Separating Design Optimization Problems into Decision-Based Design Processes

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This paper introduces the technique of separation, which replaces a design optimization problem with a set of subproblems. This separation is similar to decomposition but does not require a second-level coordination. We identify conditions under which this separation yields an exact solution and other conditions under which the error can be bounded. We show that the decision-based design framework, which seeks to find the most profitable design, can be separated into a sequence of subproblems. We also apply separation to a motor design problem and demonstrate how the surrogate constraints and objective functions affect the solution quality. These results indicate a way to apply the principles of decision-based design to design processes.

1 Introduction

Organizations that develop products and systems want to create the most valuable design that is feasible. The measurement of value, which depends upon the type of organization, may be profitability, life-cycle cost, or system effectiveness, for example. The value of the product or system that is being designed depends upon the decisions that the design engineer (or development team) makes.

The observation that engineering design requires making decisions has motivated a great deal of research, including work on decision analysis, decision theory, concept generation, modeling customer demand, multi-attribute decision-making, enterprise models, product development processes, and decentralized decision-making [1]. Design organizations can be viewed as a set of loosely-coupled decision-makers [2] that generate and share information in order to generate designs [3, 4]. The ultimate goal is to improve the quality of these decisions and increase the value of product development processes [5].

A variety of decision-making processes have been identified [6]. The two that are most relevant to engineering design are the incremental decision process model and optimization. The incremental decision process model [7] presents a structure in which a major decision is implemented as a series of small decisions. This detailed model involves iterating between the following types of activities: recognition, diagnosis, search, screen, design, judgment, analysis, bargaining, and authorization. Designers will easily recognize the similarities between this process and their own activities.

Design optimization is an important engineering design activity and a difficult mathematical problem. In general, design optimization determines values for design variables such that an objective function is optimized while performance and other constraints are satisfied [8, 9, 10]. Formal design optimization is a useful decision-making process when two conditions hold: (1) there exists enough technical knowledge to formulate a mathematical model that can express the value of a design as a mathematical function of the design variables and (2) there is a consensus on the appropriate objective function [6]. The attributes used to describe a design optimization model can be grouped into four areas: scope, variable set, objective function, and model structure [11].

The difficulty of solving large scale optimization problems and multidisciplinary optimization (MDO) problems has motivated various decomposition approaches. In general, these decomposition approaches require multiple iterations to converge to a feasible, optimal solution for a given design optimization model. Model coordination and goal coordination are two common methods for the decomposition of large scale design optimization problems [12, 13]. MDO problems have been the focus of decomposition approaches such as the bi-level integrated system synthesis (BLISS) approach [14], analytical target cascading [15, 16], collaborative optimization [17], and coupled subspace optimization (CSSO) [18, 19]. Yoshimura *et al*. [20] decompose a multi-objective optimization problem into a hierarchy of problems that have two objectives.

The decision-based design (DBD) framework [21] is an approach that explicitly addresses the challenge of creating the most profitable design. It starts with the assumption that engineering design is a decision-making process. The framework shows that possible design alternatives should be evaluated based on how they affect the value of the product. As mentioned above, a typical bottom-line measurement of value is profit. The framework also indicates that there are uncontrollable variables that affect the value of the product but notes that price is a controllable variable. The framework thus shows that the design problem is to optimize the value of the profit (the expected utility of the profit) by selecting values for all of the design variables and the price. The comprehensive nature of the DBD framework has inspired researchers to develop new design optimization models (called enterprise models) that add variables from the marketing and manufacturing domains to models with conceptual design variables and to adapt existing decomposition techniques to solve them [17, 22, 23]. These more extensive design optimization problems reflect the natural desire to handle large, complex problems in an integrated way [24].

This paper introduces an approach that replaces a design optimization problem with a set of subproblems to form a decision-based design process. In particular, this paper analyzes a version of the DBD framework, identifies conditions under which the separation is exact (the result is optimal), presents sufficient conditions for establishing bounds on the quality of a non-optimal solution, and applies the concept to a specific engineering design problem.

This paper first introduces the concept of separation. Although an in-depth analysis of the similarities and differences between separation and the related approaches is beyond the scope of this paper, these issues are discussed briefly. We then analyze the DBD framework and apply separation to a motor design optimization problem. Finally, the paper presents some more general thoughts about engineering design processes.

2 Separation

In this paper we describe an approach that replaces a design optimization problem with a set of subproblems, solves each subproblem once, and produces a feasible solution without iterative cycles. We call this approach separation. The ideal separation produces an optimal solution to the original problem. However, not all separations do.

The concept of separation is similar (but not identical) to the idea of decomposition. Both replace a large design optimization problem with a set of subproblems. In a typical decomposition approach, a second-level problem must be solved to coordinate the subproblem solutions in an iterative manner. (See Figure 1.)

