

Digital Logic Design

ENEE 244-020x

Lecture 8

Announcements

- HW3 up on course webpage. Due on Saturday, Oct. 3 @11:59pm on Canvas/ELMS

Agenda

- Last time
 - The simplification problem
 - Prime Implicants (3.3)
 - Prime Implicates (3.4)
 - Karnaugh maps (3.2)
- This time
 - Using 3,4 variable K-Maps to find minimal expressions (3.2, 3.4)
 - Minimal expressions for incomplete Boolean functions (3.5)
 - 5 and 6 variable K-Maps

Using K-Maps to Obtain Minimal Boolean Expressions

Example

		yz			
		00	01	11	10
x	0	0	0	0	1
	1	0	0	1	1

yz'
 xy } Both are prime implicants

How do we know this
is minimal?

$$f = yz' + xy$$

Finding the set of all prime implicants in an n -variable map:

- If all 2^n entries are 1, then function is equal to 1.
- For $i = 1, 2, \dots, n$
 - Search for all subcubes of dimensions $2^a \times 2^b = 2^{n-i}$ that are not totally contained within a single previously obtained subcube.
 - Each of these subcubes represents an i variable product term which implies the function.
 - Each product term is a prime implicant.

Essential Prime Implicants

- Some 1-cells appear in only one prime implicant subcube, others appear in more than one.

A Karnaugh map for three variables x , y , and z . The vertical axis is labeled x with values 0 and 1. The horizontal axis is labeled yz with values 00, 01, 11, and 10. The map contains 1s in the following cells: (0, 00), (0, 01), (1, 01), and (1, 11). Three prime implicants are circled in blue: a horizontal group of two cells at $x=0$ (00, 01), a vertical group of two cells at $yz=01$ (0, 1), and a horizontal group of two cells at $x=1$ (01, 11). The cell at (1, 01) is the only one covered by a single prime implicant, making it an essential prime implicant.

	00	01	11	10
0	1	1	0	0
1	0	1	1	0

- A 1-cell that can be in only one prime implicant is called an essential prime implicant.

Essential Prime Implicants

- Every essential prime implicant must appear in all the irredundant disjunctive normal formulas of the function.
- Hence must also appear in a minimal sum.
 - Why?

General Approach for Finding Minimal Sums

- Find all prime implicants using K-map
- Find all essential prime implicants using K-map
- **If all 1-cells are not yet covered, determine optimal choice of remaining prime implicants using K-map.

Class Exercise

Minimal Expressions of Incomplete Boolean Functions

- Recall an incomplete Boolean function has a truth table which contains dashed functional entries indicating don't-care conditions.
- Idea: Can replace don't-care entries with either 0s or 1s in order to form the largest possible subcubes.

Example

$$f(w, x, y, z) = \sum m(0, 1, 2, 5, 8, 15) + dc(6, 7, 10)$$

		<i>yz</i>			
		00	01	11	10
<i>wx</i>	00	0	1	3	2
	01	4	5	7	6
	11	12	13	15	14
	10	8	9	11	10

Example

$$f(w, x, y, z) = \sum m(0, 1, 2, 5, 8, 15) + dc(6, 7, 10)$$

		<i>yz</i>			
		00	01	11	10
<i>wx</i>	00	1	1	0	1
	01	0	1	--	--
	11	0	0	1	0
	10	1	0	0	--

Example

Step 1: Find prime implicants (pretend don't care cells set to 1)

yz

	00	01	11	10
wx 00	1	1	0	1
01	0	1	--	--
11	0	0	1	0
10	1	0	0	--

Example

assume d.c. set to 0.

Step 2: Find essential prime implicants (discount don't care cells)

		yz			
		00	01	11	10
wx	00	1	1	0	1
	01	0	1	--	--
	11	0	0	1	0
	10	1	0	0	--

Essential prime implicants: $x'z'$, xyz

Example

Step 3: Add prime implicants to cover all 1-cells (discount don't care cells)

		yz			
		00	01	11	10
wx	00	1	1	0	1
	01	0	1	--	--
	11	0	0	1	0
	10	1	0	0	--

Essential prime implicants: $x'z'$, xyz

Add: $w'y'z$

Example

Step 3: Add prime implicants to cover all 1-cells (discount don't care cells)

