

Development of the Satellite Groundtrack Interactive Display

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The Satellite Groundtrack Interactive Display (SGID) is a learning tool created for students of Orbital Mechanics. To plot groundtracks in the SGID, equations for the latitude and longitude of the subsatellite point were found as functions of the six Keplerian orbital elements; five of these are constant for a given orbit, and the sixth increases with time. A Java Applet was developed that plots latitude and longitude parametrically while displaying sliders for adjusting the values of the constant orbital elements. These can be varied to represent any closed Earth orbit. Users of the SGID are guided by a series of technical questions and preloaded example orbits.

Nomenclature

a	=	semi-major axis
E	=	eccentric anomaly
e	=	eccentricity
i	=	inclination
M	=	mean anomaly
p	=	semi-latus rectum
Q	=	rotation matrix that translates between R_{IJK} and R_{PQW}
R_{ECEF}	=	satellite position vector in Earth-Centered Earth-Fixed (ECEF) coordinates
R_{IJK}	=	satellite position vector in Earth-Centered Inertial coordinates
R_{PQW}	=	satellite position vector in perifocal coordinates
r_p	=	perigee radius
T	=	orbital period
t	=	time
z_p	=	perigee altitude
θ	=	true anomaly
θ_G	=	Greenwich sidereal time
λ	=	longitude
μ	=	Earth's gravitational parameter
φ	=	latitude
Ω	=	right ascension of the ascending node
ω	=	argument of perigee
ω_e	=	Earth's angular velocity

I. Introduction

ORBITAL Mechanics courses introduce students to the three-dimensional motion of objects in space. There are many ways to define a unique orbit; one of the most common methods is through the Keplerian orbital elements. These six elements are semi-major axis (a), eccentricity (e), inclination (i), argument of perigee (ω), right ascension of the ascending node (Ω), and true anomaly (θ). The semi-major axis is usually measured in kilometers, eccentricity is nondimensional, and the remaining elements are usually specified in degrees. It is important for students not only to be able to perform calculations on these elements, but also to understand the physical meaning of each and the effect it has on the properties of an orbit.

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For example, orbital inclination is a straightforward concept: it is the angle at which the orbit is tilted, or inclined, with respect to the equatorial plane. However, the words “argument of perigee” are not so intuitive. To communicate these concepts, textbooks often include diagrams labeling each element on an orbit. The problem with this approach is in presenting a three-dimensional concept on a two-dimensional page. These diagrams are necessary, but they should be supplemented with additional learning aids designed to reinforce students’ understanding of each element.

The Satellite Groundtrack Interactive Display (SGID) was developed by the author to provide students a means of changing each orbital element and observing the effect this has on an orbit. This tool, which is available online at <http://www.eng.umd.edu/~healy/sgid/Groundtracks.html>, shows the groundtrack of an orbit drawn over a two-dimensional map of the Earth as seen in Fig. 1. Students can adjust the orbital elements using the sliders shown at the bottom of the figure, and the display updates automatically to show the new groundtrack. The SGID includes a set of questions and preloaded orbits (Fig. 1 shows the preloaded “Circular” orbit) designed to help students explore the tool. The planning and development of the SGID will be discussed, along with the example questions, their answers, and the topics in orbital mechanics that each is meant to emphasize.

