Decision Analysis Methods for Selecting Consumer Services with Attribute Value Uncertainty

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Abstract The basic risky decision is defined as a decision for which the outcome of an uncertain event, in addition to the alternative selected, defines the final consequence. Beyond the uncertain event, additional uncertainties can enter a decision including the uncertainty in the attribute values used to assess the decision consequences. When considering the selection of consumer products and services, formal and informal reviews of products and services are often used by consumers to estimate the level of satisfaction that will be received. When developing a decision model based on these data, attribute value uncertainty is often present and should be incorporated. In this chapter, we consider the uncertainty in the attribute values used to describe the possible consequences. We present several approaches to incorporate attribute value uncertainty into the decision analysis for choosing a roofing firm based on customer review data.

Introduction

As the digital age evolves, personal opinions can be found regarding just about anything with only a few clicks of a mouse. While some opinions – fashion, entertainment, political – may be appealing only in the eye of the beholder, others can be very useful for consumers. For example: reviews and ratings from previous

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purchasers are shared for products on many retail websites; travel websites often provide a forum for past travelers to convey their experiences and reviews of a hotel or resort; and the dining experiences of past patrons at restaurants nationwide can be found in abundance. Many rely on these reviews to provide, in a qualitative sense, a measure of the satisfaction expected to be received from the product or service being considered.

While these individual qualitative reviews are certainly useful, more quantitative summaries of consumers' opinions are available from non-profit organizations such as Consumers Union and the Center for the Study of Services. These organizations survey their members to gain real world knowledge of consumer products and services. The survey results are analyzed and provided as summary statistics for a variety of performance rating criteria. Consumers can use these results when selecting a product or service provider.

From a decision analysis framework, one might model such a consumer decision as a decision under certainty with either a single or multiple attributes. The products (or service providers) are represented as n alternatives. The m attributes for each alternative are a subset of the available performance rating criteria, and the attribute values are the quantitative values based on the survey results. For each of the n alternatives, the multiple attribute values are combined using a model of the decision-maker's preferences, which yields a decision parameter value that is the basis for the final selection.

The survey attempts to assess the true values of the performance rating criteria. Because the survey is based on limited data, the summary statistics used for the performance rating criterion value is an estimate of the true value and contains uncertainty. We refer to the uncertainty associated with the attribute values as *attribute value uncertainty*. When selecting a product or service provider, the decision-maker should consider how this uncertainty affects the relative desirability of the alternatives. Through an example of selecting a roofing firm based on data published by the Center for the Study of Services, we illustrate a proposed method to incorporate the attribute value uncertainty into the analysis of a decision.

Decision Analysis and Uncertainty

Decision problems can be classified on several dimensions. First, the decision-maker can be either an individual or a group. Second, the number of attributes used to describe the set of consequences can be a single attribute or can consist of multiple attributes. And finally, a decision problem may be classified under conditions of certainty, risk, or uncertainty. These conditions may be defined as follows [1]:

1. *Decisions under certainty*: Each alternative is known to lead invariably to a specific outcome.
2. *Decisions with risk*: Each alternative leads to one of a set of possible outcomes, where each outcome occurs with a probability assumed to be known by the decision-maker. These outcomes may be the result of an uncertain future event, for example.

3. *Decisions under strict uncertainty*: Each alternative leads to one of a set of possible outcomes, though nothing is known or can be stated about the probability of the occurrence of each outcome.

Ron Howard first coined the term decision analysis in a 1966 conference talk [2] where he provided a formal procedure for the analysis of decision problems. Active work in this field had been taking place for more than a decade prior to Howard's introduction of the terminology. Notable contributions during this time include works from von Neumann and Morgenstern [3], Savage [4], and Luce and Raiffa [1]. These works provided the foundation to formally address, through analytical methods, the decision problem for which the consequence of the action cannot be realized until some uncertain event is resolved; i.e., decisions with risk. The method of expected utility theory, first formalized by von Neumann and Morgenstern [3] and later put into practical terms for multiattribute decision analysis in the award winning text of Keeney and Raiffa [5], provides a structured approach to decision analysis when uncertain events exist through the consideration of the probability distributions over the potential outcomes of the uncertain event. Consider the following example of a risky decision for which the method of expected utility theory is applicable. A family is considering one of two outings during the upcoming weekend: a visit to a local museum or attending an outdoor Major League Baseball game. An uncertain event, a weekend rain storm, whose likelihood has been described with some probability by meteorologists, may lead to unfavorable consequences if the family chooses to attend the baseball game.

The driving force behind all decisions is the decision-maker's preference structure. Models to describe one's preference structure include ordinal value functions, measureable value functions, and utility functions. Dyer [6] provides a comprehensive overview of these models, their applications and underlying assumptions, and assessment methods. In brief, ordinal value functions are applicable in decisions under certainty. They lead to a rank ordering of the decision alternatives, but do not indicate magnitude of preference among the alternatives. Measureable value functions, also applicable in decisions under certainty, provide an interval scale of measurement; that is, the decision-maker's strength of preference amongst the alternatives is captured. Finally, utility functions are applicable in decisions with risk. The utility model of one's preference structure not only considers the decision-maker's values of the potential consequences but also incorporates his psychological reactions to taking risks. See Keeney and Raiffa [5], Kirkwood [7], Dyer and Sarin [8], von Winterfeldt and Edwards [9], and Farquhar and Keller [10] for further in-depth discussions of these preference structure models.