		yz			
		00	01	11	10
wx	00	1	1	0	1
	01	0	1	--	--
	11	0	0	1	0
	10	1	0	0	--

Final minimal DNF: $x'z' + xyz + w'y'z$

Five and Six Variable K-Maps

Five Variable K-Maps

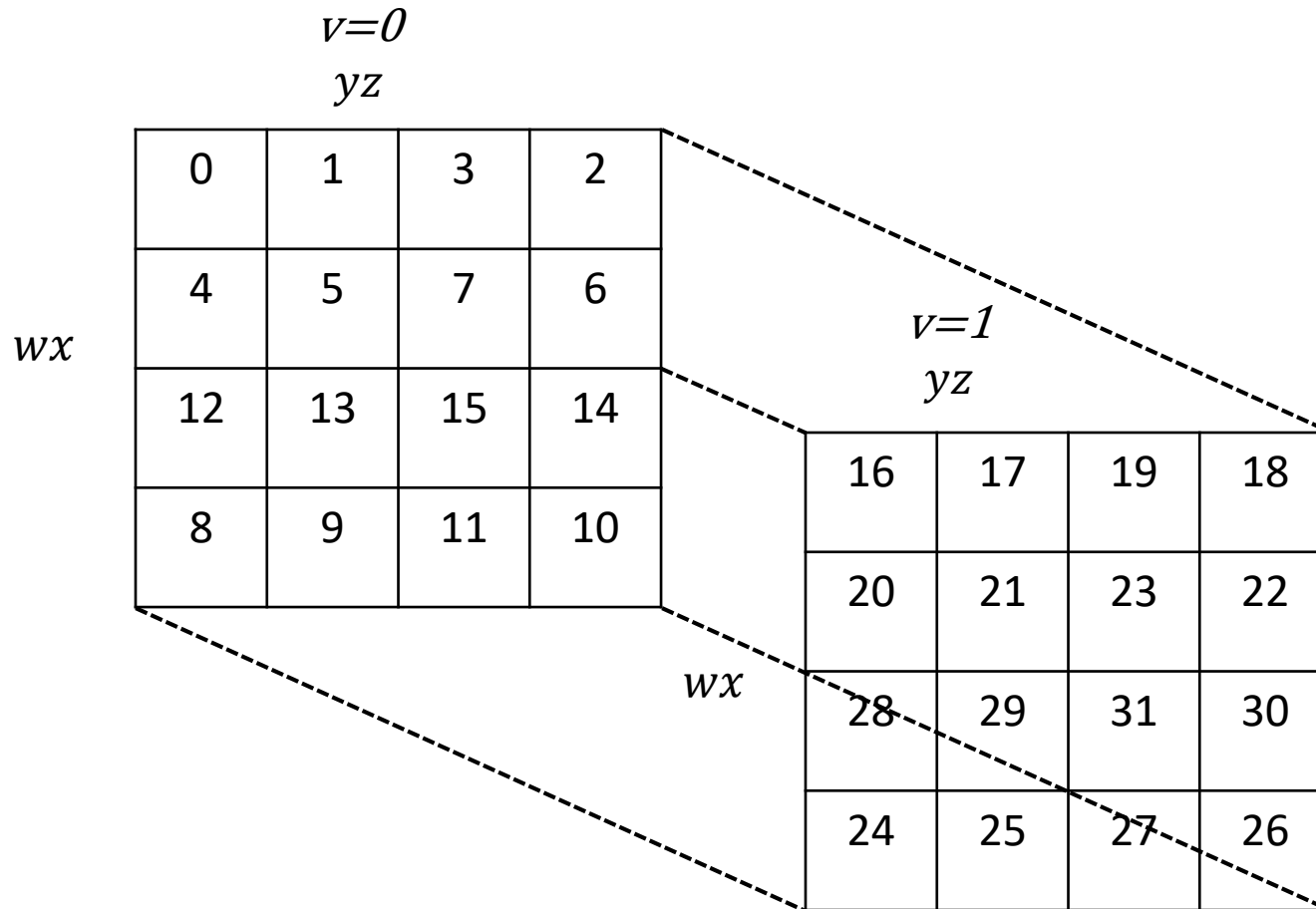
- We can visualize five-variable map in two different ways:

Five Variable K-Maps

		xyz								
		000	001	011	010		110	111	101	100
vw	00	0	1	3	2		6	7	5	4
	01	8	9	11	10		14	15	13	12
	11	24	25	27	26		30	31	29	28
	10	16	17	19	18		22	23	21	20

Subcubes: Two subcubes are possible about the mirror-image line. If there are two rectangular groupings of the same $2^a \times 2^b$ dimensions on both halves and the two groupings are the mirror image of each other.

Five Variable K-Maps



Subcubes: If each layer contains a $2^a \times 2^b$ subcube such that they can be viewed as being directly above and below each other, then the two subcubes collectively form a single subcube consisting of 2^{a+b+1} cells.

