Introduction to Secure Multi-Party Computation

Computer Systems Security

ENEE 457/CMSC 498E

Based on notes from:

Vitaly Shmatikov

Motivation

- General framework for describing computation between parties who do not trust each other
- Example: elections
  - N parties, each one has a "Yes" or "No" vote
  - Goal: determine whether the majority voted "Yes", but no voter should learn how other people voted
- Example: auctions
  - Each bidder makes an offer
    - Offer should be committing! (can't change it later)
  - Goal: determine whose offer won without revealing losing offers

More Examples

- Example: distributed data mining
  - Two companies want to compare their datasets without revealing them
    - For example, compute the intersection of two lists of names
- Example: database privacy
  - Evaluate a query on the database without revealing the query to the database owner
  - Evaluate a statistical query on the database without revealing the values of individual entries
  - Many variations

A Couple of Observations

- In all cases, we are dealing with distributed multi-party protocols
  - A protocol describes how parties are supposed to exchange messages on the network
- All of these tasks can be easily computed by a trusted third party
  - The goal of secure multi-party computation is to achieve the same result without involving a trusted third party

How to Define Security?

- Must be mathematically rigorous
- Must capture <u>all</u> realistic attacks that a malicious participant may try to stage
- Should be "abstract"
  - Based on the desired "functionality" of the protocol, not a specific protocol
  - Goal: define security for an entire class of protocols

Functionality

- K mutually distrustful parties want to jointly carry out some task
- Model this task as a function

$$f: (\{0,1\}^*)^K \rightarrow (\{0,1\}^*)^K$$

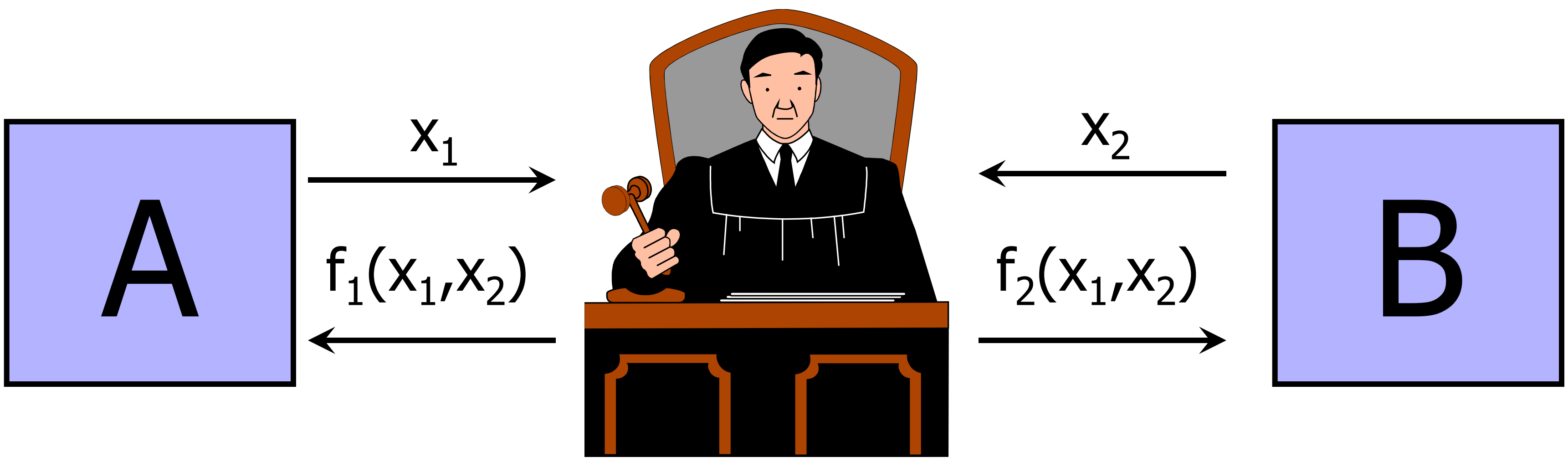
K inputs (one per party); each input is a bitstring

