RSA Encryption

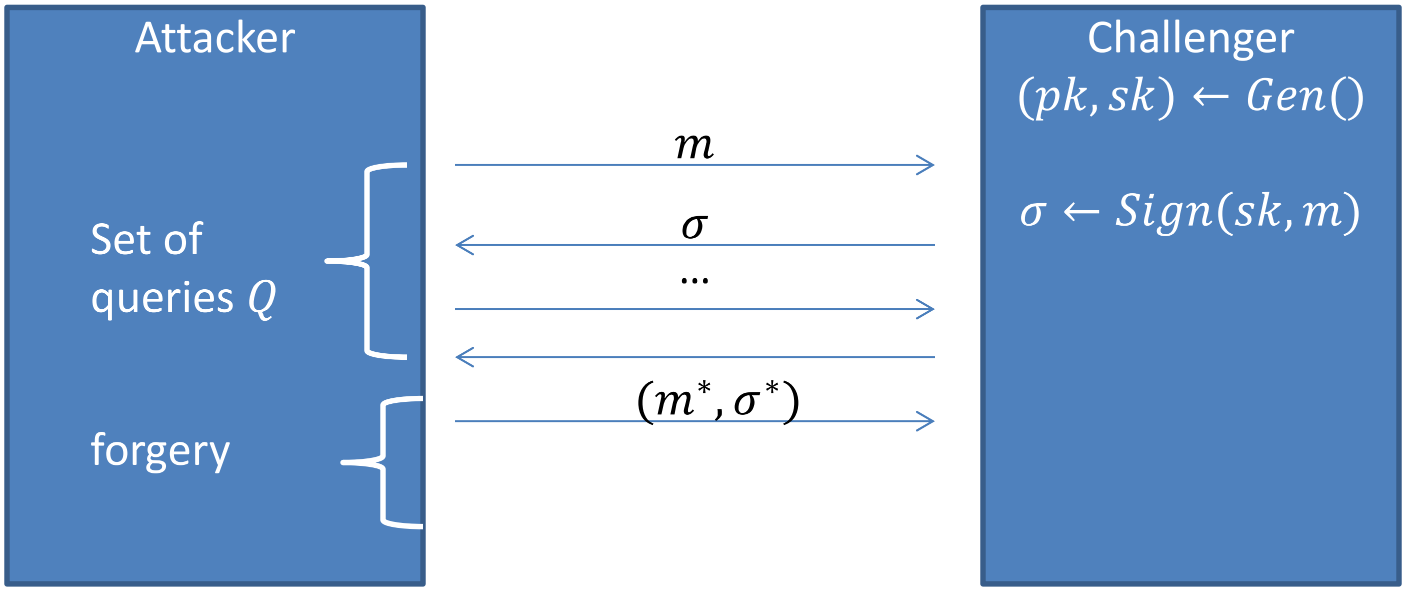
CONSTRUCTION 11.25

Let GenRSA be as in the text. Define a public-key encryption scheme as follows:

- Gen: on input 1^n run GenRSA(1^n) to obtain N, e, and d. The public key is $\langle N, e\rangle$ and the private key is $\langle N, d\rangle$.
- Enc: on input a public key $pk = \langle N, e\rangle$ and a message $m \in \mathbb{Z}_N^*$, compute the ciphertext
$$c := [m^e \bmod N].$$
- Dec: on input a private key $sk = \langle N, d\rangle$ and a ciphertext $c \in \mathbb{Z}_N^*$, compute the message
$$m := [c^d \bmod N].$$

The plain RSA encryption scheme.

Existential Unforgeability of Signatures under CMA



Attacker "wins" if :

1. $m^* \notin Q$
2. $Vrfy(pk, m^*, \sigma^*) = 1$

Security Requirement: Any efficient attacker wins with probability at most $negligible$

RSA Signatures

CONSTRUCTION 12.5

Let GenRSA be as in the text. Define a signature scheme as follows:

- Gen: on input 1^n run GenRSA(1^n) to obtain (N, e, d). The public key is $\langle N, e \rangle$ and the private key is $\langle N, d \rangle$.
- Sign: on input a private key $sk = \langle N, d \rangle$ and a message $m \in \mathbb{Z}_N^*$, compute the signature

$$\sigma := [m^d \bmod N].$$

- Vrfy: on input a public key $pk = \langle N, e \rangle$, a message $m \in \mathbb{Z}_N^*$, and a signature $\sigma \in \mathbb{Z}_N^*$, output 1 if and only if

$$m \stackrel{?}{=} [\sigma^e \bmod N].$$

The plain RSA signature scheme.