Example

$$f(v, w, x, y, z) = \sum_{v=0}^{v=1} m(1,5,9,11,13,20,21,26,27,28,29,30,31)$$

wx

0	1	0	0
0	1	0	0
0	1	0	0
0	1	1	0

wx

$v=1$
 yz

0	0	0	0
1	1	0	0
1	1	1	1
0	0	1	1

Example

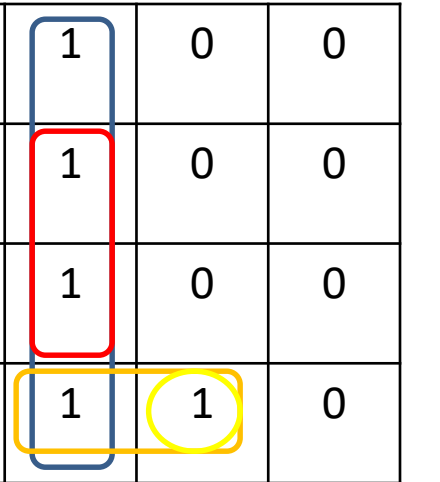
Step 1: Find all Prime Implicants.

$v=0$
 yz

wx

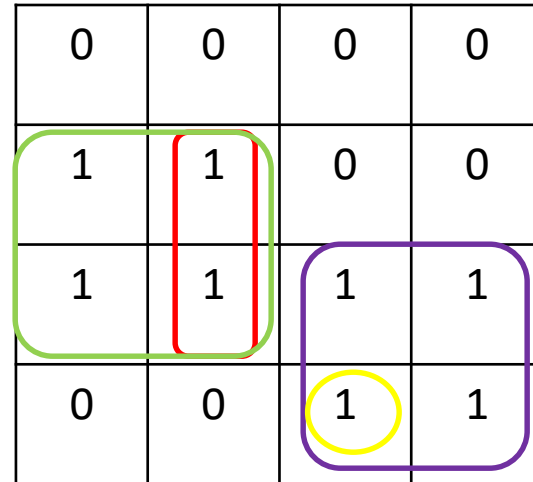
0	1	0	0
0	1	0	0
0	1	0	0
0	1	1	0

wx



$v=1$
 yz

0	0	0	0
1	1	0	0
1	1	1	1
0	0	1	1



Example

Step 2: Find all Essential Prime Implicants.

$v=0$
 yz

wx	00	0	1	0	0
	01	0	1	0	0
	11	0	1	0	0
	10	0	1	1	0

wx

$v=1$
 yz

	0	0	0	0
	1	1	0	0
	1	1	1	1
	0	0	1	1

wx

Essential Prime Implicants:

$$v'y'z$$

$$vxy'$$

$$vwy$$

Example

Step 2: Find all Essential Prime Implicants.

$v=0$
 yz

wx

0	1	0	0
0	1	0	0
0	1	0	0
0	1	1	0

$v=1$
 yz

0	0	0	0
1	1	0	0
1	1	1	1
0	0	1	1

Essential Prime Implicants:

$v'y'z$

vxy'

vwy

Example

Step 2: Find all Essential Prime Implicants.

$v=0$
 yz

wx

0	1	0	0
0	1	0	0
0	1	0	0
0	1	1	0

wx

$v=1$
 yz

0	0	0	0
1	1	0	0
1	1	1	1
0	0	1	1

Final minimal DNF:

$$v'y'z + vxy' + vwy + wx'yz$$

Six Variable K-Maps

- We can visualize a six-variable map in two different ways:

Six Variable K-Maps

		<i>xyz</i>							
		000	001	011	010	110	111	101	100
<i>uvw</i>	000	0	1	3	2	6	7	5	4
	001	8	9	11	10	14	15	13	12
	011	24	25	27	26	30	31	29	28
	010	16	17	19	18	22	23	21	20
	110	48	49	51	50	54	55	53	52
	111	56	57	59	58	62	63	61	60
	101	40	41	43	42	46	47	45	44
	100	32	33	35	34	38	39	37	36

Subcubes: If each quadrant has a rectangular grouping of dimensions $2^a \times 2^b$ and each grouping is a mirror image of the other about both the horizontal and vertical mirror-image lines.

Six Variable K-Maps

$uv=00$
 yz

	0	1	3	2
	4	5	7	6
wx	12	13	15	14
	8	9	11	10

$uv=01$
 yz

	16	17	19	18
	20	21	23	22
wx	28	29	31	30
	24	25	27	26

$uv=11$
 yz

	48	49	51	50
	52	53	55	54
wx	60	61	63	62
	56	57	59	58

$uv=10$
 yz

	32	33	35	34
	36	37	39	38
wx	44	45	47	46
	40	41	43	42

Subcubes: Subcubes occurring in corresponding positions in consecutive layers collectively form a single subcube.