K outputs

- Assume that this functionality is computable in probabilistic polynomial time

Ideal Model

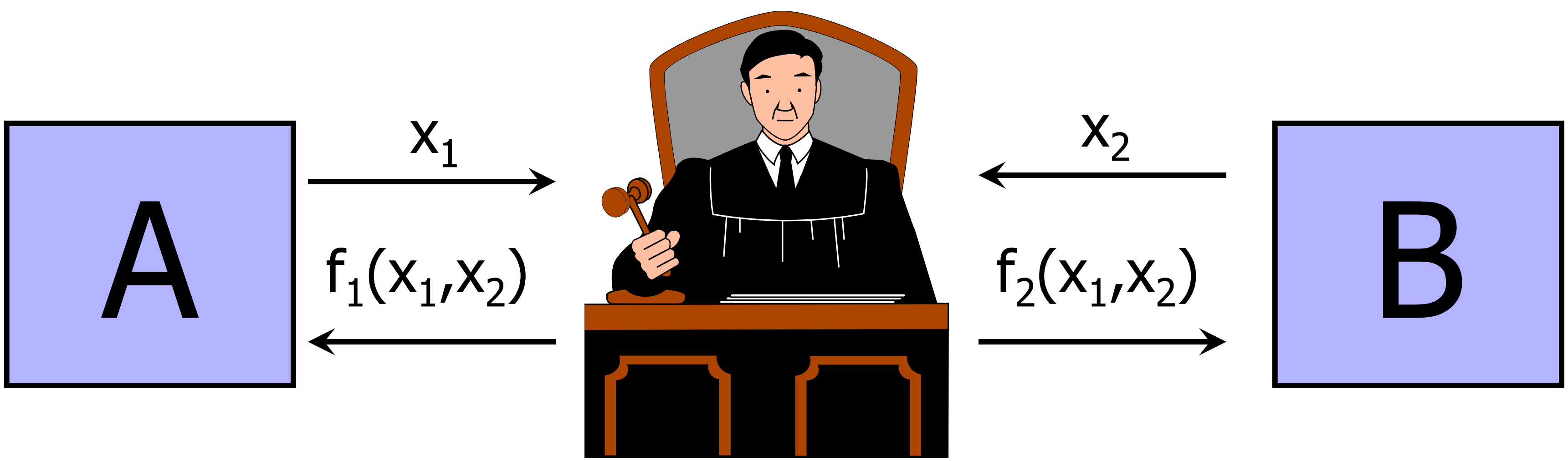
- Intuitively, we want the protocol to behave “as if” a trusted third party collected the parties’ inputs and computed the desired functionality
  - Computation in the ideal model is secure by definition!



Slightly More Formally

- A protocol is secure if it emulates an ideal setting where the parties hand their inputs to a "trusted party," who locally computes the desired outputs and hands them back to the parties

[Goldreich-Micali-Wigderson 1987]



Adversary Models

- Some of protocol participants may be corrupt
  - If all were honest, would not need secure multi-party computation
- Semi-honest (aka passive; honest-but-curious)
  - Follows protocol, but tries to learn more from received messages than he would learn in the ideal model
- Malicious
  - Deviates from the protocol in arbitrary ways, lies about his inputs, may quit at any point
- For now, we will focus on two-party protocols

Correctness and Security

- How do we argue that the real protocol "emulates" the ideal protocol?
- Correctness
  - All honest participants should receive the correct result of evaluating function f
    - Because a trusted third party would compute f correctly
- Security
  - All corrupt participants should learn no more from the protocol than what they would learn in ideal model
  - What does corrupt participant learn in ideal model?
    - His input (obviously) and the result of evaluating f

Simulation

- Corrupt participant's view of the protocol = record of messages sent and received
  - In the ideal world, view consists simply of his input and the result of evaluating f
- How to argue that real protocol does not leak more useful information than ideal-world view?
- Key idea: simulation
  - If real-world view (i.e., messages received in the real protocol) can be simulated with access only to the ideal-world view, then real-world protocol is secure
  - Simulation must be <u>indistinguishable</u> from real view

SMC Definition (First Attempt)

- Protocol for computing f(*,*) betw. A and B is secure if there exist efficient simulator algorithms S_A and S_B such that
- Correctness: for all input pairs, prot. output is correct.
- Intuition: outputs received by <u>honest</u> parties are indistinguishable from the correct result of evaluating f
- Security: $view_A$(real protocol) $\approx$ S_A (gets to query ideal functionality)
  $view_B$(real protocol) $\approx$ S_B (gets to query ideal functionality)
  - Intuition: a <u>corrupt</u> party's view of the protocol can be simulated from its input and output
- This definition does not work! Why?