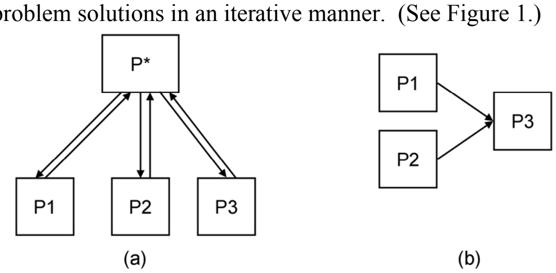

Figure 1. (a) A typical decomposition scheme has multiple first-level subproblems (P1, P2, P3) that receive inputs from a second-level problem (P*), which also coordinates their solutions. (b) Separation yields a set of subproblems. Solving one provides the input to the next.

Separation, on the other hand, does not require subsequent coordination. It is a decentralized and sequential approach related to the concept that is called factorisation in Pahl and Beitz [25]. A large problem is divided into subproblems. The solution to one subproblem will provide the inputs to one or more subsequent subproblems. However, there is no higher-level problem to coordinate the solution. Note that the separation does not have to be a simple sequence of subproblems; it may have subproblems that are solved in parallel at places. A given separation specifies a partial order in which the subproblems are solved. A different order of subproblems would be a different separation and would lead to a different solution. (Examples of this phenomenon can be found in work on sequential design decision-making [26].)

The subproblems' objective functions are surrogates for the original problem's objective function. These surrogates come from substituting simpler performance measures that are correlated with the original one, eliminating components that are not relevant to that subproblem, or from removing variables that will be determined in another subproblem.

Although it is a unique approach, separation shares some concepts and characteristics with other optimization techniques. The similarities reflect the shared strategy of dividing a large problem into smaller parts, a common approach in decision-making and optimization.

As mentioned above, separation replaces a large optimization problem with a set of smaller ones, like other decomposition approaches do. A key distinctive feature of separation is that, unlike the multiple-discipline-feasible (MDF) and individual-discipline-feasible (IDF) techniques [27-29] or concurrent subspace optimization [18, 19], separation does not iterate until the solution converges. Moreover, the subproblems in a separation do not have to correspond strictly to different disciplines.

The decentralized design that characterizes separation is also discussed by Chanron and Lewis [30], who applied concepts from game theory to study the convergence of various iterative approaches. By contrast, separation does not include iteration, as mentioned above. Moreover, separation allows one to allocate the design variables to different designers and to dictate their objective functions, instead of taking those as given, as the game theory approach does.

Like dynamic programming, separation may solve a set of subproblems and use the solution of one problem to solve another. Typically dynamic programming recursively solves a set of subproblems (corresponding to a set of possible states) starting with a trivial subproblem [31]. By contrast, separation does not contain this special recursive structure; therefore, solving a subproblem considers only the decisions that have been made.

Goal programming [32] prioritizes a set of criteria and finds a solution that meets as many high-priority goals as possible. This approach to multicriteria decision-making uses a single optimization problem that includes all the criteria. Separation, on the other hand, replaces a problem that has one (possibly complex) objective with subproblems that have different objectives. Some separations may resemble goal programming formulations. In general, however, the ordering of subproblems in a separation does not necessarily reflect the importance of their objectives.

Also, despite the similar name, separation is not the same as separable programming, a branch of mathematical programming that concerns nonlinear optimization problems in which the objective function and the constraints are sums of single-variable functions [33]. Separable programming approaches use a linear program to approximate the original problem and employ a type of simplex algorithm to find a solution. By contrast, separation replaces the original problem with a set of subproblems.

3 Separating Design Optimization Problems

Separating a design optimization problem is a modeling task that requires understanding the relationships between the design variables, constraints, and objective function. Forming a separation includes identifying the design variables, constraints, and objectives for the subproblems. Although optimization techniques exist for solving the subproblems, there are no automated methods for forming a separation.

Certain natural approaches can be identified. If the problem has a hierarchical structure, the separation can exploit that. Candidate subproblems include those that optimize intermediate values and functions of design variables that are (as a set) independent of other design variables. A separation can first set targets for intermediate values and then set values for design variables to meet these targets. Alternatively, a separation can set design variables first, using a surrogate objective function that is correlated to the ultimate objective function.

Defining surrogate objectives and appropriate constraints may require additional analysis combined with knowledge (based on experience) about which issues are the most important ones and which solutions are usually poor ones. Subproblems that correspond to different engineering disciplines or engineering tasks (as mentioned by [18]) may be useful. However, it is important to note that the subproblems do not necessarily have to correspond to different engineering disciplines. For highly coupled systems, the use of global sensitivity equations [34] may help identify appropriate subproblems and surrogate objectives. One could find a separation by applying techniques developed for decomposition approaches that rearrange the constraint-parameter incidence matrix formed by the design variables and the constraints [35-38] or the adjacency matrix of the analysis functions and the design variables [39]. Other relevant approaches include using the information gathered during Quality Function Deployment to identify the key design variables [40] and using the value of information to identify simplifications [41].

4 Separating the DBD Framework

We now consider a modified version of the DBD framework [21]. (This version ignores any uncertainties, and the demand affects the manufacturer's total lifecycle cost.) First, we will define the following notation:

m = system configuration.

M = the set of all possible configurations.

x = vector of design variables.

$X(m)$ is the set of designs that are feasible for a given configuration m.

p = selling price per unit.

a = vector of product attributes.

D = total demand over the product lifecycle (units).