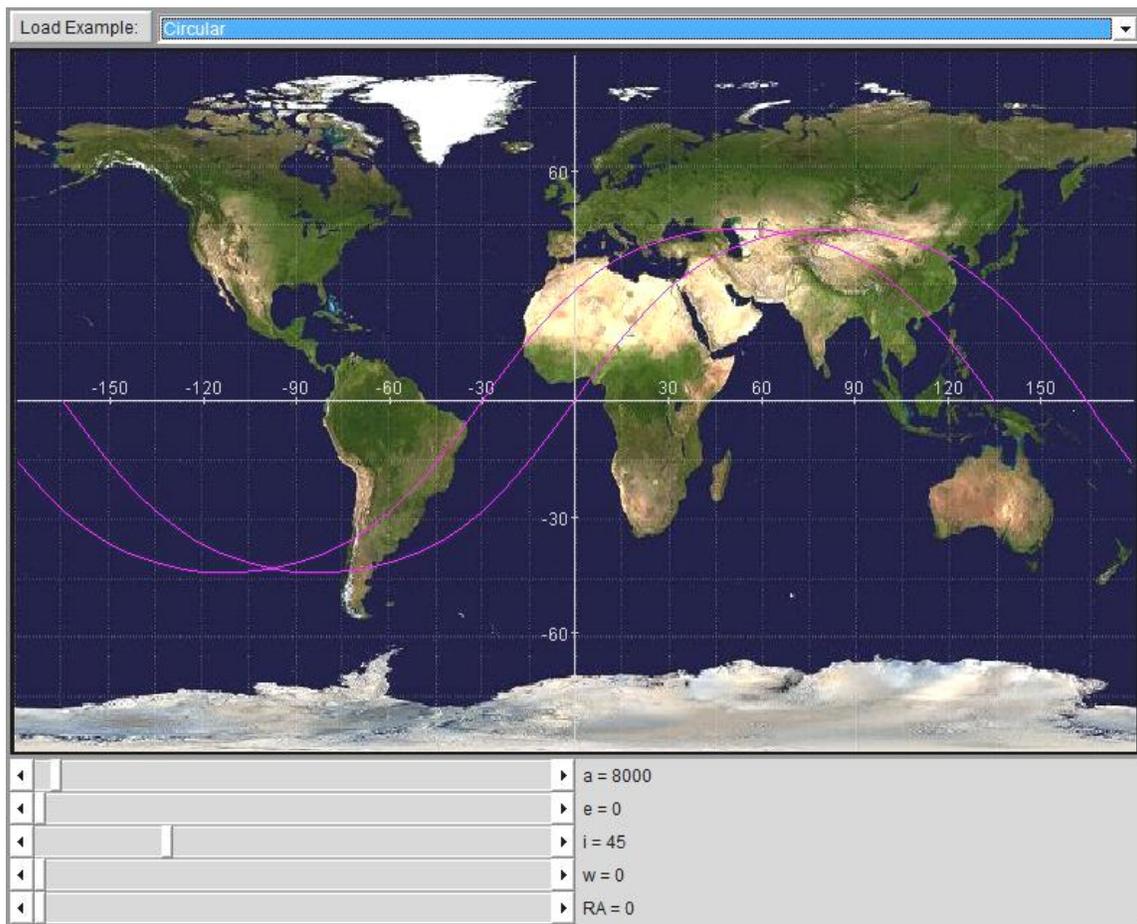


Figure 1. A screenshot of the SGID. The preloaded “Circular” orbit included in the SGID. The groundtrack drawn over the earth corresponds to the values shown on the sliders at the bottom of the image.

II. Planning the SGID

At an instant in time, a satellite in orbit is passing directly over exactly one point on the surface of the Earth. This location is called the subsatellite point, and a groundtrack is the curve formed by connecting these points over time. To plot groundtracks over a two-dimensional map of the Earth, the latitude and longitude of the subsatellite point as functions of time are needed. In this application, the latitude and longitude must be found as functions of the orbital elements. It is important to note that only five of the orbital elements are constant for a given orbit, and the sixth, true anomaly, covers a range of 0-360° during one orbit. A specific value of true anomaly indicates a specific

location on the orbit. Including a time-dependent variable such as true anomaly in the equations for latitude and longitude will create the plot of a groundtrack, instead of a single subsatellite point. This also implies that only the five constant orbital elements can be controlled through sliders in the SGID.

A. Selection of Independent Variable

Because a groundtrack can cross back on itself, as in Fig. 2, with multiple latitude values for a given longitude, it is necessary to plot latitude and longitude parametrically. Two options were considered in the selection of the independent variable for the parametric plot: true anomaly, θ , and time, t . There were several key factors considered in the choice of the independent variable. One factor was accuracy: if time were chosen, an approximation would have been necessary to solve Kepler's Equation (see Eq. (4)) for eccentric anomaly, E . An approximate solution could be found several different ways; one method, a Lagrange series, would result in visible errors in the graph for eccentricities higher than 0.6627 (Ref. 1), after which the series diverges. The choice of true anomaly requires no approximation, so that any value of e in the range $0 \leq e < 1$ is acceptable.