Randomized Ideal Functionality

- Consider a coin flipping functionality

  f()=(-,b) where b is random bit

  - f() flips a coin and tells B the result; A gets no output
- The following protocol "implements" f()
  1. A chooses bit b randomly
  2. A sends b to B
  3. B outputs b
- It is obviously insecure (why?)
- Yet it is correct and simulatable according to our attempted definition (why?)

SMC Definition

- Protocol for computing f(*,*) betw. A and B is secure if there exist efficient simulator algorithms S_A and S_B such that:
- Correctness: for all input pairs, prot. output is correct.
- Security:

$$(\text{view}_A(\text{real prot}), \text{output}_B(\text{real prot})) \approx (S_A, y_B)$$

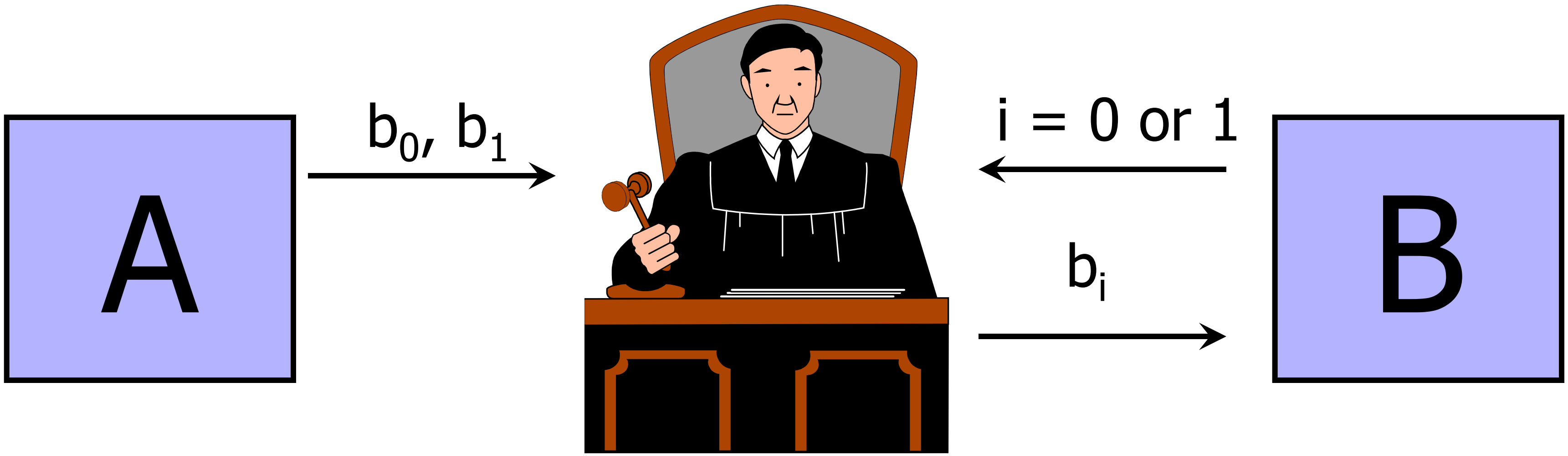
$$(\text{view}_B(\text{real prot}), \text{output}_A(\text{real prot})) \approx (S_B, y_A)$$

  - Intuition: if a corrupt party's view of the protocol is correlated with the honest party's output, the simulator must be able to capture this correlation
- Does this fix the problem with coin-flipping f?

Oblivious Transfer (OT)

[Rabin 1981]