A further aspect of uncertainty in decision making was presented in the 1960s by Daniel Ellsberg, best known in the decision analysis community for his now infamous Ellsberg Paradox (see [11] for a well described presentation). The term decision ambiguity in the decision analysis context was first defined by Ellsberg [12] and has since been generalized and elaborated by many. Frisch and Baron [13] present a nice definition: "Ambiguity is uncertainty about probability, created by missing information that is relevant and could be known." Some have used this idea to challenge the validity of utility theory; particularly as a descriptive theory though most proponents of utility theory argue that the theory was meant only as a normative one [6, 13, 14]. Others have attempted to expand utility theory to include ambiguity (see [15] as an example). In short, decision ambiguity refers to the uncertainty in describing the probability profile of a risky decision. In the previously noted example of the family outing, the decision ambiguity is the uncertainty in the probability of the weekend rainstorm described by the meteorologists.

To summarize, uncertainty in decision making is by no means a new concept. The theory of expected utility addresses the decision problem for which an uncertain future event stands between the decision at hand and the realized consequences. Decision ambiguity considers the uncertainty involved in describing the probability profile of the uncertain event in a risky decision. Other examples of uncertainty in decision making include uncertain decision-maker preference structures [16] and uncertainty in attribute weights [17–19]. Although these methods encompass many aspects of uncertainty in decision making, they all presume that the consequences (described by the attribute values) are precisely defined and neglect any uncertainty that may exist in their assessment. Little work has been published that explicitly considers the uncertainty that may be present in the assessments of the consequences (the attribute value uncertainty). Until now, this work has been limited to the use of the PROMETHEE outranking technique to incorporate attribute value uncertainty, as in Hyde et al. [20] and Zhang et al. [21].

Problem Statement

The consequence associated with any decision is the result of the selected alternative and the outcome of relevant external factors that are outside the control of the decision-maker (e.g., an uncertain future event). To illustrate this perspective, a simple decision may be represented as a decision table (Table 1). The n decision alternatives $a_1, a_2, \ldots, a_n$ are the rows in the table. The columns in the table correspond to $s_1, s_2, \ldots, s_r$, the r mutually exclusive and exhaustive possible outcomes of relevant external factors. Associated with each possible outcome is $P(s_j)$, the probability that s_j will be the true outcome. As shown in each cell of the

Table 1 General form of a decision table

		Outcomes			
		s_1	s_2	$\ldots$	s_r
Decision alternatives	a_1	$x_{1,1,1}, x_{1,1,2}, \ldots, x_{1,1,m}$	$x_{1,2,1}, x_{1,2,2}, \ldots, x_{1,2,m}$	$\ldots$	$x_{1,r,1}, x_{1,r,2}, \ldots, x_{1,r,m}$
	a_2	$x_{2,1,1}, x_{2,1,2}, \ldots, x_{2,1,m}$	$x_{2,2,1}, x_{2,2,2}, \ldots, x_{2,2,m}$	$\ldots$	$x_{2,r,1}, x_{2,r,2}, \ldots, x_{2,r,m}$
	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
	a_n	$x_{n,1,1}, x_{n,1,2}, \ldots, x_{n,1,m}$	$x_{n,2,1}, x_{n,2,2}, \ldots, x_{n,2,m}$	$\ldots$	$x_{n,r,1}, x_{n,r,2}, \ldots, x_{n,r,m}$

table, the consequence that ensues when alternative a_i is selected and s_j is the true outcome is described by m attributes and their associated attribute values x_{ij1} to x_{ijm}.

Table 1 clearly displays the components of a decision: the alternatives, the uncertain possible outcomes, and the resulting consequences described by attribute values. When the decision components are viewed as displayed in Table 1, it becomes evident that uncertainty in expressing the attribute values (that is, the uncertainty in the values of x_{ijk}) is essentially unlike uncertainty about which of the set of possible outcomes, $s_1, s_2, \ldots, s_r$, will occur (risky decision) and uncertainty in defining the probability of each outcome, $P(s_j)$ (decision ambiguity).

While it may be conceivable to model the attribute value uncertainty as an uncertain event in a risky decision, we choose to maintain a decision model that distinguishes the attribute value uncertainty as a unique component of uncertainty. The reason is that a decision-maker can control, to some extent, the amount of uncertainty in an estimate of the true value of an attribute by varying the amount of information observed in its assessment whereas the outcome of a future uncertain event cannot be controlled in this same manner.

Decision-makers often consider decision alternatives that have consequences that are described by uncertain attributes. If a decision problem includes attributes whose values are determined by means of sampling and measurement, attribute value uncertainty exists. For example, the listed fuel mileage of a new car being considered is only an estimate of the true value based on sampling and experimental evaluations which include measurement. When these attribute values are provided only as point values, the decision-maker must move forward under the assumptions that the values are accurate and that the level of uncertainty associated with each alternative is equivalent. Although the value of a new car's fuel mileage may be an innocent example, the Department of Homeland Security's selection of a radiation detection system to be installed at airports based on estimated system performance parameters is a much more serious matter. Consider also the decisions that depend upon the results of the 2010 United States census. The allocation of congressional seats and federal funding will be decided based on the estimated population within each congressional district. There is undeniably uncertainty in these estimates, and the uncertainties are not equivalent from district to district.

The selection of a product or service provider may be modeled as a multiattribute decision under certainty. That is, there are no uncertain future events that stand between the decision at hand and the realized consequences. In this case there is only a single outcome. The alternatives $a_1, a_2, \ldots, a_n$ are the n products or service providers being considered. Some of the relevant attributes have attribute value uncertainty. Because the model is a decision under certainty, either a multiattribute ordinal value function or a multiattribute measurable value function may be used to represent the preference structure. The decision-maker's goal in this situation is to select the alternative that, given his preferences, maximizes his satisfaction in the presence of attribute value uncertainty. We shall propose an approach to incorporate the attribute value uncertainty into the decision model and then consider various rules for evaluating the decision-maker's satisfaction with each alternative.