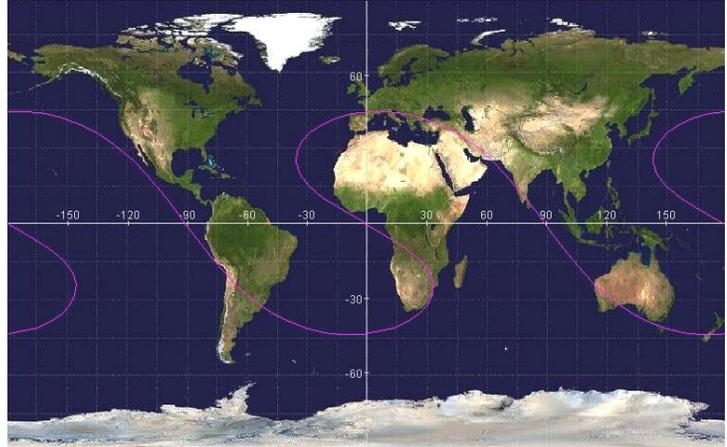


Figure 2. A groundtrack that must be plotted parametrically. An example of an orbit plotted by the SGID whose groundtrack cannot be graphed using Cartesian coordinates.

Another result of the Kepler problem is that an approximation, such as a Lagrange series, will increase the complexity of the equations and result in noticeably slower updates of the plot. This was a very significant problem, with several-second-long delays after any change to the orbital elements.

Another difference between using true anomaly and time as the independent variable is the required range to plot a single orbit. Respectively, the ranges are $0^\circ \leq \theta < 360^\circ$, and $0 \leq t \leq T$, where T is the orbital period in seconds given by Eq. (1) (Ref. 1), and μ is Earth's gravitational parameter.

$$T = \frac{2\pi a^{3/2}}{\sqrt{\mu}} \quad (1)$$

For a plot designed always to display a fixed time range, such as $0 \leq t \leq 20,000$ s, only orbits with a 20,000 second period would show exactly one groundtrack; most orbits would be displayed as a partial groundtrack or multiple groundtracks. It is convenient that the range of θ is constant for any orbit.

A final important difference between true anomaly and time is in the units of their step sizes. True anomaly has the same number of degrees in every step, whereas time has the same number of seconds in every step. For an eccentric orbit, in which a satellite travels more slowly at apogee and more quickly at perigee, a fixed interval of true anomaly will correspond to different time intervals at different points on the orbit. In other words, an animation of a satellite tracing out the groundtrack would show accurate timing if the independent variable of the plot were time, but with true anomaly as the independent variable, such an animation would not reflect the changing speed of the satellite.

Only this last argument favors time as the choice of independent variable; the others indicate that true anomaly will result in a more accurate, smoother plot. Based on these findings, true anomaly was selected as the independent variable.

B. Latitude and Longitude as Functions of the Orbital Elements

The author used the following formulas from Ref. 1 to find subsatellite point latitude and longitude using only the six orbital elements, μ , and Earth's angular velocity, ω_e . In this section, the notation R_i is used to indicate a three-dimensional rotation matrix about the i^{th} axis.

$$p = a(1 - e^2) \quad (2)$$

$$E = 2 \tan^{-1} \left(\sqrt{\frac{1-e}{1+e}} \tan \frac{\theta}{2} \right) \quad (3)$$

$$M = E - e \sin E \quad (4)$$

$$t = \frac{MT}{2\pi} \quad (5)$$

Where p is the semi-latus rectum, E is the eccentric anomaly, M is the mean anomaly, T is the period of the orbit, and t is time.

