
<td>0.1213</td>
<td>0.0416</td>
</tr>
<tr>
<td>$V_1^k$</td>
<td>0.2772</td>
<td>0.0662</td>
</tr>
<tr>
<td>$X_1^k$</td>
<td>0.5294</td>
<td>0.5310</td>
</tr>
<tr>
<td>$X_2^k$</td>
<td>0.2341</td>
<td>0.0065</td>
</tr>
<tr>
<td>$X_3^k$</td>
<td>0.1996</td>
<td>0.4464</td>
</tr>
<tr>
<td>$X_4^k$</td>
<td>0.2679</td>
<td>0.2603</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{X_1X_2}$</td>
<td>0</td>
<td>-0.5</td>
</tr>
<tr>
<td>$\rho_{X_1X_3}$</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho_{X_2X_3}$</td>
<td>0</td>
<td>-0.5</td>
</tr>
<tr>
<td>$\rho_{X_1X_3}$</td>
<td>0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Let $V_k^l$ be the AVOI for uncertain attribute $X_k^l$. As shown in Table 3, the AVOI and expected gain for the attributes for alternative 1 are clearly correlated. For case 1 alternative 2, however, all three attributes have approximately equal expected gains, but the AVOI of attribute $X_3^2$ is greater than the AVOI for attributes $X_2^2$ and $X_3^2$. The similar expected gains are not surprising considering that all three attributes have the same distribution and the utility function treats all three attributes equally. The third attribute has a greater AVOI in both cases 1 and 2 because it is the cost attribute and the utility function is nonlinear. For instance, if the value of the cost
attribute is lower than its mean (and the other two attributes are at their means), then a large increase in cost causes a small decrease in utility, so a large AVOI is possible. If the value of another attribute is lower than its mean (but the cost remains at its mean), the large increase in cost causes a larger decrease in utility, which reduces the AVOI. Of course, this depends upon the distributions; if the means for cost (attribute 3) were lower, then any changes in that attribute would have less impact on utility, which would lower the value of that information and the corresponding gain and AVOI.

Summing the AVOI values for the two cases yields 1.6295 and 0.9524 respectively. In case 1 we see that the AVOI sum is slightly higher than the VOI for the entire problem. On the other hand, in case 2, we see that the AVOI sum is less than the VOI for the entire problem. This can again be explained in terms of correlation between attributes within and across alternatives for case 2. Finding information about an alternative or its attributes reduces the overall uncertainty present in the decision problem when there is correlation present.

**Sensitivity analysis based metric for AVOI**

Here we present a heuristic to prioritize the attributes and alternatives over which value of information should be collected *without* actually evaluating the VOI as was done in the preceding paragraphs. We first recognize that the Taylor expansion of the utility function can be written as:

\[
U(\mathbf{x})|_{\mathbf{x} = \mathbf{\mu}_x} = U(\mathbf{\mu}_x) + \nabla U(\mathbf{\mu}_x)^T (\mathbf{x} - \mathbf{\mu}_x) + \frac{1}{2} (\mathbf{x} - \mathbf{\mu}_x)^T \mathbf{H}(U(\mathbf{\mu}_x))(\mathbf{x} - \mathbf{\mu}_x) \tag{11}
\]

and, the second-order terms yield the approximate value of the variance of the utility as:

\[
s^2_u \approx \sum_{i=1}^n \sum_{j=1}^n \frac{\partial U(\mathbf{\mu}_x)}{\partial x_i} \frac{\partial U(\mathbf{\mu}_x)}{\partial x_j} \sigma_{x_i x_j} \tag{13}
\]

Consider now a notional decision problem where the probability density functions of the utility function for three alternatives are as shown in Figure 1. In this case, alternative 1 has the highest expected utility, alternative 2 the second highest and alternative 3 the lowest expected utility. Value of information stems from the region where the pdf’s overlap – i.e. instances where knowledge about the realizations makes a decision maker potentially change their decision. It is easy to see that the region of overlap between alternative 1 and 2 is greater than that between alternative 1 and 3. The size of this overlap forms the basis for the heuristic we develop here.