C = lifecycle cost to manufacturer.

Π = total profit over the product lifecycle ($).

The following functions are given:

$a(x)$ relates the attributes to the design variables.

$D = q(a, p)$ relates the demand to the attributes and the price.

$C(x, D)$ relates the lifecycle cost to the design variables and the demand.

$u(\Pi)$ = utility of profit. We assume that u is monotonically increasing.

Problem P is to choose m, x, and p (the variables) to maximize the utility of the profit:

$$\begin{aligned} \max \quad & u(\Pi) \\ \text{s.t.} \quad & \Pi = Dp - C(x,D) \\ & D = q(a(x), p) \\ & m \in M \\ & x \in X(m) \\ & p \geq 0 \end{aligned} \tag{1}$$

We will separate P into two subproblems, P1 and P2. We will use a graph-like figure to represent a separation. This decision network figure has nodes that correspond to subproblems. An arc from a subproblem node indicates the variables whose values are determined by that subproblem. An arc leading into a node indicates the variables whose values are required by that subproblem. The decision networks corresponding to the original formulation and the separation are shown in Figure 2.

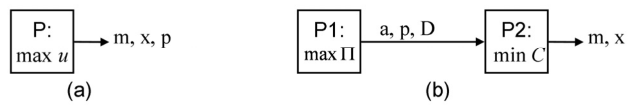

Figure 2. (a) The decision network for the integrated design optimization model. (b) The decision network for the separation.

The variables in P1, the first subproblem in our separation, are a, the vector of attributes, and the price p. Formulating this subproblem requires defining A, the set of all feasible attribute combinations. (A vector of attribute values, sometimes called "targets," is feasible if and only if there is some feasible combination of design variable values that can achieve all of those attributes simultaneously.) It also requires defining $\hat{c}(a,D)$, the approximate life cycle cost if the demand is D and the product attributes are a. Then, we can let the approximate profitability $\hat{\Pi}$ be the surrogate objective function:

$$\hat{\Pi}(a,p) = q(a,p)p - \hat{c}(a, q(a,p)) \tag{2}$$

Solving P1 provides a solution with values a^* and p^* and also yields $D^* = q(a^*, p^*)$.

$$\begin{aligned} \max \quad & \hat{\Pi}(a,p) \\ \text{s.t.} \quad & a \in A \\ & p \geq 0 \end{aligned} \tag{3}$$

The variables in P2 are m and x. Solving P2 yields the optimal values $m^* \in M$ and $x^* \in X(m^*)$:

$$\begin{aligned} \min \quad & C(x, D^*) \\ \text{s.t.} \quad & a(x) = a^* \\ & m \in M \\ & x \in X(m) \end{aligned} \tag{4}$$

The quality of this separation is determined by the set A and the approximation $\hat{c}(a,D)$. Let $A(m)$ be the set of attribute combinations that are feasible for a given configuration m in M:

$$A(m) = \{a(x) : x \in X(m)\} \tag{5}$$

If $A = \bigcup_{m \in M} A(m)$ and $\hat{c}(a,D) = \min_{m \in M, x \in X(m)} \{C(x,D) : a(x) = a\}$, then this is an exact separation. To show this, we need to show that m^*, x^*, and p^* are an optimal solution to Problem P. (The proof that this separation is exact is similar to the analysis of a Stackelberg leader-follower game.)

Suppose not. Then there exists $m' \in M$ and $x' \in X(m')$ and $p' \ge 0$ such that $a' = a(x')$, $D' = q(a', p')$, and $u(D'p' - C(x', D')) > u(D^*p^* - C(x^*, D^*))$.

Because u is monotonically increasing, $D'p' - C(x', D') > D^*p^* - C(x^*, D^*)$. Because m^* and x^* are an optimal solution for P2, we know that $C(x^*, D^*) = \hat{c}(a^*, D^*)$. Because $a' = a(x')$ and $\hat{c}(a', D') = \min_{m \in M, x \in X(m)} \{C(x, D') : a(x) = a'\}$, we know that $\hat{c}(a', D') \le C(x', D')$. Therefore,

$$\begin{aligned} \hat{\Pi}(a', p') &\ge D'p' - C(x', D') \\ &> D^*p^* - C(x^*, D^*) \\ &= D^*p^* - \hat{c}(a^*, D^*) \\ &= \hat{\Pi}(a^*, p^*) \end{aligned} \quad (6)$$

This contradicts the optimality (from P1) of a^*, p^*. Therefore, m^*, x^*, and p^* are an optimal solution to Problem P. QED.