◆ Fundamental SMC primitive

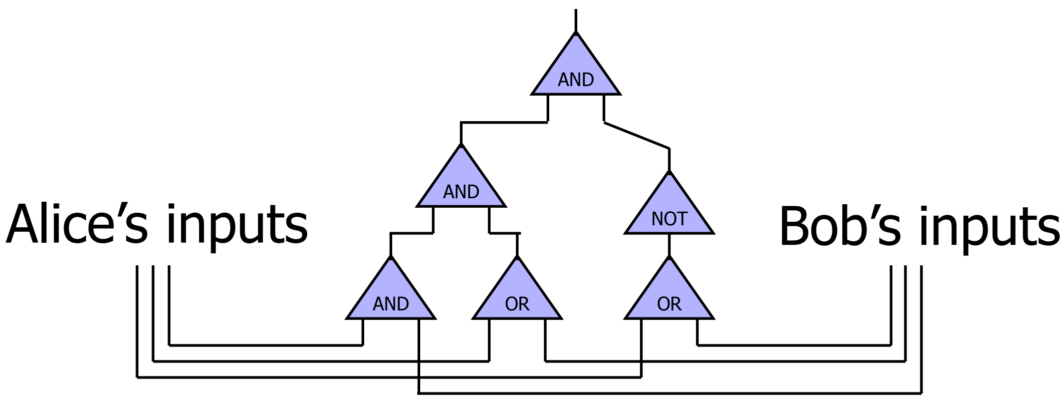


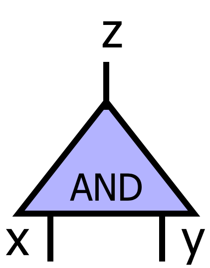
- A inputs two bits, B inputs the index of one of A's bits
- B learns his chosen bit, A learns nothing
  – A does not learn which bit B has chosen; B does not learn the value of the bit that he did not choose
- Generalizes to bitstrings, M instead of 2, etc.

Yao's Protocol

Yao’s Protocol

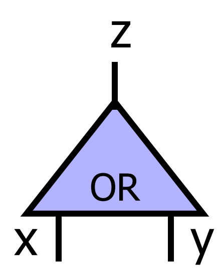
- Compute any function securely
  - … in the semi-honest model
- First, convert the function into a boolean circuit





Truth table:

x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

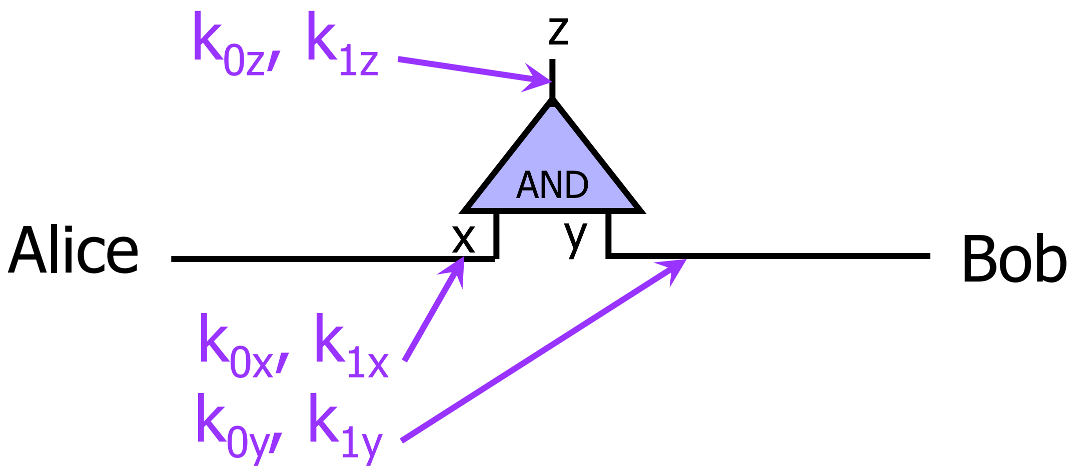


Truth table:

