Proceedings of the 2012 Industrial and Systems Engineering Research Conference
G. Lim and J.W. Herrmann, eds.

Incorporating Attribute Value Uncertainty into Decision Analysis

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Abstract

Attribute value uncertainty is present in a decision when the consequences are characterized by uncertain estimates of the underlying, unknown true values. Attributes defined based upon measurement data, experimental data, or survey data are subject to attribute value uncertainty. Uncertainty in decision modeling has received significant attention from the community, most notably the methods pertaining to risky decisions and decision ambiguity. These methods, however, proceed with only point estimates representing the attribute values, ignoring the uncertainty in the estimates, which can adversely impact the identification of the best decision alternative. This work describes a new decision analysis method that incorporates attribute value uncertainty by leveraging the systematic mechanism of the Monte Carlo method. This approach allows for the uncertainty in the attribute values to be propagated through the decision model and reflected in the resulting decision parameter. Several techniques, including stochastic dominance and majority judgment, can be used to identify the most desirable alternative based on the resulting uncertain decision parameter. The approach is illustrated by an example driven by a recent U.S. Department of Homeland Security program to investigate detection systems for screening individuals and their baggage for radioactive materials at U.S.-based international arrival airport terminals.

Keywords
Decision Analysis, Uncertainty, Homeland Security

1 Introduction

When considering major procurement decisions for equipment, organizations often experimentally evaluate the performance of the alternatives being considered before making a selection. The results of these tests, together with other characteristics of the equipment such as cost, form the basis of the selection decision. Decision analysis tools may then be used to identify the best alternative.

The evaluation goal is to determine each alternative's true performance. Because the evaluations are based on a limited number of experimental trials, however, only estimates of the true performance can be obtained. Further, these estimates contain uncertainty. To formulate a comprehensive decision model that can withstand scrutiny, the uncertainty associated with these estimates of performance must be included in the decision model.

From a decision analysis framework, a decision with multiple characteristics of interest is modeled as a multiattribute decision. The equipment performance and the other equipment characteristics of interest are the attributes. Although the values of some attributes for an alternative may be known with certainty, the values of other attributes (including those estimated by evaluation experiments, tests, surveys, and other measurements) will be uncertain. We refer to the uncertainty associated with the attribute values as *attribute value uncertainty*.

This paper proposes a method to incorporate attribute value uncertainty into the decision model through the use of the systematic mechanism of the Monte Carlo method. We present several techniques that can be used to identify the most desirable alternative based on the uncertain decision parameter that results from the propagation of the attribute value uncertainty. We conclude with an illustration of this approach driven by a recent U.S. Department of Homeland Security program to investigate detection systems for screening individuals and their baggage for radioactive and nuclear materials at U.S.-based international arrival airport terminals.

2 Uncertainty in Decision Making

Uncertainty in decision making is by no means a new concept. In the middle of the last century, the works of von Neumann and Morgenstern [1], Savage [2], and Luce and Raiffa [3] provided the foundation to formally address, through analytical methods, the decision problem in which the consequence of the action cannot be realized until some uncertain event is resolved; i.e. decisions with risk. The method of expected utility theory first formalized by von Neumann and Morgenstern [1] and later put into practical terms for multiattribute decision analysis in the award winning text of Keeney and Raiffa [4] provides a structured approach to decision analysis when uncertain events exist through the consideration of the probability distributions over the potential outcomes of the uncertain event.

A further aspect of uncertainty in decision making was presented in the 1960s by Daniel Ellsberg, best known in the decision analysis community for his now infamous Ellsberg Paradox (see [5] for a well described presentation). The term decision ambiguity in the decision analysis context was first defined by Ellsberg [6] and has since been generalized and elaborated by many. Frisch and Baron [7] present a nice definition: "Ambiguity is uncertainty about probability, created by missing information that is relevant and could be known." In short, decision ambiguity refers to the uncertainty in the probability distribution of a risky decision.

Other examples of uncertainty in decision making include uncertain decision-maker preference structures and uncertainty in attribute weights [8-11]. Although these methods encompass many aspects of uncertainty in decision making, they all presume that the consequences (described by the attribute values) are precisely defined and neglect any uncertainty that may exist in their assessment. We are unaware of any work that explicitly considers the uncertainty that may be present in the assessments of the consequences (the attribute value uncertainty).