In this situation, the decision-maker faces the risk of selecting an alternative that is not the best one, which could be identified if no attribute value uncertainty existed. The decision-maker may have to make a tradeoff, as in other settings involving risk, between alternatives whose performance is described to range from very well to poor (that is, there is a large amount of uncertainty about their performance) and other alternatives whose performance is described as neither very well nor poor (that is, there is less uncertainty about their performance). The proposed methods should help the decision-maker understand this tradeoff and make a better decision.

Approach

This section describes the approach that we will use to identify the best of a set of alternatives that have attribute value uncertainty. For any alternative, the point estimates for each attribute can be used to evaluate the decision model. The alternative with the largest resulting decision parameter value (e.g., value or measurable value in a decision under certainty, utility in a risky decision [6]) would be considered the alternative most fitting given the decision-maker's preferences. We will call this the *expected value approach*. Because this approach fails to consider the attribute value uncertainty, it may fail to select the alternative that maximizes the decision-maker's satisfaction. Thus, we propose the following approach, which augments the expected value approach by incorporating the attribute value uncertainty as follows:

1. Identify and develop the alternatives $a_1, a_2, \ldots, a_n$ and the attributes.
2. Model the uncertainty of each attribute value for each alternative. Let $F_{ij}(x)$ be the probability distribution for alternative i's value for attribute j, $i = 1, \ldots, n$, $j = 1, \ldots, m$.
3. Randomly sample the attribute values based on the associated uncertainty models to generate R realizations for each alternative. Each realization has a single value for each of the relevant attributes. Let x_{ijr} be alternative i's value for attribute j in realization r, $i = 1, \ldots, n$, $r = 1, \ldots, R$.
4. Define the multiattribute decision model based on the decision-maker's preference structure and the realizations of the attribute values. This includes defining the individual value (or utility) functions and attribute weights.
5. Propagate the attribute value uncertainty through the multiattribute decision model and onto the decision parameter. That is, for each alternative and each of its R realizations, calculate the corresponding decision parameter value. The result is a distribution of R decision parameter values for each alternative. Let y_{ir} be alternative i's value for the decision parameter in realization r, $i = 1, \ldots, n$, $r = 1, \ldots, R$.
6. Use a decision rule to identify the most desirable alternative based upon distributions of the decision parameter values.

By appropriately adjusting the definition (Step 4) and evaluation (Step 5) of the decision model, this approach can be used in both decisions under certainty and decisions with risk, including risky decisions with decision ambiguity. As Steps 1 and 4 above are the basis for developing any decision model [5] our discussion in the following Sections will focus on approaches to model the attribute value uncertainty (Step 2) and selecting an alternative based on a collection of decision parameter distributions (Step 6).

Modeling Attribute Value Uncertainty

The ideal attribute value input to the decision model is the true, but often unknown, value. As previously discussed, when attribute values are obtained based on sampling, such as surveys or measurements, the value obtained is merely an estimate of the true attribute value. This estimate contains some uncertainty that depends upon the experimental technique used to estimate the value. In this section, we describe several approaches to modeling this uncertainty. Our general decision analysis approach to incorporate attribute value uncertainty can be used with any of these approaches for modeling attribute value uncertainty.

Uncertainty and its assessment has become a popular topic in recent years. Lindley [22] suggests the reason for the peak in interest is that the rules for assessing and applying uncertainty are now understood and that past tendencies of suppressing uncertainties are no longer necessary. Of the various methods that could be leveraged to model attribute value uncertainty, we will discuss two approaches: a bootstrap approach and a Bayesian approach.

A Bootstrap Method for Modeling Attribute Value Uncertainty

The non-parametric bootstrap method relies upon resampling of the observed data to model the attribute value uncertainty. Introduced by Efron [23], the bootstrap is "a computer-based method for assigning measures of accuracy to statistical estimates" [24].

Given observed data $x_1, x_2, \ldots, x_n$, a typical application of the non-parametric bootstrap technique involves generating a sample of size n with replacement from the observed empirical distribution. Denoted by $\mathbf{x}^*$, this sample is called a bootstrap sample. From the bootstrap sample, we compute the value of the parameter of interest, denoted by θ^*. We repeat this process b times to create an approximation for the distribution of θ^* and obtain statistical properties such as the standard error, which are directly related to the parameters of the distribution that underlies the original observations.

A number of variants of the bootstrap exist beyond the non-parametric bootstrap method such as the parametric bootstrap and the Bayesian bootstrap, each of which could also be used to develop a model of the attribute value uncertainty. These variants utilize the same general resampling approach to create an approximation for the distribution of θ^*, though the Bayesian bootstrap uses a posterior probability distribution for resampling the observed data rather than the uniform distribution used in the non-parametric bootstrap resampling. The parametric bootstrap samples from an assumed parametric distribution with parameter values estimated based upon the observed data. Chernick [25] provides a summary of these and other bootstrap techniques as well as a variety of applications.

A Bayesian Model of Attribute Value Uncertainty

Another alternative in developing a model of the attribute value uncertainty is to leverage the Bayesian paradigm of inference, which describes where the likely attribute values are found using the posterior probability distribution [26]. Generally speaking, an initial degree of belief, described in terms of a probability distribution called the prior, is updated using Bayes' Theorem when new data are observed to produce a new degree of belief called the posterior distribution.

This approach allows for probabilities to be associated with the unknown parameters. That is, the resulting posterior probability distribution describes what is currently known about the parameters, where the probabilities are interpreted as representing the degree of belief that given values of the parameter is the true value [27].