$$R_{PQW} = \begin{bmatrix} \left(\frac{p \cos \theta}{1+e \cos \theta} \right) \\ \left(\frac{p \sin \theta}{1+e \cos \theta} \right) \\ (0) \end{bmatrix} \quad (6)$$

$$Q = R_3(-\Omega)R_1(-i)R_3(-\omega) \quad (7)$$

$$R_{IJK} = QR_{PQW} \quad (8)$$

$$\theta_G = \omega_e t \quad (9)$$

$$R_{ECEF} = R_3(\theta_G) R_{IJK} \quad (10)$$

Where R_{PQW} is the vector pointing to the satellite in the reference frame of the orbital plane (the perifocal coordinate system), R_{IJK} is the vector to the satellite in the Earth-Centered Inertial reference frame, and Q is the rotation matrix that translates between them. θ_G is the Greenwich sidereal time, which corresponds to the angle the Earth has rotated from a reference position, and R_{ECEF} is the satellite position vector in the Earth-Centered, Earth-Fixed reference frame.

$$\lambda = \tan^{-1} \left(\frac{R_{ECEF} [2]}{R_{ECEF} [1]} \right) \quad (11)$$

$$\varphi = \tan^{-1} \left(\frac{R_{ECEF} [3]}{\sqrt{R_{ECEF} [2]^2 + R_{ECEF} [1]^2}} \right) \quad (12)$$

Where λ is the longitude and φ is the latitude of the subsatellite point.

C. Selection of Medium for the SGID

Two main options were considered as possible methods of displaying the groundtracks. One approach involves precomputation of the groundtracks for a large but finite set of orbits. The other approach is to dynamically calculate the groundtracks every time the user adjusts an element. The benefit of precomputation is that the complicated orbital equations would not have to be evaluated while the tool is being used; instead, the groundtrack data for a given orbit would be calculated only once, then saved as an image or data file containing a set of points. Then the tool would only need to display each image in quick succession to create the illusion of motion. However, it would take massive amounts of computing time and storage space to create images for enough orbits to allow users to freely change any element at any point in the animation. For example, if the range of each of the five constant orbital elements were broken into thirty-six intervals (i.e., ten degrees per interval for the angular elements), the required number of images would be 36^5 , or 60,466,176 images! To cut down on this number, the flexibility of the tool would need to be reduced by creating a finite number of possible paths: at the start, the user could change any element, but then must proceed in changing that element until reaching the next “fork,” at which the other elements can be changed because data for the corresponding orbits are now available. Based on the limited versatility and potentially huge storage requirements of the precomputation approach, a method of dynamic groundtrack calculation was selected, with plans to assess the computation speed, and keep calculations to a minimum where possible.

Java Components for Mathematics² (JCM), written in the Java programming language, is a set of configurable applets (and their source code) capable of graphing arbitrary functions as the user provides input. The products of JCM are meant to be used for interactive learning exercises like the SGID. Among JCM’s configurable applets are the “Parametric” and “FamiliesOfGraphs” applets. In FamiliesOfGraphs, sliders like those shown in Fig. 1 are provided which can be used to adjust parameters such as orbital elements. However, the FamiliesOfGraphs applet does not support parametric plotting. Therefore, the next challenge in the creation of the SGID was to create a new applet that combines the functionality of JCM’s FamiliesOfGraphs and Parametric applets, while incorporating additional features necessary for groundtrack display.

III. Developing the Applet

There were several programming steps taken to ensure proper display of the groundtracks. Default plot properties were set, such as setting the x- and y-limits of the graph to $-180 \leq x \leq 180$, $-90 \leq y \leq 90$, corresponding to the ranges of longitude and latitude, respectively. The orbital element sliders and their ranges were added to the source code. The range $6378 \text{ km} \leq a \leq 60000 \text{ km}$ allows the user to enter semi-major axis values from the surface of the Earth up to supersynchronous altitudes, and the ranges for the other elements reflect the standard possible values for closed orbits: $0 \leq e \leq 0.99$, $0^\circ \leq i \leq 180^\circ$, $0^\circ \leq \omega \leq 360^\circ$, and $0^\circ \leq \Omega \leq 360^\circ$. The variables μ and ω_e were defined as $\mu = 398600 \text{ km}^3/\text{s}^2$ and $\omega_e = 0.00417807413^\circ/\text{s}$. The formulas established in this paper for latitude and longitude were added to the applet as a new parametric curve, and the limits of the independent variable θ were set to $-180^\circ \leq \theta \leq 180^\circ$.