x	y	z
0	0	0
0	1	1
1	0	1
1	1	1

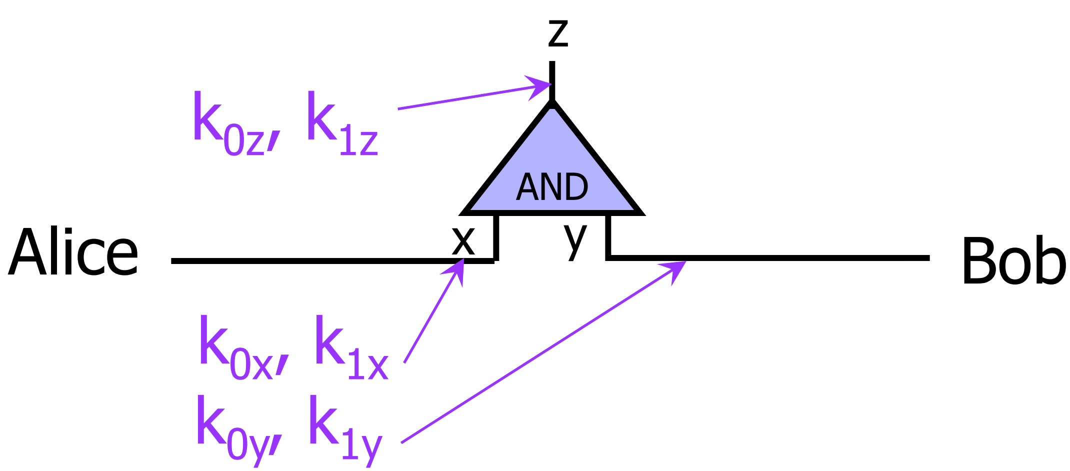
1: Pick Random Keys For Each Wire

- Next, evaluate <u>one gate</u> securely
  - Later, generalize to the entire circuit
- Alice picks two random keys for each wire
  - One key corresponds to "0", the other to "1"
  - 6 keys in total for a gate with 2 input wires



2: Encrypt Truth Table

- Alice encrypts each row of the truth table by encrypting the output-wire key with the corresponding pair of input-wire keys



Original truth table:

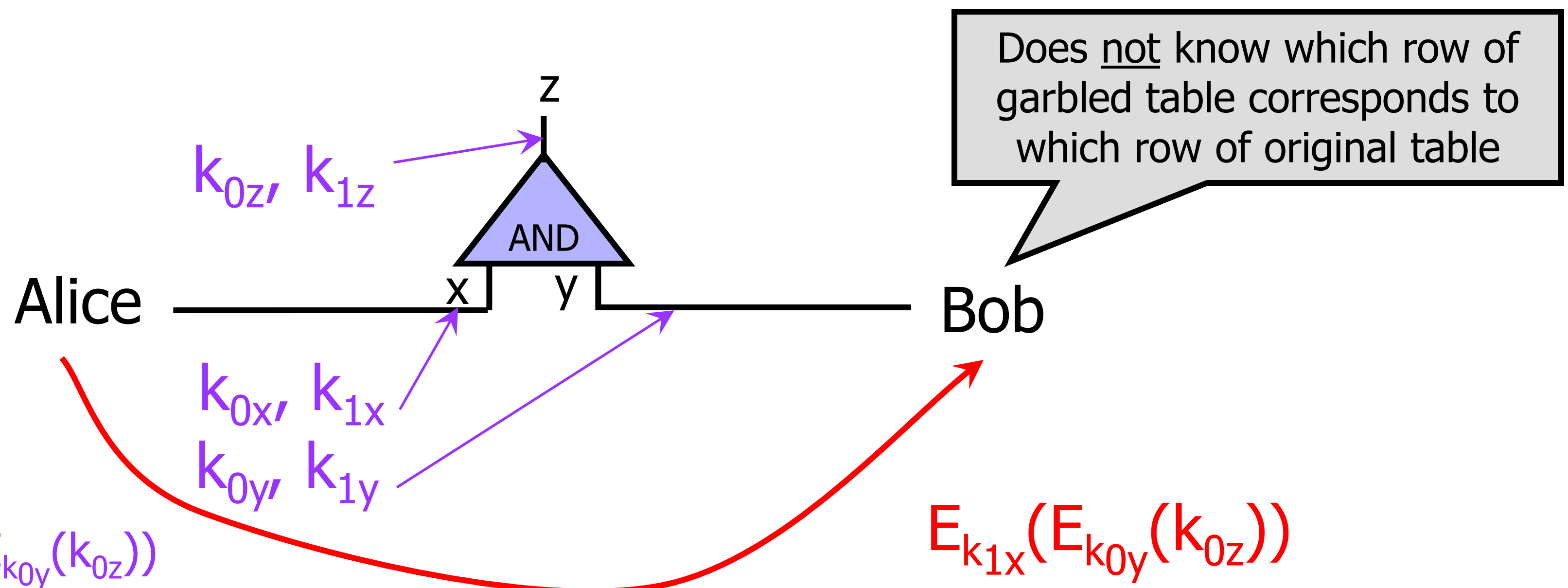
x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

Encrypted truth table:

$$E_{k_{0x}}(E_{k_{0y}}(k_{0z}))$$
$$E_{k_{0x}}(E_{k_{1y}}(k_{0z}))$$
$$E_{k_{1x}}(E_{k_{0y}}(k_{0z}))$$
$$E_{k_{1x}}(E_{k_{1y}}(k_{1z}))$$

