

Introduction to Cryptology

Lecture 8

Announcements

- HW3 due today
- HW4 up on course webpage, due Tuesday, 3/1
- Graded HW1 returned at the end of class.

Agenda

- Last time:
 - Constructing computationally secure SKE from PRG (Sec. 3.3)
 - Stream Ciphers
- This time:
 - CPA Security (Sec. 3.4)
 - Pseudorandom Functions (PRF) (Sec. 3.5)
 - Constructing CPA-secure encryption from PRF (Sec. 3.5)

CPA-Security

The CPA Indistinguishability Experiment $\text{PrivK}^{\text{cpa}}_{A,\Pi}(n)$:

1. A key k is generated by running $\text{Gen}(1^n)$.
2. The adversary A is given input 1^n and oracle access to $\text{Enc}_k(\cdot)$, and outputs a pair of messages m_0, m_1 of the same length.
3. A random bit $b \leftarrow \{0,1\}$ is chosen, and then a challenge ciphertext $c \leftarrow \text{Enc}_k(m_b)$ is computed and given to A .
4. The adversary A continues to have oracle access to $\text{Enc}_k(\cdot)$, and outputs a bit b' .
5. The output of the experiment is defined to be 1 if $b' = b$, and 0 otherwise.

CPA-Security

Definition: A private-key encryption scheme $\Pi = (Gen, Enc, Dec)$ has indistinguishable encryptions under a chosen-plaintext attack if for all ppt adversaries A there exists a negligible function $negl$ such that

$$\Pr \left[PrivK^{cpa}_{A, \Pi}(n) = 1 \right] \leq \frac{1}{2} + negl(n),$$

where the probability is taken over the random coins used by A , as well as the random coins used in the experiment.

CPA-security for multiple encryptions

Theorem: Any private-key encryption scheme that has indistinguishable encryptions under a chosen-plaintext attack also has indistinguishable multiple encryptions under a chosen-plaintext attack.

CPA-secure Encryption Must Be Probabilistic

Theorem: If $\Pi = (Gen, Enc, Dec)$ is an encryption scheme in which Enc is a deterministic function of the key and the message, then Π cannot be CPA-secure.

Why not?

Constructing CPA-Secure Encryption Scheme

Pseudorandom Function

Definition: A keyed function $F: \{0,1\}^* \times \{0,1\}^* \rightarrow \{0,1\}^*$ is a two-input function, where the first input is called the key and denoted k .

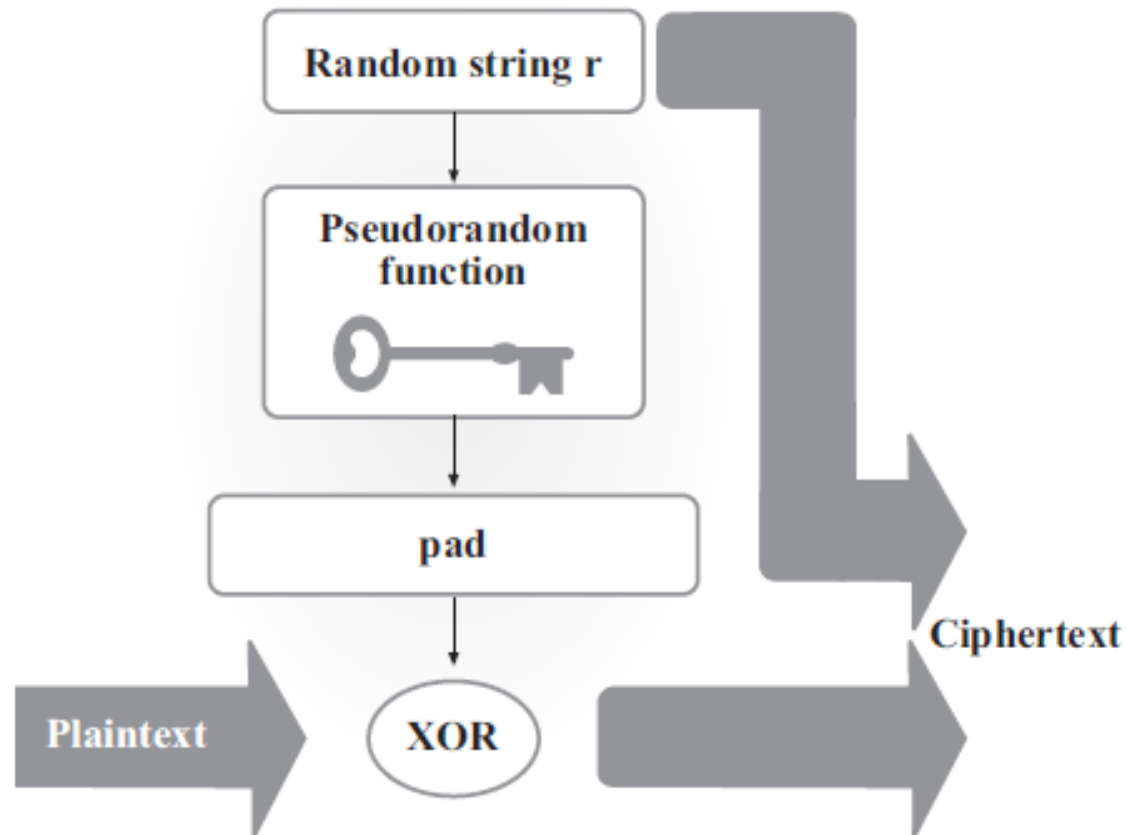
Pseudorandom Function

Definition: Let $F: \{0,1\}^* \times \{0,1\}^* \rightarrow \{0,1\}^*$ be an efficient, length-preserving, keyed function. We say that F is a pseudorandom function if for all ppt distinguishers D , there exists a negligible function $negl$ such that:

$$\left| \Pr[D^{F_k(\cdot)}(1^n) = 1] - \Pr[D^{f(\cdot)}(1^n) = 1] \right| \leq negl(n).$$

where $k \leftarrow \{0,1\}^n$ is chosen uniformly at random and f is chosen uniformly at random from the set of all functions mapping n -bit strings to n -bit strings.

Construction of CPA-Secure Encryption from PRF



Formal Description of Construction

Let F be a pseudorandom function. Define a private-key encryption scheme for messages of length n as follows:

- *Gen*: on input 1^n , choose $k \leftarrow \{0,1\}^n$ uniformly at random and output it as the key.
- *Enc*: on input a key $k \in \{0,1\}^n$ and a message $m \in \{0,1\}^n$, choose $r \leftarrow \{0,1\}^n$ uniformly at random and output the ciphertext

$$c := \langle r, F_k(r) \oplus m \rangle.$$

- *Dec*: on input a key $k \in \{0,1\}^n$ and a ciphertext $c = \langle r, s \rangle$, output the plaintext message

$$m := F_k(r) \oplus s.$$