A. Custom Functions

The built-in trigonometric functions in JCM calculate angles using units of radians. However, it is standard to specify the angular orbital elements using degrees. Therefore, it was necessary to add new trigonometric functions to the applet. For example, $\text{asind}(v)$ was defined as $\sin(v\pi/180)$, and $\text{atand}(v) = \arctan(v)*180/\pi$. However, referring again to Kepler’s Equation (Eq. (4)), the secular term E must be evaluated in radians. So, the inverse tangent in Eq. (3) and the sine in Eq. (4) use the default JCM functions, while all other angles were evaluated using the custom trigonometric functions with arguments in degrees.

Additionally, the arctangent function included in JCM has an output range of only -90° to 90° , whereas longitude ranges from -180° to 180° . A two-argument arctangent is required for this application, which can be created as follows:

$$\text{atand2}(\text{opp}, \text{adj}) = \text{adj} > 0 ? \text{atand}\left(\frac{\text{opp}}{\text{adj}}\right) : \text{opp} > 0 ? \left(\text{atand}\left(\frac{\text{opp}}{\text{adj}}\right) + 180\right) : \left(\text{atand}\left(\frac{\text{opp}}{\text{adj}}\right) - 180\right) \quad (13)$$

This formula for atand2 takes in two variables, opp and adj . If the point (adj, opp) in Fig. 3 is in quadrant 1 or 4, then adj is positive, and the function atand2 is equal to atand . If adj is negative but opp is positive (quadrant 2), then $\text{atand2} = \text{atand} + 180$. If both adj and opp are negative (quadrant 3), then $\text{atand2} = \text{atand} - 180$. As desired, the output range of atand2 is -180 to 180 degrees. This new function is used in the calculation of longitude in Eq. (11).

B. Displaying Two Orbits

As mentioned above, the range of true anomaly used in the SGID is $-180^\circ \leq \theta \leq 180^\circ$, which corresponds to one full orbit. Even if the total range were increased beyond 360° , the groundtrack graph would still display only one orbit. Any angle greater than 180° adds no new information to the graph because, for example, 181° is the same as -179° (both are in quadrant 3); increasing the range of true anomaly will only cause the graph to trace over itself.

This is important because a groundtrack representing only one orbit does not communicate very much information. An example of this is shown in Fig. 4, in which the groundtrack shown in Fig. 2 is reduced to a single orbit.

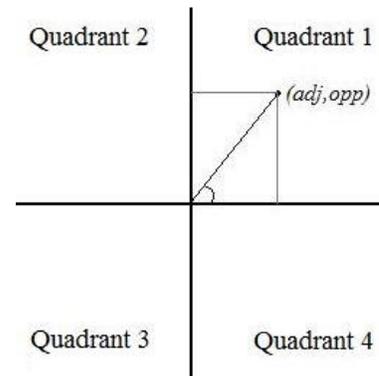


Figure 3. The four quadrants of a Cartesian coordinate system.

² <http://math.hws.edu/javamath/>

In order to display exactly two orbits in the SGID, a second parametric curve is added to the applet. The latitude used in this new curve is the same as the original, but the longitude is shifted to *begin* at the *end* of the original curve. In order to understand this transformation, see Fig. 5. The graph in part (a) shows a groundtrack of a single orbit, with its starting point ($\theta = -180^\circ$) and ending point ($\theta = 180^\circ$) labeled. There are two black arrows; the shorter arrow indicates the vector from zero longitude to the start point, and the longer arrow indicates the vector to the end point. A red “X” marks an arbitrary station along the groundtrack. In part (b), the arbitrary station is translated to its new location exactly one orbit later by subtracting the start-vector (note that the direction of the arrow has changed) and adding the end-vector. This point is indicated by the green “X.” The formula for the longitude used in the second parametric curve can be written as shown in Eq. (14):