Having identified sufficient conditions for an exact separation, we now consider an approximate separation. Suppose that the cost function $\hat{c}(a, D)$ is not exact, but we have the following error bound:

$$\left| \hat{c}(a, D) - \min_{m \in M, x \in X(m)} \{C(x, D) : a(x) = a\} \right| < \varepsilon \quad (7)$$

Then we can show that the profitability of m^*, x^*, and p^* must be within 2ε of the optimal profitability as follows. First, let $m' \in M$ and $x' \in X(m')$ and $p' \ge 0$ be an optimal solution to P. Let $a' = a(x')$ and $D' = q(a', p')$. Because m^* and x^* are an optimal solution for P2, we know that $C(x^*, D^*) = \min_{m \in M, x \in X(m)} \{C(x, D^*) : a(x) = a^*\}$. From this equality, Equation (7), and some rearranging, we have the following:

$$\begin{aligned} &\hat{c}(a^*, D^*) - C(x^*, D^*) > -\varepsilon \\ &D^*p^* - C(x^*, D^*) > D^*p^* - \hat{c}(a^*, D^*) - \varepsilon \end{aligned} \quad (8)$$

We also know that $C(x', D') \ge \min_{m \in M, x \in X(m)} \{C(x, D') : a(x) = a'\}$. From Equation (7) we know that $\min_{m \in M, x \in X(m)} \{C(x, D') : a(x) = a'\} > \hat{c}(a', D') - \varepsilon$. Combining these and rearranging terms lead to the following:

$$\begin{aligned} &C(x', D') > \hat{c}(a', D') - \varepsilon \\ &D'p' - \hat{c}(a', D') > D'p' - C(x', D') - \varepsilon \end{aligned} \quad (9)$$

Because a^* and p^* are an optimal solution to P1, we know the following:

$$D^*p^* - \hat{c}(a^*, D^*) \ge D'p' - \hat{c}(a', D') \quad (10)$$

Combining Equations (8), (9), and (10) yields the desired result, which shows that the profitability of m^*, x^*, and p^* is close to the optimal profitability:

$$D^*p^* - C(x^*, D^*) > D'p' - C(x', D') - 2\varepsilon \quad (11)$$

The analysis shows that the quality of this separation depends upon the marketing group's ability to identify feasible attribute combinations and to estimate costs. If marketing selects an infeasible attribute combination, then it will be impossible to design a satisfactory product. If the cost estimates are inaccurate, then the resulting product will be suboptimal.

5 Example: Motor Design

A universal electric motor example originally developed by Simpson [42] will be used to demonstrate the concept of separation. Simpson used this example to demonstrate new techniques in product family design. The following example ignores the product family aspect and deals with only a single motor design that should meet given power and torque requirements.

The optimization model for the universal electric motor problem includes nine variables (eight design variables and the price), four customer attributes, twenty-three intermediate engineering attributes, and seven fixed engineering parameters. Table 1 lists the design variables, their lower and upper bounds, and units. The price p is in dollars.

Table 1: Bounds on Design Variables.

Variable	Definition	Lower bound	Upper bound	units
N_c	Turns of wire (armature)	100	1500	turns
N_s	Turns of wire (stator), per pole	1	500	turns
A_{aw}	Cross sectional area of armature wire	0.01	1.0	mm2
A_{sw}	Cross sectional area of stator wire	0.01	1.0	mm2
r_o	Outer radius (stator)	0.01	0.1	m
t_s	Thickness (stator)	0.0005	0.01	m
I	Electric current	0.1	6	A
L	Stack length	0.01	0.2	m

Appendix A describes the engineering parameters and engineering attributes. The derivations of the equations and other background information on universal electric motors can be found in [42, 43]. The four customer attributes are the torque T (in Nm), the power P (in watts), the efficiency η, and the mass M (in kg). They are calculated from the design variables and the engineering attributes as follows:

$$\begin{aligned} T &= K\varphi I \\ P &= P_{\text{in}} - P_{\text{out}} \\ \eta &= P / P_{\text{in}} \\ M &= M_w + M_s + M_a \end{aligned} \quad (12)$$

As in Simpson *et al*. [43] we take as given two targets for the power and torque: $P = 300$ W and $T = 0.05$ Nm. There is also a constraint due to the geometry of the motor:

$$r_o > t_s \quad (13)$$

The cost equations were originally derived in Wassenaar and Chen [44]. We simplified the equations slightly. The design cost

C_D is assumed to be fixed at $500,000 while the material cost C_M, labor cost C_L, and capacity cost C_K vary with demand and engineering attributes. (Due to inefficiencies, the capacity cost increases quadratically when the production quantity deviates from the optimal production capacity.)

$$\begin{aligned} C_D &= 500{,}000 \\ C_M &= d\left(M_w C_c + \left(M_s + M_a\right) C_s\right) \\ C_L &= \frac{3}{7} C_M \\ C_K &= 50\left(\left(d - 500{,}000\right)/1000\right)^2 \end{aligned} \tag{14}$$

To predict demand, we used discrete choice analysis (DCA) and spline functions that we created to model customer preference. The total demand (d) is the population size (s) multiplied by the probability that a consumer will select a particular design (i.e. estimated market share). We set s = 1,000,000. The following equation shows the common DCA equations developed in [45, 46].

$$\begin{aligned} d &= s e^{v}\left[1 + e^{v}\right]^{-1} \\ v &= \Psi_1(M) + \Psi_2(\eta) + \Psi_3(P) + \Psi_4(T) + \Psi_5(p) \end{aligned} \tag{15}$$