3: Send Garbled Truth Table

- Alice randomly permutes ("garbles") encrypted truth table and sends it to Bob



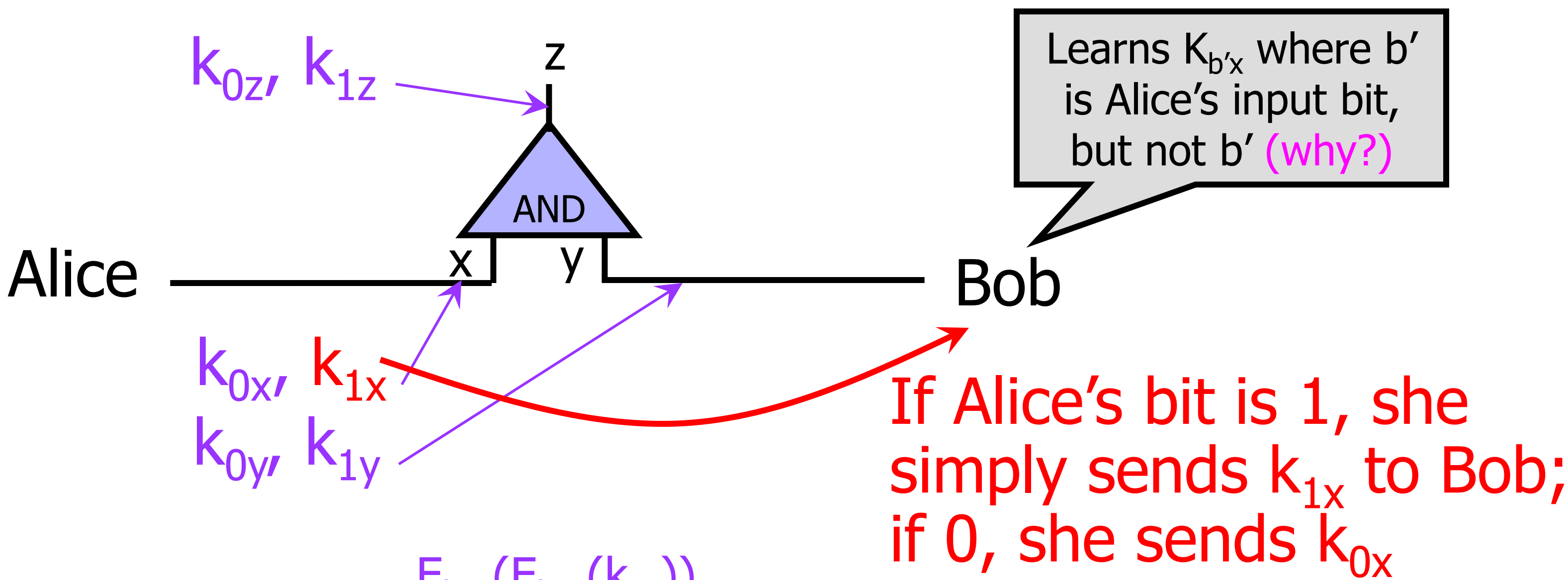
$E_{k0x}(E_{k0y}(k_{0z}))$
$E_{k0x}(E_{k1y}(k_{0z}))$
$E_{k1x}(E_{k0y}(k_{0z}))$
$E_{k1x}(E_{k1y}(k_{1z}))$

Garbled truth table:

$E_{k1x}(E_{k0y}(k_{0z}))$
$E_{k0x}(E_{k1y}(k_{0z}))$
$E_{k1x}(E_{k1y}(k_{1z}))$
$E_{k0x}(E_{k0y}(k_{0z}))$