Attacks

No message attack:

Choose $s \in Z_N^*$, compute s^e.

Ouput $(m = s^e, \sigma = s)$ as the forgery.

Attacks

Forging a signature on an arbitrary message:

To forge a signature on message m, choose arbitrary $m_1, m_2 \neq 1$ such that $m = m_1 \cdot m_2$.

Query oracle for $(m_1, \sigma_1), (m_2, \sigma_2)$.

Output (m, σ), where $\sigma = \sigma_1 \cdot \sigma_2$.

RSA-FDH

CONSTRUCTION 12.6

Let GenRSA be as in the previous sections, and construct a signature scheme as follows:

- Gen: on input 1^n, run GenRSA(1^n) to compute (N, e, d). The public key is $\langle N, e \rangle$ and the private key is $\langle N, d \rangle$.

  As part of key generation, a function $H : \{0,1\}^* \to \mathbb{Z}_N^*$ is specified, but we leave this implicit.
- Sign: on input a private key $\langle N, d \rangle$ and a message $m \in \{0,1\}^*$, compute
$$\sigma := [H(m)^d \bmod N].$$
- Vrfy: on input a public key $\langle N, e \rangle$, a message m, and a signature σ, output 1 if and only if $\sigma^e \stackrel{?}{=} H(m) \bmod N$.

The RSA-FDH signature scheme.

Certificates and Public-Key Infrastructure

A single certificate authority

- pk_{CA} must be distributed over an authenticated channel
  - Need only be carried out once
- Usually, pk_{CA} included in browser, browser programmed to automatically verify certificates as they arrive.
- To obtain certificate, must prove that url is legitimate.
- All parties must completely trust CA.

Multiple certificate authorities

- Parties can choose which CA to use to obtain a certificate.
- Parties can choose which CA's certificates to trust.
- Problem: some CA may become compromised.
- Each user must manually decide which CA to trust.

Delegation and certificate chains

- Example of certificate chain:

$$pk_A, cert_{B \to A}, pk_B, cert_{C \to B}$$

Need only trust Charlie in the above example.

- Certificate asserts that legitimate party holds public key and *that the party is trusted to issue other certificates*.
  - Delegation of CA's ability to issue certificates

The "web of trust" model

- Model is used by PGP ("pretty good privacy") email encryption software for distribution of public keys.
- Anyone can issue certificates to anyone else
- Each user must decide who to trust
- Example:
  - Alice holds pk_1, pk_2, pk_3 for users C_1, C_2, C_3
  - Bob has certificates $cert_{C_1 \rightarrow B}, cert_{C_3 \rightarrow B}, cert_{C_4 \rightarrow B}$
- Public keys and certificates can be stored in a central database.

Invalidating Certificates

- Expiration: Include expiration date as part of the certificate.
  - Very coarse grained method. E.g. employee leaves company but certificate does not expire for a year.
- Revocation
  - CA includes a serial number in every certiciate it issues.
  - At the end of each day, the CA will generate a certificate revocation list (CRL) with the serial numbers of all revoked certificates.
  - CA will sign the CRL and the current date.
  - Signed CRL is then widely distributed.

Putting it all together: SSL/TLS

- TLS: Transport Layer Security Protocol
  - Protocol used by browser when connecting via https
- Standardized protocol based on a precursor called SSL (Secure Socket Layer).
  - Latest SSL version: SSL 3.0
  - TLS version 1.0 released in 1999
  - TLS version 1.1 in 2006
  - TLS version 1.2 (current) in 2008
  - 50% of browsers still use TLS 1.0
- Allows a client (web browser) and a server (website) to agree on a set of shared keys and then use those keys to encrypt and authenticate their subsequent communication.
- Two parts:
  - Handshake protocol performs authenticated key exchange to establish the shared keys
  - Record-layer protocol uses shared keys to encrypt/authenticated the communication.
- Typically used for authentication of servers to clients (usually only servers—websites—have certificates).