We consider the utility difference between alternative 2 and 1, \(U(X^2) - U(X^1)\) and evaluate the contribution of each attribute to the variance of this difference in a sensitivity metric, \(\xi_k\).

\[
\xi_k = \sum_{i=1}^n \sum_{j=1}^n \sigma_{x_i x_j} I^k(i,j) \frac{\partial U(\mathbf{\mu}_x)}{\partial x_i} \frac{\partial U(\mathbf{\mu}_x)}{\partial x_j} \tag{14}
\]

Where \(I^k(i,j)\) is an indicator function that takes the value of 1 when either \(i\) or \(j\) equals \(k\) and 0 otherwise. Clearly, the sensitivity value thus calculated is proportional to the variance of that attribute, increases if the partial derivative of utility function is higher and is zero if the attribute is deterministic. All these properties are desirable properties of the metric.

**Table 4: Comparison of the sensitivity metric with AVOI.**

<table>
<thead>
<tr>
<th>Attribute (X^k)</th>
<th>(\xi_k) Case 1</th>
<th>AVOI Case 1</th>
<th>(\xi_k) Case 2</th>
<th>AVOI Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X^1)</td>
<td>0.0009</td>
<td>0.1213</td>
<td>-0.0005</td>
<td>0.0416</td>
</tr>
<tr>
<td>(X^2)</td>
<td>0.0019</td>
<td>0.2772</td>
<td>0.0001</td>
<td>0.0652</td>
</tr>
<tr>
<td>(X^3)</td>
<td>0.0036</td>
<td>0.5294</td>
<td>0.0036</td>
<td>0.5310</td>
</tr>
<tr>
<td>(X^4)</td>
<td>0.0027</td>
<td>0.2341</td>
<td>0.0006</td>
<td>0.0087</td>
</tr>
<tr>
<td>(X^5)</td>
<td>0.0027</td>
<td>0.1996</td>
<td>0.0002</td>
<td>0.0446</td>
</tr>
<tr>
<td>(X^6)</td>
<td>0.0027</td>
<td>0.2679</td>
<td>0.0027</td>
<td>0.2603</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.78</td>
<td></td>
<td>0.93</td>
<td></td>
</tr>
</tbody>
</table>

We notice from the relative values of the sensitivity metric \(\xi_k\) and its correlation with AVOI that this metric is a strong indicator of the VOI. In each case, the most important attribute is identified directly from the parameters of the problem. This information can be calculated relatively easily without performing expensive AVOI calculation.

**4. CASE STUDY: VEHICLE ALIGNMENT**

In this example decision problem, we consider the front wheel alignment of automobiles in an assembly line undertaken by the automaker. In mass production of
automobiles, proper wheel alignment is an important but challenging step. Wheel alignment is measured using the metrics of camber, caster and toe angles. Figures 2 and 3 illustrate camber and toe on a typical vehicle, and these are the only two metrics we will consider for the rest of our discussion. Consequently, automakers are very interested in ensuring that such vehicles do not reach the market, otherwise there are direct repercussions on vehicle safety, warranty costs and, in case of widespread issues, loss of customer confidence.

Poor wheel alignment affects ride performance, accelerates tire wear and contributes to accidents. Consequently, automakers are very interested in ensuring that such vehicles do not reach the market, otherwise there are direct repercussions on vehicle safety, warranty costs and, in case of widespread issues, loss of customer confidence.

The decision problem involves selecting one from a set of three available aligners (Figure 4). The attributes are (1) camber, (2) toe and (3) the cost (per vehicle) of alignment. The cost includes labor and costs associated with cycle time. The joint probability distribution of the attributes associated with each aligner is shown in Table 5. These are baseline values that are representative of typical values seen in most vehicles. The strength and sign of correlation between the attributes for the same aligner is chosen for demonstration purposes. The attributes of different aligners are independent. Further, it is assumed that cost is independent of camber and toe, and the performance of the third aligner is considered deterministic.