3 Problem Statement

This paper considers a decision that has n decision alternatives $a_1, a_2, ..., a_n$ (which form the set A). The consequences that result when alternative a_i is selected are described by m attributes and their associated attribute values x_{i1} to x_{im} . The attribute value uncertainty is the uncertainty in the values of x_{ij} .

Decision-makers often consider decision alternatives that have consequences that are described by uncertain attributes value estimates. If a decision problem includes attributes whose values are determined by means of sampling and measurement, attribute value uncertainty exists. When these attribute values are provided only as point values, the decision-maker must move forward under the assumptions that the values are accurate and that the level of uncertainty associated with each alternative is equivalent.

Scientists and engineers are trained to quantify and report the uncertainties in assessments, including measured physical quantities such as mass and performance characteristics such as the probability of system failure. These uncertainties may be developed through a variety of techniques including data-based methods and subjective expert opinions.

A comprehensive decision model for the selection from $a_1, a_2, \ldots, a_n$ of the alternative that, given the decision-maker's preferences, maximizes his satisfaction must include the uncertainty in the attribute values used to describe each alternative. We shall propose an approach to incorporate the attribute value uncertainty into the decision model and then consider various rules for evaluating the decision-maker's satisfaction with each alternative.

In this situation, the decision-maker faces the risk of selecting an alternative that is not the best one, which could be identified if no attribute value uncertainty existed. The decision-maker may have to make a tradeoff, as in other settings involving risk, between alternatives whose performance is described to range from very well to poor (that is, there is a large amount of uncertainty about their performance) and other alternatives whose performance is described as neither very well nor poor (that is, there is less uncertainty about their performance). The proposed methods should help the decision-maker understand this tradeoff and make a better decision.

4 Approach

This section describes the approach that we will use to identify the best of a set of alternatives that have attribute value uncertainty. For any alternative, the point estimates for each attribute can be used to evaluate the decision model. The alternative with the largest resulting decision parameter value (e.g. value or measurable value in a decision under certainty, utility in a risky decision [12]) would be considered the alternative most fitting given the decision-maker's preferences. We will call this the *expected value approach*. Because this approach fails to consider the attribute value uncertainty, it may fail to select the alternative that maximizes the decision-maker's satisfaction. Thus, we propose the following approach, which augments the expected value approach by incorporating the attribute value uncertainty as follows:

1. Identify and develop the alternatives $a_1, a_2, \ldots, a_n$ and the attributes.
2. Model the uncertainty of each attribute value for each alternative. Let $F_{ij}(x)$ be the probability distribution for x_{ij}, alternative *i*'s value for attribute *j*, $i = 1, \ldots, n$, $j = 1, \ldots, m$.
3. Randomly sample the attribute values based on the associated uncertainty models to generate *R* realizations for each alternative. Each realization has a single value for each of the relevant attributes. Let x_{ijr} be alternative *i*'s value for attribute *j* in realization *r*, $i = 1, \ldots, n$, $j = 1, \ldots, m$, $r = 1, \ldots, R$.
4. Define the decision model based on the decision-maker's preference structure and the realizations of the attribute values. This includes defining the individual value functions and attribute weights.
5. Propagate the attribute value uncertainty through the decision model and onto the decision parameter. That is, for each alternative and each of its *R* realizations, calculate the corresponding decision parameter value. The result is a distribution of *R* decision parameter values for each alternative. Let y_{ir} be alternative *i*'s value for the decision parameter in realization *r*, $i = 1, \ldots, n$, $r = 1, \ldots, R$.
6. Use a decision rule to identify the most desirable alternative based upon distributions of the decision parameter values.

As Step 1 above is the basis for developing any decision model [4] our discussion will focus on approaches to model the attribute value uncertainty (Step 2) and selecting an alternative based on a collection of decision parameter distributions (Step 6).

4.1 Modeling Attribute Value Uncertainty

Several approaches that may be used to model attribute value uncertainty include the parametric frequentist approach, the non-parametric bootstrap approach, and the Bayesian (subjective probability) approach. Our general decision analysis approach to incorporate attribute value uncertainty can be used with any of these approaches of modeling attribute value uncertainty, which we briefly describe here.