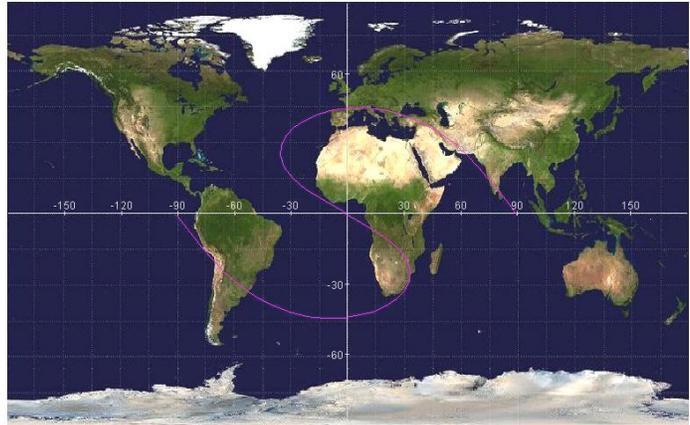


Figure 4. A groundtrack showing only one orbit. Compare to Fig. 2, which shows the same groundtrack with two orbits.

$$\lambda_2 = \lambda - \lambda(\theta = -180^\circ) + \lambda(\theta = 180^\circ) \tag{14}$$

It is possible for the formula shown in Eq. (14) to produce values of longitude outside the range of the graph. Any values greater than 180° or less than -180° must be shifted by 360° to be displayed properly. A new function, *abs2*, was written to accomplish this:

$$\text{abs2}(v) = v > 180 ? (v - 360) : v < -180 ? (v + 360) : v \tag{15}$$

The final formula for the longitude used to plot the second orbit is given in Eq. (16):

$$\lambda_2 = \text{abs2}(\lambda - \lambda(\theta = -180^\circ) + \lambda(\theta = 180^\circ)) \tag{16}$$

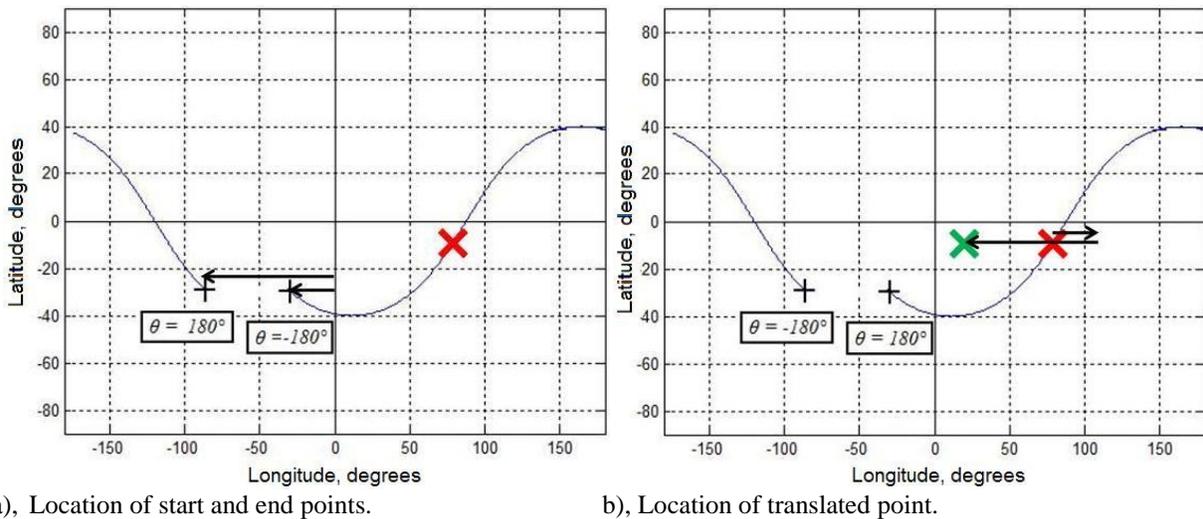


Figure 5. Transformation of longitude for displaying two orbits. These graphs illustrate the procedure for calculating the translated longitude of a subsatellite point after one full orbit.

