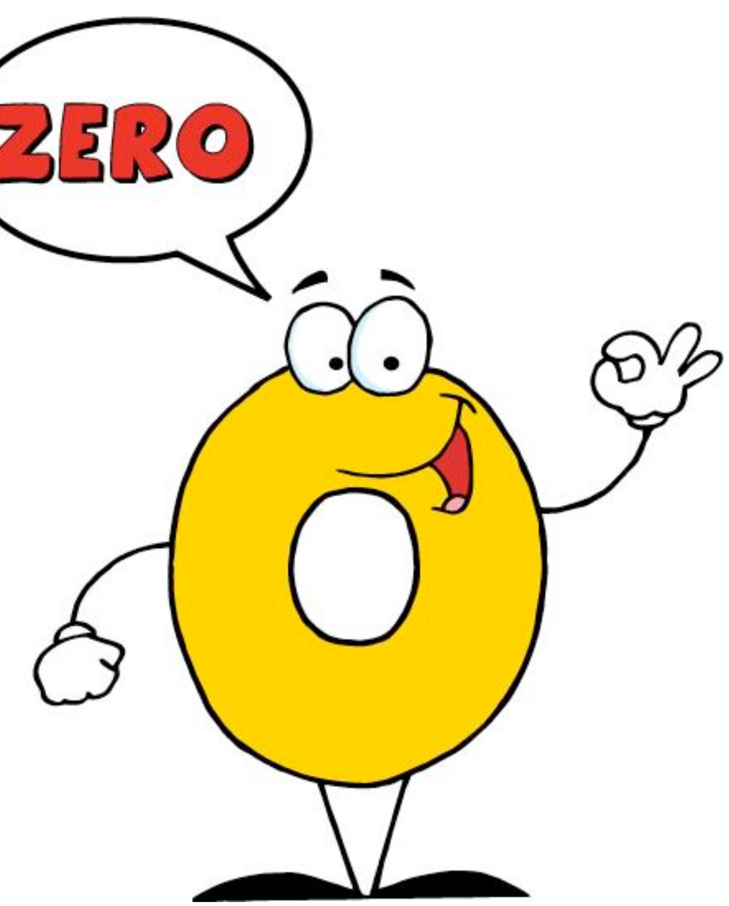
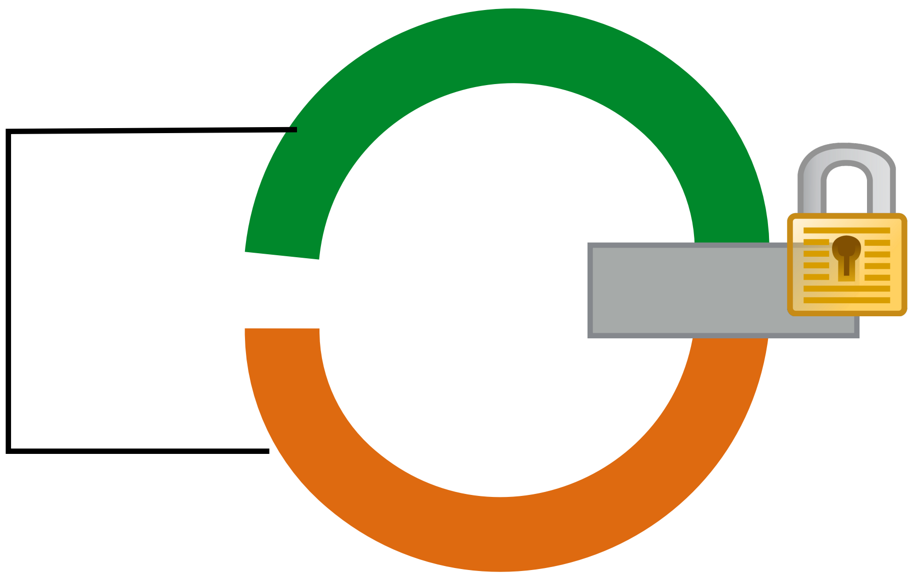
Zero-knowledge proofs



- Goal: P proves to V that some statement is true
    - ***Without*** conveying additional information
- In general, probabilistic
    - Repeat a bunch of times as proof

Example 1: Hallway password



- Does Peggy have the password?
- Both stand in the entrance.
    - When Victor isn't looking, Peggy picks one hall
    - Victor then yells "GREEN" or "ORANGE"
    - Peggy must come back via the chosen color
- Repeating many times "proves" Peggy has password
    - With high probability