The posterior distribution provides a method to model one's knowledge of the true value of the attribute. This model captures the uncertainty in the attribute value estimate provided by the sampling or measurements.

Selection of an Alternative

Traditional decision analysis approaches clearly identify the most desirable alternative. This property should not be lost when expanding the model to be more comprehensive by including attribute value uncertainty. The result of propagating uncertainty is a set of decision parameter values that are described by distributions. Thus, selecting an alternative changes from a simple ordering exercise to a comparison of distributions. This section discusses three approaches to compare the resulting decision parameter distributions: Rank 1, Stochastic Dominance, and Majority Judgment.

Rank 1

In each realization, each alternative has one value for the decision parameter. If we consider the realizations one at a time and examine the decision parameters for all of the alternatives in that realization, then the alternatives can be ranked by the decision parameter, and the most desirable alternative (the one ranked first) can be identified. (If multiple alternatives tie for first in a realization, all of those so tied are considered as ranked first.) The number of realizations in which an alternative is ranked first (its *rank 1 value*) describes the relative desirability of that alternative. An alternative's rank 1 value can vary from 0 (it is never ranked first) to R (it ranked first in every realization). We use this value in the decision rule that selects the alternative with the greatest rank 1 value.

Stochastic Dominance

Our second approach builds upon the concept of stochastic dominance for comparing distributions. In the following discussion Y_i and Y_j represent the decision parameters for alternatives i and j respectively. The distributions of these parameters are the ones generated by the R realizations.

Hadar and Russell [28] discuss stochastic dominance as an approach to predicting a decision-maker's choice between two uncertain events without knowledge of the decision-maker's utility function. They define two types of stochastic dominance: first-degree stochastic dominance and second-degree stochastic dominance which are presented below.

First-degree stochastic dominance: Y_i stochastically dominates Y_j in the *first degree* if and only if

$$P\left[Y_i \leq y\right] \leq P\left[Y_j \leq y\right] \qquad \forall y \tag{1}$$

That is, the value of the cumulative distribution for Y_i never exceeds that of Y_j for all $y \in Y$.

Second-degree stochastic dominance: When the support of Y_i and Y_j is contained within the closed interval [a, b], Y_i stochastically dominates Y_j in the *second degree* if and only if

$$\int_a^t P\left[Y_i \leq y\right] dy \leq \int_a^t P\left[Y_j \leq y\right] dy \qquad \forall t \in [a,b] \tag{2}$$

That is, the area under the cumulative distribution for Y_i is less than or equal to that of Y_j for all $t \in [a,b]$.

First-degree stochastic dominance is relevant in the absence of any restrictions on the unknown utility function other than monotonicity. Second-degree stochastic dominance is more restrictive in that the results apply only when the unknown utility functions are concave, indicating a risk-averse decision-maker. Under these restrictions, if Y_i is found to stochastically dominate Y_j in either the first or second degree then alternative i is preferred to alternative j because alternative i will have a greater expected utility.

If a single Y_i is identified to stochastically dominate (first- or second-degree) Y_j, for all j, $i \neq j$, and, in at least one case, the inequality in Eqs. 1 or 2 is found to be a strict inequality then Hadar and Russell have shown that alternative i can be selected with few underlying assumptions.

We use the idea of stochastic dominance as a decision rule to select the alternative with a set of decision parameter values that stochastically dominates all others. It should be noted, however, that this rule may not produce a solution, and thus an alternative would not be identified for selection.

In particular, consider the decision parameter values for alternatives i and j. Both are sets of R values generated as discussed in Step 5 of the approach. Alternative i dominates alternative j based upon the ideas of first-degree stochastic dominance if, for all values y, the number of values of the decision parameter Y_i that are not greater than y is less than or equal to the number of values of the decision parameter Y_j that are not greater than y.

Let $Z_i = \{y_{i[1]}, y_{i[2]}, \ldots, y_{i[R]}\}$ be the ordered set of the R decision parameter values generated as discussed in Step 5 of the approach for alternative i where $y_{i[1]} \leq y_{i[2]} \leq \ldots \leq y_{i[R]}$. Let $f_i(y)$ be the number of decision parameter values in Z_i that are less than or equal to y. Note that this is a step function that increases at each value in the set Z_i. Let a and b be the lower bound and the upper bound on the decision parameter values across all of the alternatives. Alternative i dominates alternative j based upon the ideas of second-degree stochastic dominance if the following condition holds:

$$\int_a^t f_i(y)dy \leq \int_a^t f_j(y)dy \quad \forall t \in [a,b] \tag{3}$$

Because $f_i(y)$ and $f_j(y)$ are step functions, it is easy to calculate these integrals for any value of t, and this condition holds for all $t \in [a,b]$ if it holds for all $t \in Z_i \cup Z_j$.

Majority Judgment

By considering the decision parameter value resulting from each of the R realizations as a score assigned by an individual judge or voter, the problem of selecting an alternative based on distributions of decision parameter values may be viewed as one of social choice. A consensus value for each alternative that appropriately represents the message of all judges is sought in comparing and selecting the most desirable alternative. While many models of social choice exist, we consider the method of Majority Judgment.