C. Preloaded Example Orbits

One of the features of JCM is the ability to include a drop-down menu of examples at the top of an applet. For the SGID, the author customized this capability to read in example orbits from strings with this format: “Orbit

Name; a, e, i, ω, Ω ". These example orbits can be defined on the web page that calls the applet, so that a professor including the SGID on a web site could specify the orbits which appear on the menu. The preloaded orbits on the SGID web site are a selection of typical orbits that every student should be familiar with (Circular, Molniya, Sun-Synchronous, and Geosynchronous), along with a few examples which are referenced in the guided learning questions.

IV. SGID Guided Learning Questions

The SGID website presents the applet described above along with a series of technical and conceptual questions. These questions were designed to guide students in their use of the applet, and they cover several introductory orbital mechanics topics. This section describes each question and discusses their solutions.

A. Start from the Circular Example. What is the approximate subsatellite point when the satellite starts its orbit?

Answer: Noticing from Fig. 1 that the Circular Example is a direct orbit ($i < 90^\circ$), which travels from west to east, the start of the orbit can be found at the point $(-165, 0)$.

1. Now change ω to 250° . What is the new subsatellite point? Answer: (75, 42)
2. Now change i to 85° . What is the new subsatellite point? Answer: (28, 67)
3. Now change Ω to 300° . What is the new subsatellite point? Answer: $(-30, 67)$

The purpose of this question is to introduce students to locating a subsatellite point on the axes of the graph and experimenting with adjusting the sliders for the first time. When changing the argument of perigee, students will notice a snake-like motion of the curve. Changing the inclination will stretch the curve vertically. Increasing the right ascension of the ascending node will shift the curve eastward.

B. All groundtracks plotted with this tool represent two orbits. In which of the first four examples (Circular, Molniya, Sun-Synchronous, and Geosynchronous) would the groundtrack change if three orbits were shown? When would a repeating ground track be useful?

Answer: Because the Circular and Sun-Synchronous groundtracks have visible starting and ending points, an additional orbit would extend the groundtrack at one of these points. The Molniya and Geosynchronous groundtracks are closed curves, and a third orbit would simply trace over the existing graph.

When a satellite is meant to provide coverage of a certain ground station, a repeating groundtrack ensures that the satellite and ground station will be in contact with each other on a regular schedule.

C. What is the orbital period, in hours, of the Molniya orbit? The Geosynchronous orbit?

Answer: Using Eq. (1) and the displayed values of semi-major axis, the period of the Molniya orbit is $T_M = 2\pi(26561.7^{3/2})/\sqrt{398600} = 43082 \text{ s} = 11.97 \text{ hr}$, and the Geosynchronous period is $T_G = 2\pi(42164.2^{3/2})/\sqrt{398600} = 86164 \text{ s} = 23.93 \text{ hr}$.

This is the first question that requires students to read values from the sliders and perform calculations, using the equations they have learned in class or from a textbook. Students will find that the Molniya and Geosynchronous periods are about 12 hours and 24 hours respectively. This reiterates the concept that the period required for a repeating groundtrack is dependent on the period of the Earth's rotation.

1. What two values of Ω will produce a Molniya groundtrack that hovers over a 60° longitude?

Answer: 240° and 60°

This question is meant to reinforce the student's understanding of the connection between the location of a ground station and the selection of a satellite's orbit.

D. Load the Perigee Example and calculate the perigee radius and perigee altitude for this orbit. What is the physical meaning of your results?

Answer: Using the equations $r_p = a(1-e)$ and $z_p = r_p - R_E$, where $R_E = 6378 \text{ km}$, students will find that $r_p = 8000(1-0.7) = 2400 \text{ km}$ and $z_p = 2400 - 6378 = -3978 \text{ km}$. A negative altitude at perigee means that the orbit is intersecting Earth.

This question shows that even though the SGID will plot any selection of orbital elements, not all combinations of elements represent a viable Earth orbit. Students are reminded that orbits exist in three dimensions, and some information (such as intersection with the Earth) may be lost in a two-dimensional representation such as a groundtrack.