The attraction value ν is calculated from the following spline functions for the mass, efficiency, power, torque, and price:

$$\begin{aligned} \Psi_1(M) &= 0.5\left(1 - M\right) \\ \Psi_2(\eta) &= \eta - 0.5 \\ \Psi_3(P) &= -\left(1 - \frac{P}{300}\right)^2 \\ \Psi_4(T) &= -\left(1 - \frac{T}{0.05}\right)^2 \\ \Psi_5(p) &= \frac{25 - 4p}{15} \end{aligned} \tag{16}$$

The profit Π of a motor design is a function of the demand (d), price (p), and the costs discussed above.

$$\Pi = dp - \left(C_D + C_M + C_L + C_K\right) \tag{17}$$

This formulation is related to the notation of Section 4 as follows. The set of configurations has only one element, so the configuration is given. The set $X(m)$ is defined by the upper and lower bounds, the engineering attributes, and the geometry constraint shown in Equation (13). The attributes a are the torque, power, efficiency, and mass. The demand function $q(a, p)$ is determined by the spline functions and the demand functions in Equations (15) and (16). The cost function $C(x, D)$ is determined by the sum of the costs described by Equation (14). Finally, the utility $u(\Pi) = \Pi$.

We conducted numerical tests using different separations of the motor design problem in order to compare their solution quality to the solutions found by solving all-at-once formulations of the problem. (Note that one could consider the all-at-once formulations as "trivial" separations.) The decision networks corresponding to the formulations and separations are shown in Figure 3.

The first formulation (A1) is an all-at-once formulation that determines values for the design variables and price in order to maximize profit. Note that the terms ψ_3 and ψ_4 in the demand model penalize deviations from the power and torque targets.

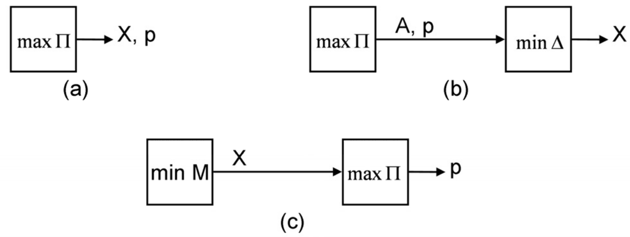

Figure 3. Decision networks (X = the vector of design variables). (a) The all-at-once formulations (A1 and A2) maximize profit. (b) Separation S1 finds the most profitable attribute values and price and then sets the design variables to satisfy them. (c) Separation S2 finds the best design and then sets the price to maximize profit.

The second formulation (A2) is an all-at-once formulation that determines values for the design variables and price in order to maximize profit while enforcing the power and torque requirements (by including them as equality constraints).

Our first separation (S1) has two subproblems, like the one analyzed in Section 4. The first subproblem determines values for the mass, efficiency, and price in order to maximize profit while enforcing the power and torque requirements. This subproblem requires a surrogate cost function that relates the total cost to the customer attributes (power, torque, mass, and efficiency) and price. The first key issue is the material cost, which is a function of the three components' masses, which are not available in this subproblem. Therefore, in the objective function, we replace C_M with $\bar{C}_M = dM\bar{C}$, where $\bar{C}$ is some "average" material cost. Other surrogate cost functions might be possible.

The relationship between mass and efficiency is another important issue. Not all combinations of values for mass and efficiency are feasible; in general, a higher efficiency motor will require more mass. Creating a surrogate constraint for first subproblem in S1 is important to finding a practical solution (one that can be realized) and is critical to employing this particular separation. We will consider two different surrogate constraints in our experimental results.

After solving the first subproblem in S1, we need to determine values for the eight design variables in order to minimize the deviation from the four attribute targets ($M^*, \eta^*, P^* = 300$ and $T^* = 0.05$). We can immediately satisfy the efficiency target by setting the current equal to the value $I^* = P^* / \left(V_t \eta^*\right)$. The second subproblem in S1 then finds values for the other seven design variables in order to minimize a deviation (or loss) function Δ that includes deviations from a target total resistance, the target torque, and the target mass. Achieving these three targets will satisfy all four customer attribute targets.

$$\begin{aligned} \Delta_1 &= \left(\frac{\left(R_a + R_s\right) I^{*2}}{P^*/\eta^* - P^* - 2I^*} - 1\right)^2 \\ \Delta_2 &= \left(\frac{T}{T^*} - 1\right)^2 \\ \Delta_3 &= \left(\frac{M}{M^*} - 1\right)^2 \\ \Delta &= \Delta_1 + \Delta_2 + \Delta_3 \end{aligned} \tag{18}$$

The second separation (S2) also has two subproblems. The first subproblem in S2 determines values for the eight design variables while satisfying the power and torque requirements. Different versions of this subproblem use different objective functions, including minimizing mass, maximizing efficiency, and minimizing material cost. Given values for the design variables, which set the four customer attributes, the second subproblem in S2 determines the price in order to maximize profit.

6 Experimental Results

As mentioned above, the purpose of the numerical experiment was to compare the quality of the solutions that the separations generate to those of the all-at-once formulations and to get some insight into the computational effort. All of the optimization problems were solved using the fmincon function in the MATLAB optimization toolbox. Ten initial designs (listed in Appendix B) were found by solving the second subproblem in S1 for ten different randomly-generated combinations of the four customer attributes (power, torque, efficiency, and mass).