In an attempt to identify a model of social choice that overcomes the shortcomings displayed by traditional social choice models such as the Borda and Condorcent methods, Balinski and Laraki [29, 30] propose the method of Majority Judgment. The Majority Judgment method relies upon the middlemost interval to identify a social grading function that has desirable functional properties, provides protection against outcome manipulation by individual voters or judges, and overcomes many of the shortcomings of traditional social choice models. When considering $y_{i[1]}, y_{i[2]}, \ldots, y_{i[R]}$ ordered scores for a given alternative i, the *majority-grade* is defined to be the median score, $y_{i[(R+1)/2]}$, when R is odd and the lower bound of the middlemost interval, $y_{i[R/2]}$, when R is even where

$y_{i[1]} \leq y_{i[2]} \leq \ldots \leq y_{i[R]}$. The Majority Judgment method identifies the alternative with the largest majority-grade as the most desirable alternative in the social choice context. If multiple alternatives have the same largest majority-grade, then a single majority-grade value is removed from the set of scores for each alternative in the tie, and the majority-grade of the new distributions are calculated. If a tie again occurs, this process is repeated until a single alternative has the largest majority-grade. The method extends this concept to provide a complete rank-ordering termed the *majority-ranking*.

As the majority-grade is defined based upon the middle-most interval, it emphasizes the significance of place in order rather than magnitude. That is, it is robust against extreme scores. As the goal of the decision problem at hand is to identify the most desirable alternative given the set of considered alternatives this trait makes the method of Majority Judgment an attractive rule for selecting an alternative. Further, the majority-ranking provides a ranking between any two alternatives that are dependent upon the grades of only those two alternatives. In other words, the majority-ranking is independent of irrelevant alternatives (Arrow's IIA).

To select a decision alternative based on the Majority Judgment method where the decision parameters are described by distributions, a majority-grade is computed for each alternative by considering the decision parameter value resulting from each of the R realizations as an individual score. Specifically, the median (if R is odd) or the lower bound of the middlemost interval (if R is even) of the distribution of decision parameter values is computed for each alternative. The alternative with the largest majority-grade is then identified as the most desirable alternative. If a tie exists, the tie-breaking procedure defined by the method of Majority Judgment is used to identify the single most desirable alternative.

Application

When homeowners require a repairman or other services, they often seek reviews and recommendations for potential service providers. One source for such information is the Center for the Study of Services, who publishes quarterly periodicals in several major metropolitan areas. These periodicals provide ratings for various consumer services. The Spring/Summer 2011 edition of the Washington Consumers' Checkbook [31] provides an extensive review of roofing firms in the Washington, D.C., metropolitan area. We will consider the problem of selecting a roofing firm using the data in the Washington Consumers' Checkbook to illustrate the proposed approaches for making decisions in the presence of attribute value uncertainty. As the purpose of this demonstration is to illustrate the application of the decision analysis method rather than to endorse any particular service provider, the roofing firm names as provided by the Washington Consumers' Checkbook review have been replaced by numeric ID codes.

The Washington Consumers' Checkbook review includes ten performance rating criteria for each of 94 roofing firms. The results of the review were obtained through a survey of the organization's members. The ten performance rating criteria are:

1. Work performed properly on first attempt
2. Began and completed work promptly
3. Provided cost information early
4. Neatness of work
5. Expert advice on service options and costs
6. Overall performance
7. Percentage of customers rating a firm "adequate" or "superior" for "overall performance"
8. Number of complaints (and rate) filed with local government
9. Number of complaints (and rate) filed with the Better Business Bureau
10. Percent of $5,000 job the firm allows the customer to pay upon completion

For each of criteria 1 through 6, the measure provided is the proportion of customers surveyed who rated the firm as "superior". For criteria 8 and 9, the complaint rate is the ratio of the number of complaints filed to the number of full-time employees performing residential work. This measure is an attempt to account for the exposure of a company, whereas a larger company that performs more work experiences greater exposure to incur complaints. In addition to the performance rating criteria, the number of survey responses is noted for each roofing firm.

Roofing Firm Decision Model

The decision of selecting a roofing firm from the firms reviewed in the Washington Consumers' Checkbook was modeled as a multiattribute decision with certainty. There are no uncertain events in this decision model, but uncertainty is prevalent in the review's estimates of the performance criteria values that are attributes of the decision model.

The alternatives considered in this decision model are the roofing firms. Of the 94 firms included in the survey, seven were removed from consideration due to incomplete data, so $n = 87$ decision alternatives remained.

For this demonstration, we considered the following $m = 4$ attributes:

X_1: Work performed properly on first attempt
X_2: Began and completed work promptly
X_3: Neatness of work
X_4: Percent of $5,000 job the firm allows the customer to pay upon completion

The survey results for the performance rating criteria were used as estimated values of the attributes included in the decision model. The estimates for attributes X_1, X_2, X_3 are provided as the proportion of customers surveyed who rated the firm "superior" for each performance criteria. These attributes are random variables consisting of a collection of Bernoulli trials: the performance criterion was rated by

Table 2 Summary statistics for the distribution of data across the 87 firms considered in the decision model

	Mean	Std dev	Min	Median	Max
Number of survey responses	54.17	66.30	10	29	390
X_1: Work performed properly on first attempt	0.74	0.16	0.23	0.79	1.00
X_2: Began and completed work promptly	0.74	0.16	0.28	0.77	1.00
X_3: Neatness of work	0.76	0.16	0.27	0.79	1.00
X_4: Percent of job firm allows paid after completion	0.77	0.19	0.33	0.67	1.00

each survey respondent as either superior or not. Thus, for each of the i roofing firms ($i = 1, 2, \ldots, 87$), each of the X_j performance criteria ($j = 1, 2, 3$) can be described by a binomial random variable with parameters p_{ij} and k_i, where p_{ij} is the proportion of the k_i survey respondents who provided a rating of "superior" for the performance criteria. For these attributes, a larger value is preferred.

Attribute X_4 is not random; it is provided by the roofing firm. The attribute is the percentage of a $5,000 job that the firm will allow the customer to pay upon completion of the job. The value is considered to be a constant for each firm. Larger values for X_4 are preferred because, if the value is small, the customer must pay more upfront, which increases the customer's financial risk. There were seven firms (of the 94 firms included in the survey) for which no value for this attribute was obtained. These seven firms were removed from the alternatives considered in the decision analysis model. Summary statistics of the distribution for the four attributes and the number of survey responses across the 87 firms considered are provided in Table 2.