E. Load the Retrograde Example. Watch what happens when you change inclination from 105° to 75° . Which groundtrack covers more of the Earth's surface during the two orbits plotted? Look up retrograde orbits and briefly explain how one orbit can go farther and "faster" in reference to the Earth than another, while their orbital speeds are the same.

Answer: As shown in Fig. 6 (a), when $i = 105^\circ$, the satellite crosses the Earth about three and a half times. When $i = 75^\circ$ as shown in part (b), the satellite only crosses about half of the Earth. The retrograde ($i = 105^\circ$) orbit covers much more land. This is because the direction of retrograde orbital motion opposes the direction of the Earth's rotation. The orbit's speed relative to the inertial reference frame is constant despite any inclination changes, but the retrograde orbit covers more of the Earth's surface than a direct orbit because its speed relative to the rotating Earth is greater.

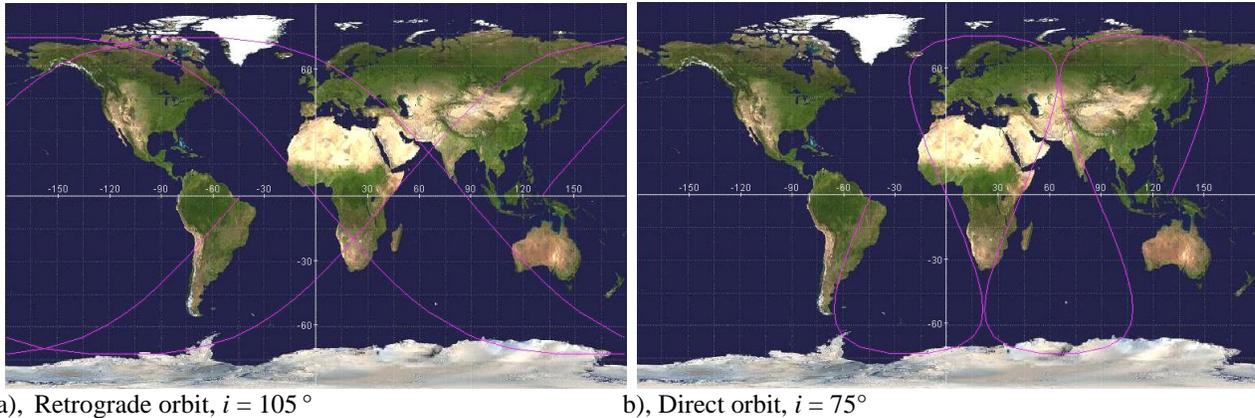


Figure 6. Two nearly identical orbits. *These plots show an orbit before and after a 30° inclination change, going from retrograde to prograde.*

F. You want to design an orbit that will enable a satellite to take scans of 90% of the Earth's surface in its lifetime. Starting from the Circular orbit, vary one orbital element to achieve this. Which element is critical, and what should its value be?

Answer: An inclination of 85° will ensure that over time, the satellite will see all the Earth's surface except at the poles. A near-polar orbit is often chosen for satellites that must take data or images of the Earth.

G. Design a second orbit so that a satellite will never see more than 10% of the Earth's surface. Starting from the Circular orbit, vary one orbital element to achieve this. Which element is critical (different than your previous answer), and what should its value be?

Answer: One approach to this problem would be to decrease the satellite's inclination, but this is not an acceptable response, because the element changed must be "different than your previous answer." Instead, the problem is solved by changing the semi-major axis to a geosynchronous altitude ($a = 42164$ km), which will cause the satellite to perform "figure-eights" over a fixed longitude, never seeing most of the planet.

V. Conclusion

Due to the flexibility of the orbital element sliders, the capability to preload examples, and the ease of access to Java applets, the SGID has the potential to teach students about many more topics in Orbital Mechanics than are covered in these example questions. Professors and students are encouraged to experiment with finding interesting groundtracks, imagine practical applications, and write their own questions. In fact, users are encouraged to create their own "learning tool" to examine any topic in orbital mechanics that interests them; the development of a project like the SGID is an excellent way to test one's knowledge of satellite motion while helping others learn.

Acknowledgments

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