Example 2: Two baseballs

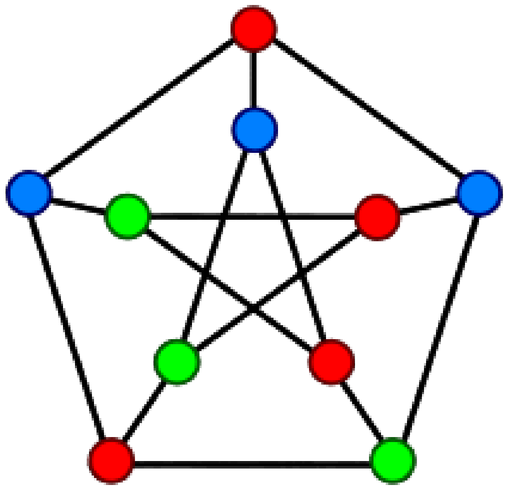
- Peggy has two baseballs: One red, one green
  - Otherwise identical
- Victor is color-blind, thinks they are the same
- Peggy places them in Victor's hands
  - Victor puts them behind his back, may switch
  - Peggy tells whether he switched
  - As before, repeat many times

Security properties

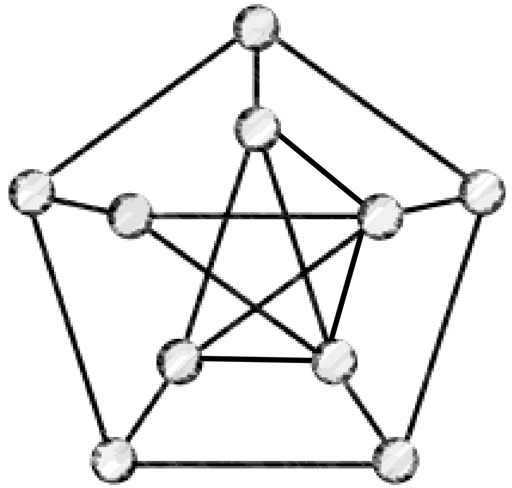
- Complete: Honest V will be convinced by honest P
- Sound: Honest V can't* be convinced by cheating P
- Proves nothing to outside observers either way
  - Peggy and Victor can **collude** by ***precomputing***
- Peggy could cheat with a time machine
  - Victor gets the same info either way
  - Implies that real protocol does not leak

Defining 3COL:

- G=(V,E) is 3-colorable iff there exists a mapping $\Phi: V \rightarrow \{1,2,3\}$ so that $\Phi(u) \neq \Phi(v)\ \ \forall (u,v) \in E$ (Φ is called “a 3-coloring of G”).
- 3COL={ G: G is 3-colorable} (NP Complete)