A multiattribute measurable value function was used to represent the decision-maker's preference structure. The preference model used in this demonstration represent the preference structure of the author. We assume the preference structure is such that attributes X_1, X_2, X_3, X_4 are mutually preference independent and mutually difference independent. Therefore, the multiattribute measureable value function can be represented by the sum of single attribute measureable value functions [8] as displayed in Eq. 4.

$$v(x_1, x_2, x_3, x_4) = \sum_{i=1}^{4} \lambda_i v_i(x_i) \tag{4}$$

Here $\sum_{i=1}^{4} \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1$ and the individual measureable value functions $v_i(x_i)$ are scaled such that, for x_i^*, the most preferred outcome, $v_i(x_i^*) = 1$ and, for x_i^0, the least preferred outcome, $v_i(x_i^o) = 0$.

Expected Value Approach

One may employ the performance ratings provided by the survey in conjunction with Eq. 4 to evaluate the multiattribute measureable value model. As described

Table 3 Additive individual measureable value functions and weights for the expected value approach

Attribute	$v_i(x_i)$	λ_i
X_1: Work performed properly on first attempt	$v_1(x_1) = -\frac{1}{3.634}\left(1 - e^{(x_1-0.23)/0.502}\right)$	0.476
X_2: Began and completed work promptly	$v_2(x_2) = 1 - e^{-(x_2-0.28)/0.866}$	0.190
X_3: Neatness of work	$v_3(x_3) = x_3/0.73 - 0.37$	0.286
X_4: Percent of price for a $5,000 job the firm allows the customer to pay upon completion of job	$v_4(x_4) = \frac{1}{0.909}\left(1 - e^{-(x_4-0.33)/0.280}\right)$	0.048

Table 4 Attribute values and resulting decision parameter value for the top 10 roofing firm alternatives based on the expected value approach

Roofing firm ID	n	x_1	x_2	x_3	x_4	Value
Firm 29	24	1.00	0.92	0.96	1.00	0.9837
Firm 84	82	0.99	0.99	0.99	1.00	0.9835
Firm 57	23	0.95	0.96	0.96	1.00	0.9263
Firm 28	54	0.96	0.92	0.92	1.00	0.9216
Firm 90	36	0.94	0.88	0.97	0.95	0.9183
Firm 8	347	0.95	0.94	0.93	1.00	0.9145
Firm 91	49	0.96	0.73	0.89	1.00	0.9089
Firm 93	13	0.92	1.00	0.92	0.67	0.8679
Firm 71	89	0.93	0.74	0.84	1.00	0.8570
Firm 35	23	0.91	0.83	0.91	0.66	0.8530

by Keeney and Raiffa [5], the alternative with the largest resulting value would be considered to be the alternative most fitting given the decision-maker's preferences.

The individual measureable value functions $v_i(x_i)$, $i = 1, 2, 3, 4$, that were considered in the value analysis were developed by utilizing an augmentation to the midvalue splitting technique that leverages an analytical exponential form [7] based on the attribute value ranges displayed in Table 2. The swing weighting procedure [32] was used to develop the associated weights λ_i, $i = 1, 2, 3, 4$, for each individual measureable value function. The individual measureable value functions and associated weights are provided in Table 3.

Based on these defined individual measureable functions and associated weights, Eq. 4 was evaluated for each alternative. Table 4 displays the top 10 alternatives resulting from the analysis utilizing the expected value approach. All results are displayed graphically in Fig. 1. Roofing Firm 29, whose value equals 0.9837, is the most desirable alternative. This firm is followed closely by Roofing Firm 84, whose value equals 0.9835.

