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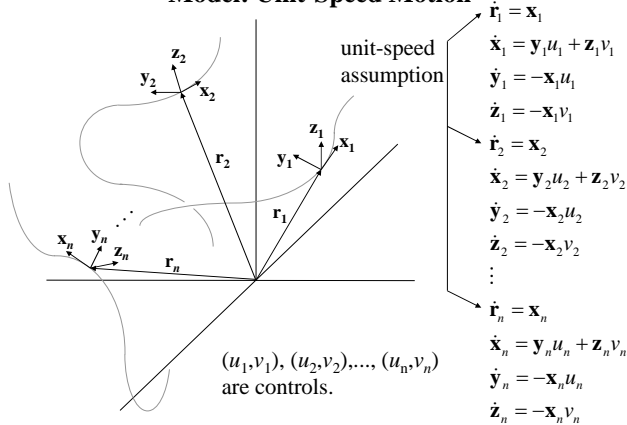
Naval Research Lab (NRL) collaborators:  
 Jeff Heyer, Larry Schuette, David Tremper

ISR collaborator: Fumin Zhang

## Abstract

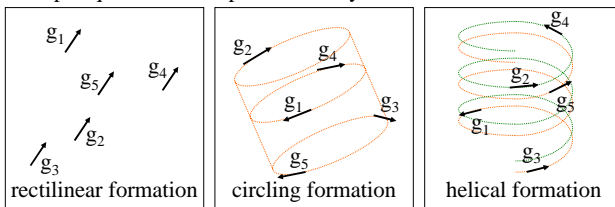
- Modeling and analysis of novel control laws for vehicles moving at constant speed.
- Practical motivation: coordinating the flight of meter-scale UAVs (unmanned aerial vehicles). Possible implications for UUV, UGV, or USV swarms, or biological swarming/schooling systems.
- Objective: UAV formation demo in collaboration with NRL.

## Model: Unit-Speed Motion



## Equilibrium Shapes

- The control laws are assumed to be invariant under rigid motions in three-dimensional space.
- Therefore, we can define appropriate shape variables (which capture relative distances and angles between vehicles).
- Shape equilibria correspond to steady-state formations.



## Rectilinear Control Law

$$u_j = \frac{1}{n} \sum_{k \neq j} \left[ -\eta \left( \frac{\mathbf{r}_{jk}}{|\mathbf{r}_{jk}|} \cdot \mathbf{x}_j \right) \left( \frac{\mathbf{r}_{jk}}{|\mathbf{r}_{jk}|} \cdot \mathbf{y}_j \right) + f(|\mathbf{r}_{jk}|) \left( \frac{\mathbf{r}_{jk}}{|\mathbf{r}_{jk}|} \cdot \mathbf{y}_j \right) + \mu \mathbf{x}_k \cdot \mathbf{y}_j \right]$$

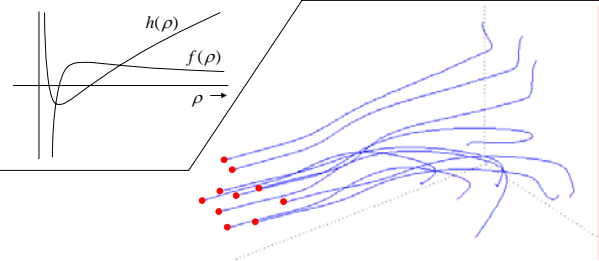
Align vehicle  $j$  perpendicular to the baseline between vehicles  $j$  and  $k$ .      Steer toward or away from vehicle  $k$  to maintain appropriate separation.      Align vehicle  $j$  with vehicle  $k$ .