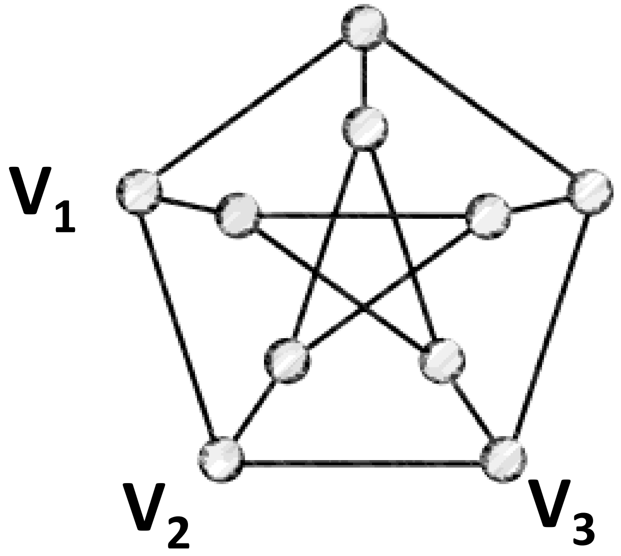
$\in$ 3COL



NOT $\in$ 3COL

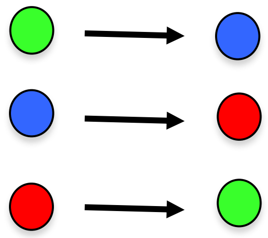
ZK Protocol for 3COL

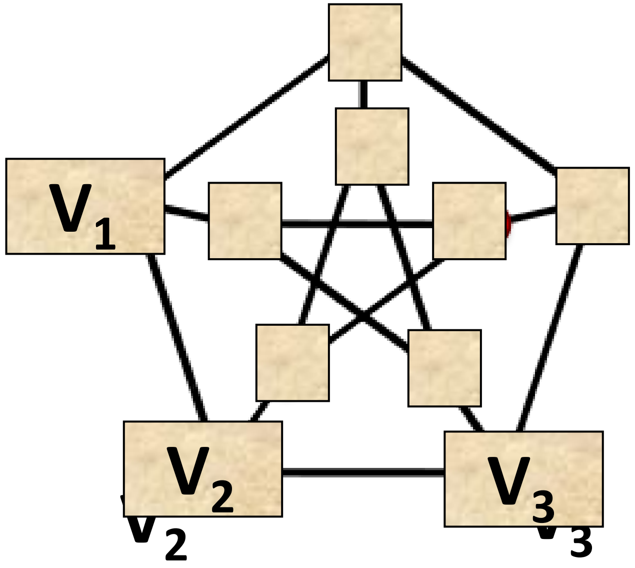
- Common input: Graph G=(V,E)
- P: knows a 3-coloring, wants to prove G is 3-colorable



ZK Protocol for 3COL

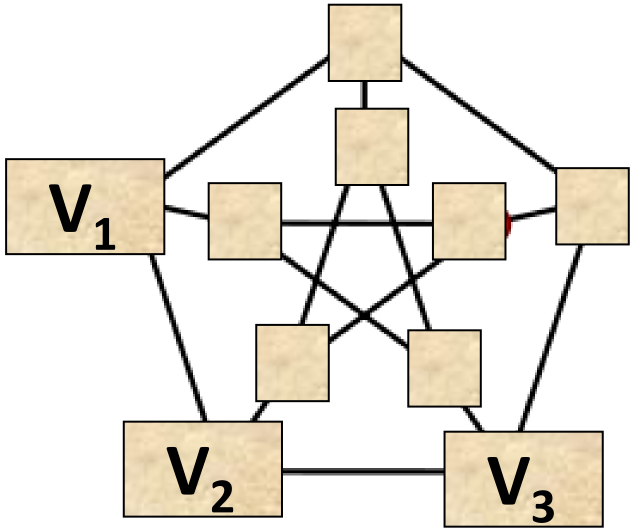
- Common input: Graph G=(V,E)
- P: knows a 3-coloring, wants to prove G is 3-colorable
- P chooses a random color permutation
- P permutes colors accordingly
- Puts all the nodes in “envelops”….
- ….And sends them to verifier





ZK Protocol for 3COL

- Verifier gets envelops V_1 V_2 V_3 …
- Chooses a random edge , eg. (V2,V3) , and sends to prover
- Prover checks that the two nodes are indeed an edge…
- … then opens the envelops of to reveal colors



ZK Protocol for 3COL

- Verifier gets envelops

- Chooses a random edge , eg. (V2,V3) , and sends to prover
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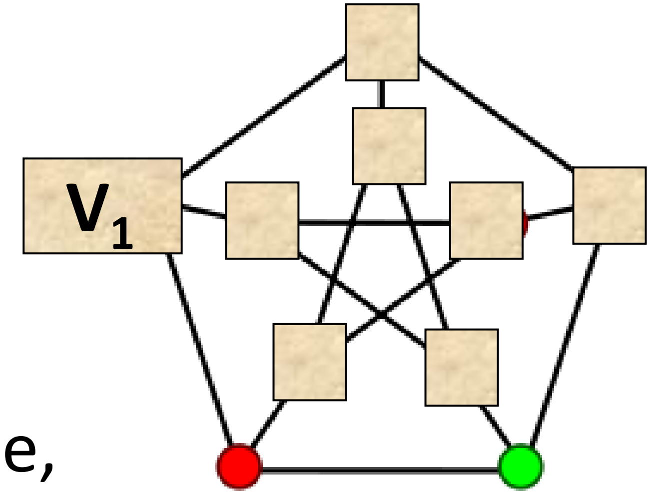
- Verifier accepts if colors are different



(We saw formal description in class last time,

using **commitments** for envelops)

Theorem: If the protocol uses perfectly binding commitments, then this is a zero knowledge protocol with completeness 1 and soundness $1-1/|E|$

Proof sketch:

- Completeness: If G is 3-colorable and both P and V follow the specified protocol, V will always accept
- Soundness: Suppose G is not 3-colorable. Then no matter what any cheating prover P* puts in the envelopes, there will be at least one edge (u,v) in E that is colored badly. That edge is picked by V with probability $1/|E|$.
    - Note: this uses the fact that the commitments are binding even against a computationally unbounded prover