The frequentist's statistical confidence interval [13] is based upon the observed data and provides an interval within which the true value of the quantity or parameter of interest is believed to fall. The $(1-\alpha)\times 100\%$ confidence interval, where the value of α is often a small value such as 0.05, indicates that if the experiment were repeated a large number of times and a $(1-\alpha)\times 100\%$ confidence interval were calculated in each case, the true value of the parameter of interest will be captured by $(1-\alpha)\times 100\%$ of these intervals on average. For further general discussion of statistical confidence intervals, see Hahn and Meeker [14].

The non-parametric bootstrap method relies upon resampling the observed data to model the attribute value uncertainty. Introduced by Efron [15], the bootstrap is "a computer-based method for assigning measures of accuracy to statistical estimates" [16]. Given observed data $x_1, x_2, \ldots, x_n$, a typical application of the bootstrap technique involves generating a bootstrap sample of size *n* with replacement from the observed empirical distribution, computing the value of the parameter of interest, repeating this process to create an approximation for the distribution of the parameter, and obtaining its desired statistical properties.

An alternative to the frequentist view, the Bayesian (or subjective) interpretation is that a probability is a measure of degree of belief [17], which allows for probabilities to be associated with the unknown parameters. Generally speaking, an initial degree of belief, described in terms of a probability distribution called the prior, is updated using Bayes' Theorem when new data are observed to produce a new degree of belief called the posterior distribution. The posterior distribution provides a method to model one's knowledge of the true value of the attribute and captures the uncertainty in the attribute value estimate provided by the sampling or measurement results [18].

4.2 Selecting an Alternative

Traditional decision analysis approaches clearly identify the most desirable alternative. This property should not be lost when expanding the model to be more comprehensive by including attribute value uncertainty. The result of propagating attribute value uncertainty is a set of decision parameter values that are described by distributions. Thus, selecting an alternative changes from a simple ordering exercise to a comparison of distributions. This section discusses three approaches to compare the resulting decision parameter distributions: rank 1, stochastic dominance, and majority judgment.

In each realization, each alternative has one value for the decision parameter. If we consider the realizations one at a time and examine the decision parameters for all of the alternatives in that realization, then the alternatives can be ranked by the decision parameter, and the most desirable alternative (the one ranked first) can be identified. (If multiple alternatives tie for first in a realization, all of those so tied are considered as ranked first.) The number of realizations in which an alternative is ranked first (its *rank 1 value*) describes the relative desirability of that alternative. An alternative's rank 1 value can vary from 0 (it is never ranked first) to R (it ranked first in every realization). We use this value in the decision rule that selects the alternative with the greatest rank 1 value.

Our second approach builds upon the concept of stochastic dominance for comparing distributions. In the following discussion Y_i and Y_j represent the decision parameters for alternatives i and j respectively. The distributions of these parameters are the ones generated by the R realizations.

Hadar and Russell [19] discuss stochastic dominance as an approach to predicting a decision-maker's choice between two uncertain events without knowledge of the decision-maker's utility function. They define two types of stochastic dominance: first-degree stochastic dominance and second-degree stochastic dominance.

First- degree stochastic dominance: Y_i stochastically dominates Y_j in the *first degree* if and only if

$$P\left[Y_i \le y\right] \le P\left[Y_j \le y\right] \qquad \forall y \tag{1}$$

That is, the value of the cumulative distribution for Y_i never exceeds that of Y_j for all $y \in Y$.

Second-degree stochastic dominance: When the support of Y_i and Y_j are contained in the closed interval $\left[a,b\right]$, Y_i stochastically dominates Y_j in the *second degree* if and only if

$$\int_a^t P\left[Y_i \le y\right]dy \le \int_a^t P\left[Y_j \le y\right]dy \qquad \forall t \in \left[a,b\right] \tag{2}$$

That is, the area under the cumulative distribution for Y_i is less than or equal to that of Y_j for all $t \in \left[a,b\right]$.

First-degree stochastic dominance is relevant in the absence of any restrictions on the unknown utility function other than monotonicity. Second-degree stochastic dominance is more restrictive in that the results apply only when the unknown utility functions are concave, indicating a risk-averse decision-maker. Under these restrictions, if Y_i is found to stochastically dominate Y_j in either the first or second degree then alternative i is preferred to alternative j because alternative i will have a greater expected utility.