For separation S1, we considered four scenarios formed by combining two different sets of surrogate constraints with two values for average material cost. The average material cost $\bar{C}$ was set to \$1.5 per kilogram and \$2 per kilogram. (Note that both values are between the parameters C_c and C_s.) The first set of surrogate constraints (CS1) had the following equations:

$$\begin{aligned} &\eta \le 0.97 \\ &M \ge 0.15 + 0.05/(1-\eta) \end{aligned} \qquad (19)$$

The second set of surrogate constraints (CS2) had the following equations:

$$\begin{aligned} &\eta \le 0.97 \\ &M \ge 0.02/(1-\eta) \end{aligned} \qquad (20)$$

Tables 2, 3, and 4 show the experimental results for each separation. (Because the subproblems have locally optimal solutions, we solved them with multiple initial points and report the best solution that was found.) The profit of the solution to A1 is slightly higher than the profit of the A2 solution (which is taken as the benchmark), but the A1 solution (P = 315 W and T = 0.0472 Nm) also misses the power and torque targets. The A2 formulation requires many more iterations. The quality of the solution found by separation S1 depends greatly upon the surrogate constraint set. The best solution is found using CS2, which allows mass to become smaller (which is desirable) and thus includes more of the solution space. Of course, it takes more effort to search this larger space. Changing the average material cost does not affect the solution quality as much. Separation S2 shows that, in this case, designs that maximize efficiency (one solution reached nearly 96%) are not as profitable as designs that minimize the material cost or the mass (which are closely related). Note that the high-efficiency solution has a very large mass, which increases costs and reduces profit significantly compared to the low-mass and low-cost designs.

Considering separation S1 in light of the results in Section 4, we note that the constraint sets do not include all of the feasible attribute combinations; indeed, some more profitable combinations are left out. (That is, the set A is incomplete.) Moreover, using a simpler material cost function ($\bar{C}_M = dM\bar{C}$) introduces an approximation in the surrogate objective function $\hat{\Pi}$. Thus, this separation does not satisfy the conditions for an exact separation. In the worst case (when the stator and armature have a mass of 4.5 kg, the windings have no mass, the efficiency equals 0.96, and the price equals 0 to increase demand), the difference between $\bar{C}_M$ and C_M is over \$2,980,000. For the best solution found for formulation A1, when $\bar{C}$ = \$1.5 per kilogram, the difference between $\bar{C}_M$ and C_M is only \$23,082.

Table 2: Results for each formulation and separation

	Scenario	Function Evaluations (average)	Profit (\$)	Deviation from A2 (%)
A1		579	4,000,518	0.29
A2	P = 300, T = 0.05	65037	3,989,027	-
S1	CS1, $\bar{C}$ = 1.5	181	3,317,975	16.82
	CS1, $\bar{C}$ = 2	168	3,580,730	10.24
	CS2, $\bar{C}$ = 1.5	306	3,935,065	1.35
	CS2, $\bar{C}$ = 2	306	3,935,521	1.34
S2	Max Efficiency	312	3,040,692	23.77
	Min Cost	554	3,379,202	15.29
	Min Mass	834	3,379,029	15.29

Table 3: Best design found in each formulation and separation

	N_c	N_s	A_{aw}	A_{sw}
A1	610.6097	285.0453	0.184783	0.184783
A2	655.1343	305.7057	0.180135	0.018042
S1.1	971.5219	56.1535	0.247552	0.094276
S1.2	483.5268	223.3108	1.0000	0.054794
S1.3	391.2846	234.6498	0.235729	0.153379
S1.4	377.6240	177.9033	0.167358	0.172152
S2.1	1280.116	385.2213	0.902201	1.0000
S2.2	374.4015	144.2613	0.042322	0.042731
S2.3	375.9264	143.2410	0.045773	0.039134

	r_o	t_s	l	L	p
A1	0.0100	0.004451	3.184614	0.0100	9.09
A2	0.0100	0.004445	3.054586	0.0100	9.08
S1.1	0.015036	0.0100	3.394331	0.032943	8.42
S1.2	0.018205	0.0100	3.483778	0.0100	8.41
S1.3	0.011727	0.003904	3.056057	0.015274	9.03
S1.4	0.010499	0.001617	3.101400	0.018867	9.01
S2.1	0.010704	0.007590	2.729624	0.0100	8.83
S2.2	0.0100	0.004645	6.0000	0.0100	7.78
S2.3	0.0100	0.004630	6.0000	0.0100	7.78

Table 4: Attributes of best design found in each formulation and separation

	T	P	η	M
A1	0.0472	315	0.8608	0.1026
A2	0.05	300	0.8540	0.1063
S1.1	0.05	300	0.7686	0.3661
S1.2	0.05	300	0.7498	0.3074

S1.3	0.05	300	0.8536	0.1366
S1.4	0.05	300	0.8411	0.1259
S2.1	0.05	300	0.9557	0.5583
S2.2	0.05	300	0.4348	0.0330
S2.3	0.05	300	0.4348	0.0331

7 Discussion: Engineering Design Process

The results above show that a design optimization problem can be replaced by a set of subproblems. We now turn to engineering design processes. Separation provides a perspective in which engineering design processes can be considered as heuristics for the problem of finding the most valuable design. From this perspective, separation is a model for a certain class of engineering design processes.