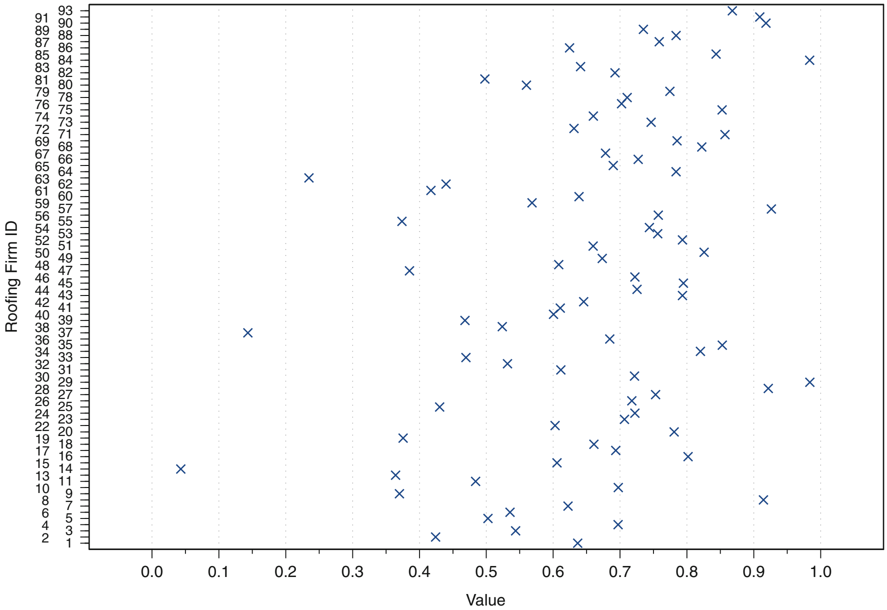

Fig. 1 Decision model results using the expected value approach

Incorporating Attribute Value Uncertainty

To describe the uncertainty in attributes X_1, X_2, and X_3 we chose to use a Bayesian approach. For each alternative and each of these attributes, we began with the assumption – or prior knowledge – that the true value of the attribute lies between 0 and 1 with equal likelihood. This is represented by the *Uniform* (0, 1) prior distribution, which is equivalent to a *Beta* (1, 1) distribution. Observations from a *Binomial* (k_i, p_{ij}) distribution were used to update the prior distribution. In this case, the observations were the estimates of the X_j performance criteria ($j = 1$, 2, 3) obtained by the Washington Consumers' Checkbook review for the $i = 1$, 2, …, 87 roofing firms. Given this new information along with the prior distribution, the knowledge about the unknown parameter p was updated to create a posterior distribution. Because the *Beta* (α, β) distribution is the conjugate prior to the *Binomial* (n, p) distribution, the posterior distribution is the $Beta(1 + k_i p_{ij}, 1 + k_i(1 - p_{ij}))$ distribution. This posterior distribution describes the uncertainty in each attribute for each alternative.

Given the posterior distributions for each attribute for each alternative, we drew $R = 1000$ random samples from each of these distributions. Summary statistics for these random realizations for each attribute across all alternatives are presented in Table 5.

Table 5 Summary statistics for the 1000 random realizations across all alternatives

	Mean	Std dev	Min	Median	Max
X_1: Work performed properly on first attempt	0.73	0.17	0.027	0.76	1.00
X_2: Began and completed work promptly	0.73	0.17	0.050	0.76	1.00
X_3: Neatness of work	0.74	0.17	0.049	0.78	1.00
X_4: Percent of job firm allows paid after completion	0.77	0.19	0.33	0.67	1.00

Table 6 Additive individual measureable value functions and weights when considering attribute value uncertainty

Attribute	$v_i(x_i)$	λ_i
X_1: Work performed properly on first attempt	$v_1(x_1) = -\frac{1}{4.53}\left(1 - e^{(x_1-0.027)/0.570}\right)$	0.476
X_2: Began and completed work promptly	$v_2(x_2) = \frac{1}{0.954}\left(1 - e^{-(x_2-0.050)/0.309}\right)$	0.190
X_3: Neatness of work	$v_3(x_3) = x_3/0.951 - 0.05$	0.286
X_4: Percent of price for a \$5,000 job the firm allows the customer to pay upon completion of job	$v_4(x_4) = \frac{1}{0.909}\left(1 - e^{-(x_4-0.33)/0.280}\right)$	0.048

Based on the distributions of the random realizations for each attribute across all alternatives, summarized by the ranges displayed in Table 5, the individual measureable value functions $v_i(x_i)$, $i = 1, 2, 3, 4$, and associated weights λ_i, $i = 1, 2, 3, 4$, were redefined by again using an augmentation to the midvalue splitting technique and the swing weighting procedure. The redefined individual measureable value functions and weights are provided in Table 6.

Provided these defined individual measureable functions and associated weights, Eq. 4 was evaluated for each alternative for each of the 1000 random realizations. The result is a distribution of 1000 overall decision parameter values for each roofing firm.

When analyzing the resulting 87 distributions of decision parameter values, we first found the minimum value of every alternative's decision parameter and identified the greatest of these minimum values. We then determined that 59 alternatives were dominated in the following way: for each of these 59 alternatives, the maximum value of its decision parameter was less than the greatest minimum value. This left 28 non-dominated alternatives. The non-dominated alternatives and their associated value distributions are displayed in Fig. 2.

Results

Given the distributions of decision parameter values for the non-dominated roofing firms, we applied the Rank 1, Stochastic Dominance, and Majority Judgment decision rules. Table 7 lists the Rank 1 and Majority Judgment results for the six firms that had the most value in the expected value approach (see Table 4). The

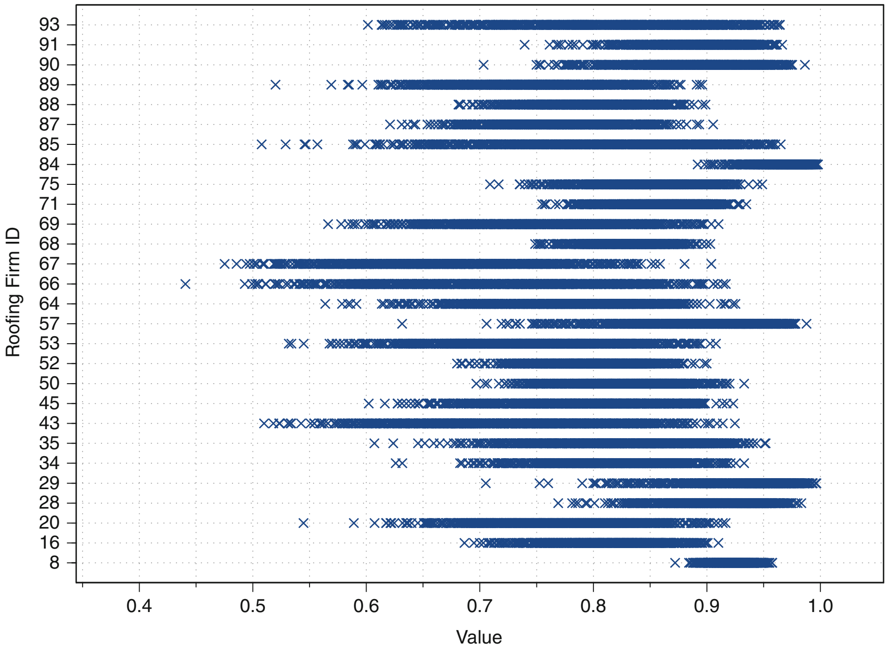

Fig. 2 Decision parameter distributions for the 28 non-dominated alternatives

Table 7 Results for each decision rule considered. For each firm, the table lists its rank when using the different decision rules. For the expected value approach, the value in *parenthesis* is the overall value. For the rank 1 decision rule, the value in parenthesis is the number of times that firm was ranked first. For Majority Judgment, the value in *parenthesis* is the majority-grade