If there exists a single Y_i that stochastically dominates (first- or second-degree) Y_j, for all $j, i \neq j$, and, in at least one case, the inequality in Equation (1) or (2) is found to be a strict inequality, then alternative i can be selected with few underlying assumptions [19].

We will use the idea of stochastic dominance as a decision rule to select the alternative with a set of decision parameter values that stochastically dominates the others. It should be noted, however, that this rule may not produce a solution, and thus an alternative would not be identified for selection.

In particular, consider the decision parameter values for alternatives i and j. Both are sets of R values generated as discussed in Step 5 of the approach. Alternative i dominates alternative j if, for all values y, the number of values of the decision parameter Y_i that are not greater than y is less than or equal to the number of values of the decision parameter Y_j that are not greater than y. Similar conditions based on the idea of second-degree stochastic dominance can be verified computationally as well, but the details of this procedure are beyond the scope of this paper.

Finally, the majority judgment rule selects the alternative with the largest *majority-grade* as the most desirable alternative. The majority-grade is computed as the median (if R is odd) or the lower bound of the middlemost interval (if R is even) of the distribution of the R decision parameter values for each alternative. For breaking ties, we follow the scheme developed by Balinski and Laraki [20, 21]. If multiple alternatives have the same largest majority-grade, then a single majority-grade value is removed from the R realizations of the decision parameter for each alternative in the tie, and the majority-grade of the new distributions are calculated. If a tie again occurs, this process is repeated until a single alternative has the largest majority-grade.

5 Application

In 2008 the Domestic Nuclear Detection Office (DNDO) of the U.S. Department of Homeland Security (DHS) was congressionally mandated to work with the U.S. Customs and Border Protection (CBP) on the evaluation and improvements of the current radiation detection systems in U.S. based international airports. As a result of this mandate, the PaxBag pilot program was initiated to identify the best possible system design for detecting, identifying and localizing illicit radiological or nuclear (rad/nuc) material entering the United States through international passenger and baggage screening. Such a system would be implemented and operated by the CBP, who has complete control of the international arrival terminal, including the ability to control passenger flow in and about the terminal. This challenge was met by testing and evaluating, in a laboratory environment, available radiation detection equipment suitable for such an application, followed by an operational demonstration of the system that displayed the strongest potential for improved capability over currently deployed technology. The work presented in this paper focuses on the decision process to select the system to put forth for the operational demonstration. While the goal of the PaxBag pilot program was to develop a general solution, the discussion of this work is based on an evaluation pertaining to the Seattle-Tacoma (Sea-Tac) International Airport.

Although the analysis presented here is based upon actual data collected and the general preferences of the decision-makers involved, the results may not reflect the decision implemented by the DNDO and CBP as not all methods presented in this work were implemented in the original analysis. This is particularly true of the method presented to incorporate attribute value uncertainty, which had not been developed at the time the original analysis was performed.

Passengers arriving at U.S.-based international airport terminals are subject to two mandatory encounters with CBP agents. As illustrated in Figure 1, passengers exit the aircraft and choose one of several queues prior to the first CBP encounter at the immigration and passport control station. Upon exiting the passport control station, passengers proceed to the luggage carousel to claim any luggage they may have. With luggage in tow, passengers proceed to the second CBP encounter at one of several customs declaration stations before exiting the international arrival terminal for the main terminal of the airport. During each of these encounters, or at any other time within the international arrival terminal, passengers may be subject to further interrogation as deemed necessary.

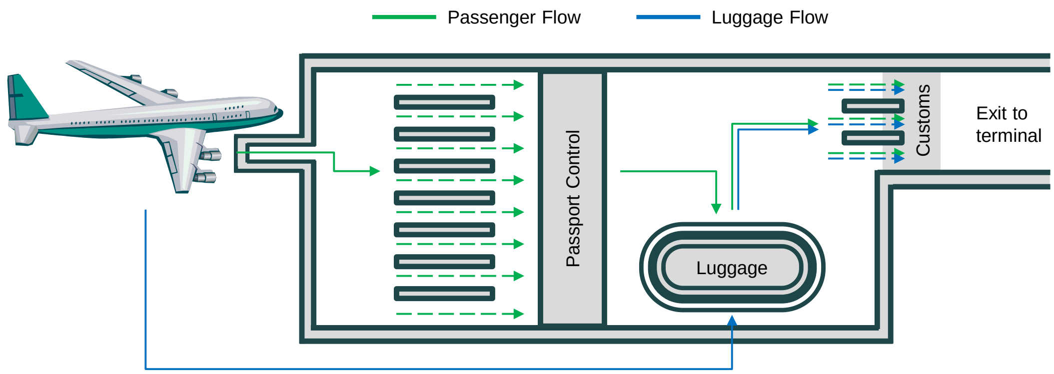

Figure 1: Generic passenger and luggage flow at a U.S.-based international arrival terminal