Now we consider the case where the output alignment metrics of a vehicle from each aligner can be predicted. The variability in the outputs is a function of the aligner state, algorithm used for alignment, time spent during alignment, and variability in BIW (body-in-white) dimensions. We consider a smart factory where all the BIW dimensions are available and the aligner state is known. Let us now consider a consultant that can predict accurately the values of the final alignment metrics. Clearly, since there is overlap between the attribute

![Figure 2: Positive and negative camber in a typical automobile as viewed from the front.](image)

![Figure 3: Illustration of toe out and toe in, in a typical vehicle as viewed from the top of the vehicle.](image)

![Figure 4: Aligner selection decision problem.](image)

<table>
<thead>
<tr>
<th>Aligner</th>
<th>Mean</th>
<th>Covariance Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\mathbf{\mu}^1 = [1 \ 0.5 \ 20]^T$</td>
<td>$\Sigma^1 =$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$\mathbf{\mu}^2 = [0.75 \ 0.75 \ 17]^T$</td>
<td>$\Sigma^2 =$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$\mathbf{x}^3 = [1.5 \ 1 \ 12]^T$</td>
<td>Deterministic</td>
</tr>
</tbody>
</table>
distributions of the three alternative aligners, all of them have a non-zero probability of being the best. Therefore, this information has value. The automaker is interested in determining the value of this information and which uncertain attributes have the most value.

The parameters of the multiattribute utility function of the automaker are shown in Table 6. The single attribute utility functions are exponential and the automaker prefers less of each attribute to more.

### Table 6: Utility function parameters for the alignment problem

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Min value</th>
<th>Max value</th>
<th>Risk tolerance</th>
<th>Scaling constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$ (camber, degrees)</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>$X_2$ (toe, degrees)</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>$X_3$ (cost, $/car)</td>
<td>0</td>
<td>30</td>
<td>15</td>
<td>0.4</td>
</tr>
</tbody>
</table>

### Results

The expected utility of aligner 1 is 0.752, that of aligner 2 is 0.794 and that of aligner 3 is 0.709. Therefore, under the uncertainty present in the decision problem, aligner 2 should be selected and the utility of the decision situation is 0.794. From the sensitivity information given in Table 7, we see that the maximum benefit is achieved by collecting information on the cost associated with using aligner 1, followed by that on aligner 2. We confirm by evaluating the attribute-specific value of information (AVOI) that this is indeed correct. It should be noted that the computational effort for calculating AVOI is very high, while the sensitivity metrics $\xi_k$ can be calculated quickly.

### Table 7: Sensitivity metric $\xi_k$ and AVOI for the aligner problem.

<table>
<thead>
<tr>
<th>Attribute $X^I_k$</th>
<th>$\xi_k$</th>
<th>AVOI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1^1$</td>
<td>0.0013</td>
<td>0.1136</td>
</tr>
<tr>
<td>$X_1^2$</td>
<td>$\approx 0$</td>
<td>0.0437</td>
</tr>
<tr>
<td>$X_2^1$</td>
<td>0.0484</td>
<td>1.0487</td>
</tr>
<tr>
<td>$X_2^2$</td>
<td>0.0003</td>
<td>0.0662</td>
</tr>
<tr>
<td>$X_3^1$</td>
<td>0.0003</td>
<td>0.0676</td>
</tr>
<tr>
<td>$X_3^2$</td>
<td>0.0023</td>
<td>0.8503</td>
</tr>
<tr>
<td>Comp. time</td>
<td>0.1648 sec</td>
<td>210.8 sec</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.76</td>
<td></td>
</tr>
</tbody>
</table>

The total value of information for all the uncertainties is $2.26 and the corresponding utility with information is 0.815. This leads to a utility gain of 0.021. From the results it is clear that if there is an opportunity to acquire information about the uncertain attributes, the uncertainty in cost should be prioritized. In this case, since there is correlation within an alternative but no correlation between alternatives, we see that the sum of AVOI is similar to the VOI for all the attributes together.