We will use the term *progressive design process* to describe an engineering design process that creates a product or system design through a series of distinct phases. (Thus, this term would not cover prototype-based design processes that iterate through generate-build-test cycles.) The phases generate intermediate results by making decisions about different aspects of the design and generating increasingly detailed information. (The name reflects the similarity to a progressive die, which makes an increasingly complex part through a series of punches.) Pahl and Beitz [25], Asimow [47], Ullman [48], and Ulrich and Eppinger [49] are among those presenting progressive design processes.

Progressive design processes emphasize the movement from one phase to another and the intermediate results that are generated. A progressive design process can be viewed as a heuristic for the value optimization problem discussed at the opening of this paper. For instance, if we consider the design process presented by Pahl and Beitz [25], one part of the process is described as optimizing the principle (or concept); another optimizes the layout, form, and material; and another optimizes the production. Moreover, the process is based on a general problem-solving process and ends with a “solution.” It seems clear that the entire process is concerned with finding a feasible and valuable system design, even if optimality is not guaranteed.

Previous research has developed models of design processes that focus on the activities that need to be done, as in Gantt charts, the PERT and critical path methods, IDEF, the design structure matrix, Petri nets, and signposting [50]. Such models have been used to estimate the cost and duration of design processes [51-55]. The approach taken in this paper provides a way to consider the quality of the design process: how good is the solution that it creates? Answering this question would seem to be a way to extend the principles of decision-based design (including the idea that design should find the most valuable product) from a single decision to a design process.

This paper has presented two ways to evaluate the quality of a progressive design process by modeling it as a separation of a design optimization problem. The separation of the DBD framework corresponds to a simple design process in which marketing experts determine the product’s price and the attribute values that the product should have; then the engineers have to find the lowest cost design that can meet these targets. Moreover, it indicates mathematically that a progressive design process is a reasonable way to design a product or system, provided that the subproblems are appropriately formulated. It is not necessary to formulate and solve the problem as an integrated whole. The motor design results give additional examples of separations and demonstrate the importance of choosing appropriate surrogate constraints and objective functions.

The proposal to use separations to evaluate design decision-making is in the spirit of research into using game theory concepts to represent design processes, including [26, 30, 56, 57, 58]. Some separations correspond exactly to cooperative games, non-cooperative games, and Stackelberg games. However, separations are not limited to these special cases. The analysis of separations studies not only changes in the structure of the separation but also changes to the subproblems’ constraints and objectives; these are not taken as given.

This perspective of engineering design is not in conflict with the use of concurrent engineering, in which cross-functional teams consider downstream issues (especially those related to manufacturing) throughout the entire design process. The use of concurrent engineering creates a new separation by modifying the objectives and constraints used to make design decisions and by changing when decisions are made (e.g., some process design activities may be started earlier). However, there is still a separation because the design process is still divided into different subproblems.

Finally, we recognize that creating a separation that corresponds to a real product development process and analyzing its quality are difficult challenges. We are still learning how to do both of these steps, and the results presented here are only the beginning of studying this approach. Progress toward this goal will help us better understand and improve product development processes.

In particular, the analysis of Section 4 assumed that there was no uncertainty in order to simplify the exposition. Considering the expected utility of profit or changing other aspects of the original problem formulation would lead to different conditions for exact and approximate separations.

8 Conclusions

This paper introduces an approach for solving design optimization problems by replacing them with a set of subproblems, which we call a separation. Separation provides a different way to find solutions to design optimization problems. However, a separation must be carefully designed to provide a valuable solution. This paper has shown how separation can be used to solve the decision-based design framework and a motor design problem. If the subproblems are correctly formulated, the separation yields an optimal solution. The quality of approximate separations depends upon the constraints and objectives used in the subproblems. Because it avoids iteration of decomposition, a separation may reduce the time needed to find a feasible solution, which could be useful when development time is limited and the designer is willing to accept a suboptimal, feasible design. Such a separation is helpful. However, a separation that fails to find a feasible solution must be replaced with a better separation.

The usefulness of some separations is not meant to justify all heuristic design methods. Instead, it highlights the need to evaluate engineering design processes as a whole and to validate individual design tools and methods by considering their role as heuristics for subproblems in the separation of the design optimization problem. They can be evaluated properly only in the context of the design process in which they are used. Otherwise, they may be finding excellent solutions to the wrong problem. In the future perhaps we will see a careful analysis of various design methods that considers their usefulness as part of a design process and the quality of the solutions that are generated.

Adopting a general concept of optimization as a way to view progressive design processes places this research among other work that views design as a mathematical problem-solving process or a rational decision-making process. However, there is also value in other perspectives, including those that view design as a creative process, a cognitive process with divergent and convergent thinking, or a social process involving teams and various languages or representations for communication. (See, for example, Dym *et al*. [59] for more about these other perspectives.) Future research will need to study the relationships between these perspectives and the one taken in this paper.