For the purposes of this decision analysis, a detection system is defined to contain two subsystems: passport control and customs declaration. Each subsystem will have one of three radiation detection sensor technologies: Permanently Mounted Pedestrian Portal 1 (PMPP 1), Permanently Mounted Pedestrian Portal 2 (PMPP 2), or Relocateable Stanchion System 4 (RS 4), and these sensors will be placed either at the passport control (customs declaration) booths or in the queues prior to the booths. The operational modes to be used with this sensor technology will be defined by the speed at which passengers (controlled by the CBP agents) move past the sensor. Each subsystem will use one of the following operational modes: a fast (2.5 m/s) walk, a slow (0.5 m/s) walk, a short (30 sec) dwell, or a long (2 min) dwell. Thus, there are 24 combinations of sensor technologies, sensor locations, and operational modes for each subsystem, which yields a total of 576 combinations. The detection system alternatives are the 576 possible combinations of these choices. The goal of the decision analysis is to select the detection system alternative that maximizes the value function as defined by the DNDO and CBP preference structure. Note that each detection system alternative includes one combination of sensor technology, sensor location, and operational mode for the passport control subsystem and one for the customs declaration subsystem.

This decision was modeled as a multiattribute decision under certainty, and thus a value function was used. It was assumed that the preference structure is such that the attributes are mutually preference independent and mutually difference independent; thus the multiattribute value function can be represented by the sum of single attribute value functions [22].

In the decision model, the three main objectives were (1) maximize material interdiction, (2) minimize the operational impact, and (3) minimize the cost. Sub-objectives and attributes were developed to support the main objectives. The first objective (which is explained in the next paragraph) comprised 45% of the total value. The second objective, which comprised another 45% of the total value, was a function of the additional delays that passengers may experience, the additional CBP officers needed to operate the detection system, and scores that reflect the effort of detection system set-up, interoperability, and required facility modifications. The third objective (cost) was 10% of the total value.

Those attributes of direct interest to this research were the sensor technology performance measures that supported the material interdiction objective. The decision model considered the performance of sensor technologies to detect and identify eleven rad/nuc sources of specific interest to DNDO, both carried by passengers and packed in baggage. Each of the 22 attributes was the probability that the alternative will successfully detect and identify that source. The value functions used take into account the fact that passengers will go through both detection subsystems, while checked baggage will go through only the second subsystem. In the overall value function, some sources had greater weights than others, but this depended upon the type of rad/nuc material and not upon whether it was carried by passengers or packed in baggage.

The ability of each sensor technology to detect and identify these sources under the different operational modes was experimentally evaluated in a laboratory setting, and the results were expressed as a probability. As these probabilities were based on experimental evaluations from a limited number of trials, they were only estimates for the sensor's true capabilities. All other attributes (those that supported the second and third objectives) were considered to contain no uncertainty.

To describe the uncertainty in the performance attributes we used a Bayesian approach. For each alternative and each of these attributes, we began with the assumption – or prior knowledge – that the true value of the attribute (the probability that a particular sensor technology can successfully detect and identify a particular source) lies between 0 and 1 with equal likelihood. This was represented by the $Uniform(0,1)$ prior distribution, which is equivalent to a $Beta(1,1)$ distribution. For each sensor technology and operational mode combination i and source j, the test results were treated as observations from a $Binomial(n_{ij}, p_{ij})$ distribution and were used to update the prior distribution. n_{ij} denotes the number of trials of sensor technology and operational mode combination i on source j, and p_{ij} denotes the fraction of those trials that the sensor technology successfully detected and identified the source. Given this information, the knowledge about the unknown detection probability (the prior distribution) was updated to create a posterior distribution. Because the $Beta(\alpha,\beta)$ distribution is the conjugate prior to the $Binomial(n,p)$ distribution, the posterior distribution is the $Beta(1+n_{ij}p_{ij}, 1+n_{ij}(1-p_{ij}))$ distribution. This posterior distribution describes the alternative's attribute value uncertainty.