4: Send Keys For Alice's Inputs

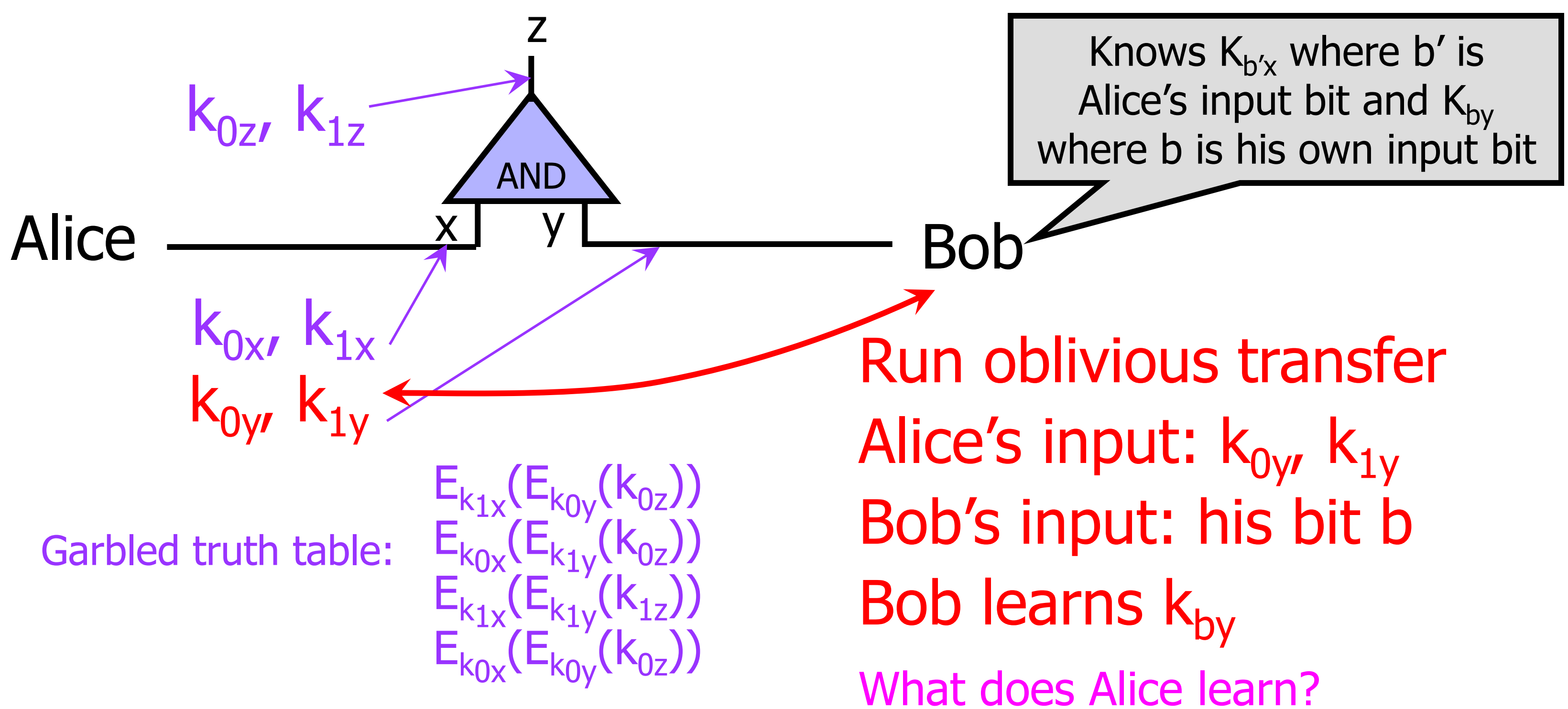
- Alice sends the key corresponding to her input bit
  - Keys are random, so Bob does not learn what this bit is



Garbled truth table:
$E_{k_{1x}}(E_{k_{0y}}(k_{0z}))$
$E_{k_{0x}}(E_{k_{1y}}(k_{0z}))$
$E_{k_{1x}}(E_{k_{1y}}(k_{1z}))$
$E_{k_{0x}}(E_{k_{0y}}(k_{0z}))$