In our generic examples described earlier and in the alignment problem we notice that the AVOI for the cost attributes is greater than the AVOI for the other attributes. This can be attributed to the non-linearity of the utility function as lower-valued realizations of the cost random variable allows for addition of VOI cost without affecting utility too much. We investigated the effect of changing the scaling constant associated with the cost attribute in the decision maker’s utility function. Figure 5 shows that, as $k_{\text{cost}}$ increases, the AVOI for the cost attributes for both cost attributes increased. This is expected since the decision maker values cost more. It is seen, however, that at a value of about 0.55, the AVOI for attribute $X_3^2$ becomes higher than that for $X_3^1$. This may be explained as follows: as the decision maker values cost more, the higher variance of $X_3^2$ influences the decision even more. As a result, the corresponding AVOI increases substantially.

![Figure 5: Effect of change in the scaling constant for cost on AVOI.](image)

![Figure 6: Effect of correlation between alternatives on VOI.](image)

We also investigated the effect of the correlation between alternatives on AVOI by increasing the correlation coefficient.
between the toe and camber attributes of aligner 1 and 2. Figure 6 shows the impact of this change: the value of information monotonically decreased as the correlation increased. This suggests that, when the attributes are correlated, collecting information becomes less valuable as the alternatives improve or worsen together. It is important to note that the trend continues for negative values of correlation between the alternatives. Negative correlation implies that the alternatives have a higher probability of crossing-over (see Figure 1), which explains the trend.

5. DISCUSSION

These results provide some insights into the benefits of different ways for evaluating the AVOI. The financial approach presented in this paper provides a way to determine whether collecting the information is worth the cost, but this approach requires the most computational effort, as shown in Table 7. The sensitivity metric is much easier to compute and was correlated with AVOI in the examples that we considered. The expected gain requires calculating the expected utility in both cases (with and without the attribute information) but is more strongly correlated with AVOI than the sensitivity metric (as shown in Table 3). Still, it is possible that attributes may have the same expected utility gain but different AVOI values as we showed in Section 3.

The results of the sensitivity analyses provided some empirical evidence, shown in Figures 5 and 6, to support our intuition about the impact of changing the utility function and the correlation among uncertain attributes. If the weight on cost in the decision-maker’s utility function increases, then the VOI generally goes up. Similarly, increasing correlation decreases value of information. In general, these effects are a result of whether the highest rated alternatives are brought closer by the change or driven further apart, as illustrated in Figure 1.

6. SUMMARY AND CONCLUSIONS

Most engineering problems involve multiple uncertain attributes that a decision-maker must balance to make a decision. Often, there is a possibility to reduce or eliminate some uncertainty through tests or detailed analysis. However, it is not always possible to eliminate uncertainty in all relevant attributes simultaneously. Engineers are therefore naturally interested in identifying where (about which attributes) reducing uncertainty will have the maximum benefit. This paper formulated the problem of calculating the expected value of perfect information about one or more uncertain attributes in a multiattribute decision problem and presented an approach for calculating it. The approach allows us to individually calculate the effect of the uncertainty resolution in one attribute, and we call this attribute-specific value of information (AVOI). This paper also presented two other approaches for comparing different attributes to determine which ones have the greatest AVOI: the sensitivity metric and the expected gain, which were both correlated with AVOI in the examples that we considered. The sensitivity metric, based on the first-order second moment method is able to identify attributes with the highest AVOI with minimal computational effort.

In the future, it would be valuable to identify special cases where calculating the AVOI is simpler, to develop approaches for assessing the value of imperfect information (due to inaccurate information sources), and to express the expected gain in non-monetary terms. Finally, it will be interesting to extend the treatment presented in this paper to non-normal attributes and other general utility functional forms.

References