Given the posterior distributions for each attribute for each alternative, we drew R = 1000 random samples from each of these distributions. The individual value functions and overall decision parameter were evaluated for each alternative for each of these 1000 realized samples. While our method to incorporate attribute value uncertainty prescribes that the individual value functions and associated weights be re-evaluated based on the range of performance observed in the random sampling, the DNDO and CBP decision-makers were not available to make these updates, thus the originally defined functions were used.

When analyzing the resulting 576 distributions of decision parameter values, we first found the minimum value of every alternative's decision parameter and identified the greatest of these minimum values. We then determined that 540 alternatives were dominated by one or more other alternatives in the following way: for each of these 540 alternatives, the maximum value of its decision parameter was less than the greatest minimum value. This left 36 non-dominated alternatives. The non-dominated alternatives and their associated value distributions are displayed in Figure 2. These 36 alternatives are the focus of the remainder of the decision analysis.

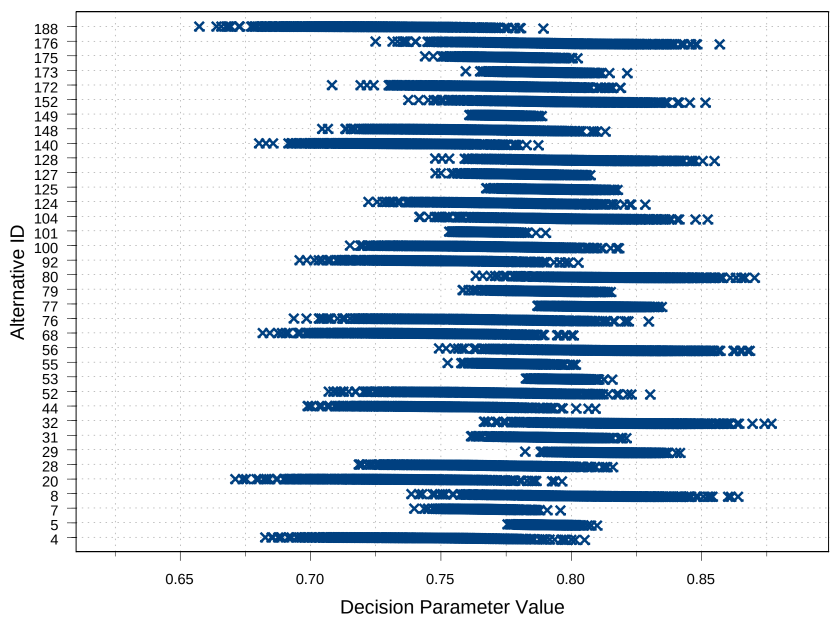

Figure 2: Decision parameter distributions for the 36 non-dominated alternatives

Given the distributions of decision parameter values for the 36 non-dominated alternatives, we applied the rank 1, majority judgment, and stochastic dominance selection rules to aid in the identification of the most desirable alternative. The order of the top 6 most desirable alternatives given the DNDO and CBP preference structure are identical under both the rank 1 and the majority judgment decision rules (see Table 1). Empirical cumulative distribution functions for these six alternatives are displayed in Figure 3. Among these six, no alternative stochastically dominates all others.

Table 1: The six best alternatives using the rank 1 and majority judgment decision rules

Alternative ID	Alternative Description	Rank 1 Value	Majority Grade
80	PMPP 1 - Passport Booth - 30 sec; PMPP 2 - Customs Booth - 30 sec	329	0.819
32	PMPP 1 - Passport Booth - 2 min; PMPP 2 - Customs Booth - 30 sec	273	0.818
29	PMPP 1 - Passport Booth - 2 min; PMPP 2 - Customs Booth - 0.5 m/s	233	0.812
77	PMPP 1 - Passport Booth - 30 sec; PMPP 2 - Customs Booth - 0.5 m/s	143	0.811
56	PMPP 1 - Passport Booth - 2.5 m/s; PMPP 2 - Customs Booth - 30 sec	20	0.809
128	PMPP 2 - Passport Booth - 2 min; PMPP 2 - Customs Booth - 30 sec	1	0.802