$$v_j = \frac{1}{n} \sum_{k \neq j} \left[ -\eta \left( \frac{\mathbf{r}_{jk}}{|\mathbf{r}_{jk}|} \cdot \mathbf{x}_j \right) \left( \frac{\mathbf{r}_{jk}}{|\mathbf{r}_{jk}|} \cdot \mathbf{z}_j \right) + f(|\mathbf{r}_{jk}|) \left( \frac{\mathbf{r}_{jk}}{|\mathbf{r}_{jk}|} \cdot \mathbf{z}_j \right) + \mu \mathbf{x}_k \cdot \mathbf{z}_j \right]$$

$$\mathbf{r}_{jk} = \mathbf{r}_k - \mathbf{r}_j, \quad f(|\mathbf{r}_{jk}|) = \alpha \left[ 1 - \left( \frac{r_o}{|\mathbf{r}_{jk}|} \right)^2 \right], \quad \mu > \frac{\eta}{2} > 0, \quad \alpha > 0.$$

Global convergence result for  $n = 2$  (Justh, Krishnaprasad 2004) using the Lyapunov function:  $V = -\ln(1 + \mathbf{x}_2 \cdot \mathbf{x}_1) + h(|\mathbf{r}_2 - \mathbf{r}_1|)$

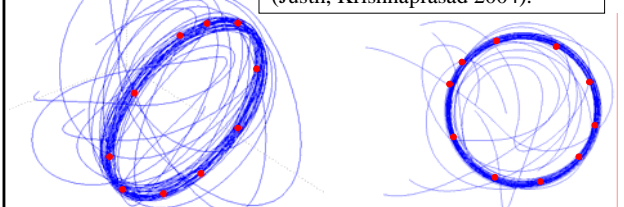
penalizes heading-direction differences      penalizes inter-vehicle distances which are too large or small



## Circling Control Law

$$u_j = \frac{1}{n} \sum_{k \neq j} \left\{ -\eta \left( \frac{\mathbf{r}_{jk}}{|\mathbf{r}_{jk}|} \cdot \mathbf{x}_j \right) \left( \frac{\mathbf{r}_{jk}}{|\mathbf{r}_{jk}|} \cdot \mathbf{y}_j \right) + f(|\mathbf{r}_{jk}|) \left( \frac{\mathbf{r}_{jk}}{|\mathbf{r}_{jk}|} \cdot \mathbf{y}_j \right) + \mu \left[ -\mathbf{x}_k \cdot \mathbf{y}_j + 2 \left( \frac{\mathbf{r}_{jk}}{|\mathbf{r}_{jk}|} \cdot \mathbf{x}_k \right) \left( \frac{\mathbf{r}_{jk}}{|\mathbf{r}_{jk}|} \cdot \mathbf{y}_j \right) \right] \right\}$$

Global convergence result for  $n = 2$  (Justh, Krishnaprasad 2004).

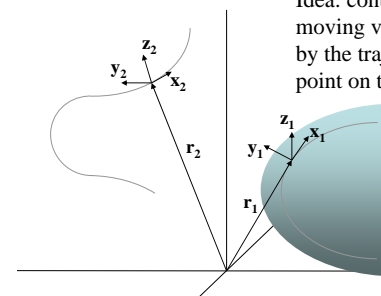


## Gyroscopic Interaction Laws

- In our formation control model, particles (i.e., vehicles) interact through **gyroscopic forces**.
- Gyroscopic forces do no mechanical work: the kinetic energy (and hence the speed) of each particle remains constant.
- The physically relevant quantities are  $\mathbf{r}_j$  (the position) and  $\mathbf{x}_j$  (the unit tangent vector to the trajectory),  $j=1, \dots, n$ , which imposes constraints on the form of  $(u_j, v_j)$ ,  $j=1, \dots, n$ .

## Future Research: Boundary-Following

Idea: control inputs for the moving vehicle are determined by the trajectory of the closest point on the obstacle surface.



Goals: boundary following and non-collision.

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## References

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- E.W. Justh and P.S. Krishnaprasad, "Equilibria and steering laws for planar formations," *Systems and Control Letters*, in press, 2004 (see also ISR TR 2002-38, 2002).
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