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Appendix A. engineering parameters and attributes
Engineering Parameters

Length of air gap l_g = 7.0 × 10-4 m

Terminal voltage V_t = 115 V

Resistivity of copper ρ = 1.69 × 10-8 $Ohms \bullet m$

Permeability of free space μ_o = 4 π × 10-7 H/m

Number of stator poles p_{st} = 2

Cost of copper C_c = 2.2051 $\$/kg$

Cost of steel C_s = 0.882 $\$/kg$

Density of copper δ_c = 8,960 kg/m^3

Density of steel δ_s = 7,861.09 kg/m^3

Engineering Attributes

Magnetizing intensity [Ampere turns/ m]

$$H = N_c I / (l_c + l_r + 2l_g)$$

Mean path length within the stator [m]

$$l_c = \pi(2r_o + t_s)/2$$

Diameter of armature [m] $l_r = 2(r_o - t_s - l_g)$

Input power [W] $P_{in} = V_t I$

Power losses due to copper and brushes [W]

$$P_{out} = I^2(R_a + R_s) + 2I$$

Armature wire length [m] $l_{aw} = (2L + 2l_r)N_c$

Stator wire length [m] $l_{sw} = p_{st}(2L + 4(r_o - t_s))N_s$

Armature wire resistance [Ohm] $R_a = \rho l_{aw} / A_{aw} \times 10^6$

Stator wire resistance [Ohm] $R_s = \rho l_{sw} / A_{sw} \times 10^6$

Mass windings [kg] $M_w = (l_{aw}A_{aw} + l_{sw}A_{sw})\delta_c \times 10^{-6}$

Mass of stator [kg] $M_s = \pi L(r_o^2 - (r_o - t_s)^2)\delta_s$

Mass of armature [kg] $M_a = \pi L(r_o - t_s - l_g)^2 \delta_s$

Motor constant [dimensionless] $K = N_c / \pi$

Magneto magnetic force [A turns] $\Im = N_s I$

Magnetic flux [Wb] $\varphi = \Im / \Re$

Total reluctance [A turns/ Wb] $\Re = \Re_s + \Re_a + 2\Re_g$

Stator reluctance [A turns/ Wb] $\Re_s = l_c / (2\mu_{steel}\mu_o A_s)$

Armature reluctance [A turns/ Wb]

$$\Re_a = l_r / (\mu_{steel}\mu_o A_a)$$

Reluctance of one air gap [A turns/ Wb]

$$\Re_g = l_g / (\mu_o A_g)$$

Cross sectional area of stator [m^2] $A_s = t_s L$

Cross sectional area of armature [m^2] $A_a = l_r L$

Cross sectional area of air gap [m^2] $A_g = l_r L$

Relative permeability of steel [dimensionless]

$$\mu_{steel} = -0.2279H^2 + 52.411H + 3115.8 \quad H \le 220$$

$$\mu_{steel} = 11633.5 - 1486.33\ln(H) \quad 220 < H \le 1000$$

$$\mu_{steel} = 1000 \quad H > 1000$$

APPENDIX B. Initial Designs.

Table B.1: Initial designs for Separations S1 and S2.

N_c	N_s	A_{aw}	A_{sw}	r_o	t_s	I	L	p
622.4461	10.2789	0.1798	0.0855	0.0296	0.0087	6.0175	0.0253	7
971.5237	41.0603	0.1669	0.2469	0.0299	0.0036	3.1220	0.0143	7
622.4460	10.2836	0.2004	0.1590	0.0294	0.0051	6.0691	0.0254	7
971.4546	52.0387	0.2522	0.9947	0.0113	0.0021	3.1499	0.0273	7
373.0639	21.7458	0.2813	1	0.0159	0.0008	4.0786	0.0721	7
971.5356	48.7206	0.2976	0.9760	0.0142	0.0025	2.1452	0.0454	7
383.8721	33.1335	0.2371	1	0.0103	0.0005	3.8343	0.0794	7
971.5287	56.2628	0.2115	0.2162	0.0123	0.0013	2.6033	0.0304	7
483.5892	223.6510	0.0644	1	0.0243	0.0005	3.3765	0.0100	7
970.7074	128.5235	0.2849	0.2776	0.0181	0.0098	2.6027	0.0154	7

Figure 1. (a) A typical decomposition scheme has multiple first-level subproblems (P1, P2, P3) that receive inputs from a second-level problem (P*), which also coordinates their solutions. (b) Separation yields a set of subproblems. Solving one provides the input to the next.

Figure 2. (a) The decision network for the integrated design optimization model. (b) The decision network for the separation.

Figure 3. Decision networks (X = the vector of design variables). (a) The all-at-once formulations (A1 and A2) maximize profit. (b) Separation S1 finds the most profitable attribute values and price and then sets the design variables to satisfy them. (c) Separation S2 finds the best design and then sets the price to maximize profit.

Table 1: Bounds on Design Variables.

Table 2: Results for each formulation and separation

Table 3: Best design found in each formulation and separation

Table 4: Attributes of best design found in each formulation and separation

Table B.1: Initial designs for Separations S1 and S2.