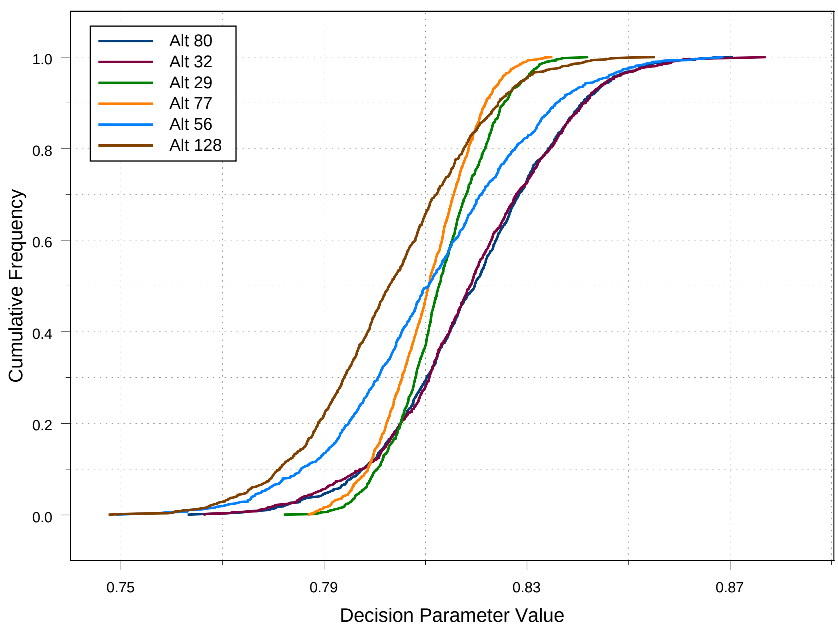

Figure 3: The distributions of the decision parameter values for the six alternatives with the greatest median values

Both the rank 1 and majority judgment selection rules identify alternative 80 as the most desirable, closely followed by alternative 32. From the empirical cumulative distribution functions, it is seen that these two alternatives have a nearly identical distribution of decision parameter values. Because they do not stochastically dominate all other alternatives, the decision-makers are faced with making a tradeoff between these alternatives (which have greater uncertainty) and alternatives 29 and 77, which have less uncertainty. In this case, the empirical cumulative distribution curves of alternatives 80 and 32 fall largely to the right of all other alternatives. Further, the DNDO and CBP decision-makers should take comfort in the fact that the top five alternatives displayed in Table 1 utilize the same sensor technologies (PMPP 1 and PMPP 2) and locations (within booths). The only difference between these alternatives is the operational modes. The inferiority of the alternatives that use the RS 4 technology reflects the overall value function (which includes cost and operational impact measures) and does not necessarily imply that this technology is less able to detect rad/nuc sources.

6 Summary and Conclusions

This paper presented an approach for making decisions when uncertainty exists in the values of the attributes being used to compare the alternatives and derive a decision parameter. This type of uncertainty is different from uncertainty about future events (risky decisions) and uncertainty about the probabilities of future events (decision ambiguity). Ignoring attribute value uncertainty, especially when it varies between alternatives, could lead to poor decisions.

The method presented here requires modeling the uncertainty about the attribute values and then propagating the uncertainty to determine the uncertainty in the decision parameter. The method is a Monte Carlo approach that randomly samples values of the uncertain attributes and computes the corresponding values of the decision parameter. Because it is not limited to specific types of distributions or decision models, it is a very general approach that can be used in a wide variety of settings. This paper presented three decision rules (rank 1, majority judgment, and stochastic dominance) for selecting an alternative based on its decision parameter distribution.

Unfortunately, it does require many samples of the uncertain attributes, which could be computationally expensive. The decision-maker must choose a decision rule to compare the distributions of the alternatives' decision parameters, and different rules may identify different alternatives as the "best."

This paper has used the example of selecting a radiation detection system to demonstrate the approach, but the approach can be used in any setting with attribute value uncertainty, including other models of attribute value uncertainty, other forms of the decision parameter, and other decision rules beyond those presented here.

The next question to consider is that of experimental design: when planning to initially obtain, or if the decision-maker has the opportunity to get more information about some attributes for some alternatives, which information would be most valuable? In this case, information provides value by removing uncertainty about an alternative's decision parameter estimate.

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