Roofing firm ID	n	Expected value	Rank 1	Majority judgment
Firm 29	24	1 (0.9837)	2 (175)	2 (0.9426)
Firm 84	82	2 (0.9835)	1 (740)	1 (0.9724)
Firm 57	23	3 (0.9263)	3 (27)	5 (0.9007)
Firm 28	54	4 (0.9216)	4 (23)	4 (0.9164)
Firm 90	36	5 (0.9183)	5 (13)	6 (0.8980)
Firm 8	347	6 (0.9145)	6 (11)	3 (0.9234)

results for the Stochastic Dominance decision rule are best displayed graphically as empirical cumulative distribution curves, which are displayed in Fig. 3 for the top roofing firms.

As seen in Table 7, when considering the more comprehensive decision model that incorporates the attribute value uncertainty, the Rank 1 and Majority Judgment decision rules identify Roofing Firm 84 as the most desirable alternative with Roofing Firm 29 identified as the second most desirable alternative. (The expected value approach identified Roofing Firm 29 as the most desirable alternative, with Roofing Firm 84 as the second largest value.) In 1000 realizations, Roofing Firm

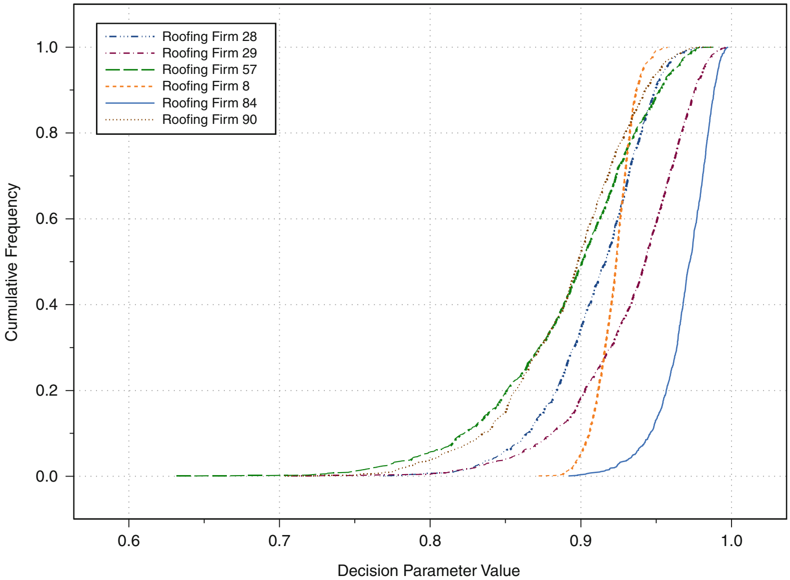

Fig. 3 Empirical cumulative distribution curves for the top roofing firms. Roofing Firm 84 is shown to stochastically dominate all other firms in the first degree

84 was the most desirable option 740 times, and Roofing Firm 29 was the most desirable option only 175 times. The majority-grade in the Majority Judgment selection method for Roofing Firm 84 was 0.9724, while the majority-grade for Roofing Firm 29 was 0.9426.

As shown in Fig. 3, the empirical cumulative distribution curve for Roofing Firm 84 never exceeds that of any other alternative, so Roofing Firm 84 stochastically dominates all other alternatives in the first degree. Thus Roofing Firm 84 is deemed to be the most desirable alternative using the Stochastic Dominance decision rule. This result is consistent with the results obtained by the other decision rules that consider the attribute value uncertainty. Further, the fact that we found one alternative that stochastically dominates all of the other alternatives in the first degree is an extremely powerful result, as Hadar and Russell [28] have shown that this is the decision-maker's most preferred alternative regardless of his underlying utility function.

Summary and Conclusions

This chapter presented an approach for making decisions when uncertainty exists in the values of the attributes being used to compare the alternatives and derive a decision parameter. This type of uncertainty is different from uncertainty about future events (risky decision) and uncertainty about the probabilities of future events (decision ambiguity). Ignoring this uncertainty, especially when it varies between alternatives, could lead to poor decisions.

The method presented here requires modeling the uncertainty about the attribute values and then propagating that uncertainty to determine the uncertainty in the decision parameter. The method is a Monte Carlo approach that randomly samples values of the uncertain attributes and computes the corresponding values of the decision parameter. Because it is not limited to specific types of distributions or decision models, it is a very general approach that can be used in a wide variety of settings. This chapter presented three decision rules (Rank 1, Majority Judgment, and Stochastic Dominance) for selecting an alternative based on its decision parameter distribution.

Unfortunately, it does require many samples of the uncertain attributes, which could be computationally expensive. The decision-maker must choose a decision rule to compare the distributions of the alternatives' decision parameters, and different rules may identify different alternatives as the "best".

This chapter has used the example of selecting a roofing firm to demonstrate the approach, but the approach can be used in any setting with attribute value uncertainty, including other models of attribute value uncertainty, other forms of the decision parameter, and other decision rules beyond those presented here.

The next research question to consider is that of experimental design: when planning to initially obtain, or if the decision-maker has the opportunity to get more information about some attributes for some alternatives, which information would be most valuable? In this case, information provides value by removing uncertainty about an alternative's decision parameter estimate.

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