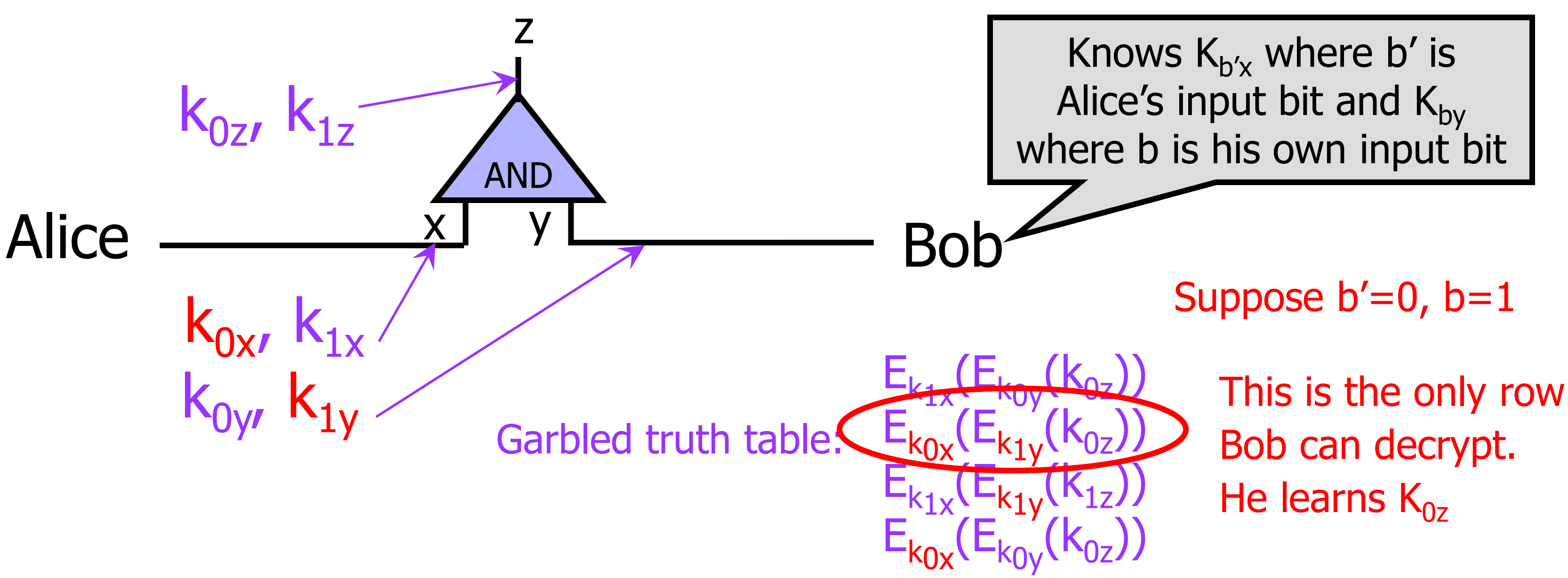
5: Use OT on Keys for Bob’s Input

- Alice and Bob run oblivious transfer protocol
  - Alice’s input is the two keys corresponding to Bob’s wire
  - Bob’s input into OT is simply his 1-bit input on that wire



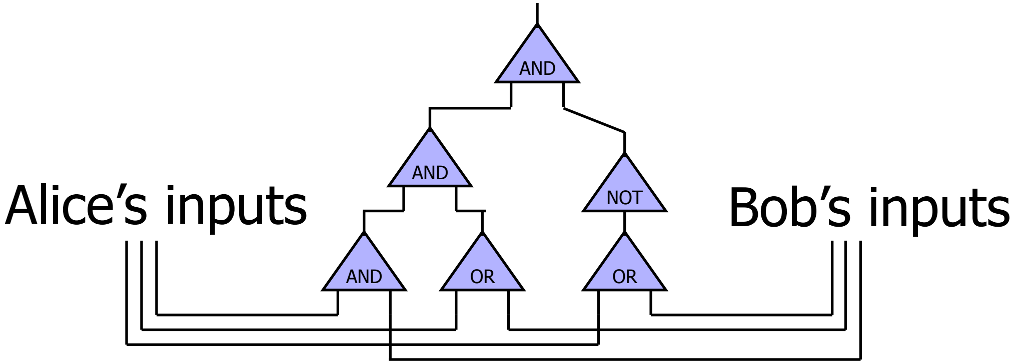
6: Evaluate Garbled Gate

- Using the two keys that he learned, Bob decrypts exactly one of the output-wire keys
  - Bob does not learn if this key corresponds to 0 or 1
    - Why is this important?



7: Evaluate Entire Circuit

- In this way, Bob evaluates entire garbled circuit
  - For each wire in the circuit, Bob learns only one key
  - It corresponds to 0 or 1 (Bob does not know which)
    - Therefore, Bob does not learn intermediate values (why?)



- Bob tells Alice the key for the final output wire and she tells him if it corresponds to 0 or 1
  - Bob does not tell her intermediate wire keys (why?)

Brief Discussion of Yao's Protocol

- Function must be converted into a circuit
  - For many functions, circuit will be huge
- If m gates in the circuit and n inputs, then need 4m encryptions and n oblivious transfers
  - Oblivious transfers for all inputs can be done in parallel
- Yao's construction gives a constant-round protocol for secure computation of any function in the semi-honest model
  - Number of rounds does not depend on the number of inputs or the size of the circuit!