

ENEE244-020x

Digital Logic Design

Lecture 5

Announcements

- HW 2 up on course webpage, due on Wednesday, Sept. 23 @11:59pm
Extended.

Agenda

- Last time:
 - Boolean Algebra axioms and theorems (2.3-2.4)
- This time:
 - Boolean Functions and Complements (2.5)
 - Simplification of Boolean Formulas via Algebraic Manipulation (2.5)
 - Canonical and Standard Forms (2.6)
 - Digital Logic Gates (2.8)

A Two-Valued Boolean Algebra

- $B = \{0,1\}$
- $(+) = \text{OR}, (\cdot) = \text{AND}$
- $\overline{1} = 0, \overline{0} = 1$

Can verify that all the postulates hold by “perfect induction.”
[Checking that equality holds for all possible variable settings]

OR

		x	
		0	1
y	0	0	1
	1	1	1

AND

		x	
		0	1
y	0	0	0
	1	0	1

Example:

Distributivity: $x + (y \cdot z) = (x + y) \cdot (x + z)$

1. Show that if the left hand side is true then the right hand side is true
2. If the right hand side is true then the left hand side is true.

Logic Diagrams

Equation Complementation

- For every Boolean function f there is an associated complementary function f' in which $f'(x_1, \dots, x_n) = 1$ iff $f(x_1, \dots, x_n) = 0$.
- Example:

$$f = w'xz' + w(x + y'z)$$
$$f' = (w'xz' + w(x + y'z))'$$

Equation Complementation

- Use DeMorgan's Law to simplify:

$$\begin{aligned}f' &= (w'xz' + w(x + y'z))' \\&= (\overline{w'xz'})'(w(x + \overline{y'z}))' \\&= (w + x' + z)(w(x + \overline{y'z}))' \\&= (w + x' + z)(w' + (x + y'z)') \\&= (w + x' + z)(w' + x'(y + z'))\end{aligned}$$

$$\begin{aligned}(x+y)' &= x' \cdot y' \\(x \cdot y)' &= x' + y'\end{aligned}$$

Theorems Specific to Two-Valued Boolean Algebras

Shannon's Expansion Theorem

$$(a) f(x_1, \dots, x_i, \dots, x_n) = \overset{f(\vec{x})|_{x_i=1}}{\nearrow} x_i \cdot f(x_1, \dots, 1, \dots, x_n) + \overset{f(\vec{x})|_{x_i=0}}{\searrow} x'_i \cdot f(x_1, \dots, 0, \dots, x_n)$$

$$(b) f(x_1, \dots, x_i, \dots, x_n) = (\overline{x}_i + f(x_1, \dots, 1, \dots, x_n))(x_i + f(x_1, \dots, 0, \dots, x_n))$$

$$\nwarrow f(\vec{x})|_{x_i=1}$$

$$\swarrow f(\vec{x})|_{x_i=0}$$

Example of Equation Simplification (Using Expansion about a Variable)

$$\begin{aligned}
 f(x, y, z) &= \overbrace{(x + xy)(x' + y) + \boxed{yz}}^{g(x, y)} \\
 &= x \underbrace{(1 + 1 \cdot y)(0 + y)}_{f(x, y, z)|_{x=1}} + x' \underbrace{(0 + 0 \cdot y)(1 + y)}_{f(x, y, z)|_{x=0}} + \boxed{yz}
 \end{aligned}$$

$xy + yz$

$$\begin{aligned}
 &xy + xyz + x'yz \\
 &xy + (x + x')yz \\
 &\boxed{xy + yz}
 \end{aligned}$$

Shannon's Reduction Theorems

$$\underbrace{(a) \ x_i \cdot f(x_1, \dots, x_i, \dots, x_n)}_{g(\vec{x})} = x_i \cdot f(x_1, \dots, 1, \dots, x_n)$$

$$(b) \ x_i + f(x_1, \dots, x_i, \dots, x_n) = x_i + f(x_1, \dots, 0, \dots, x_n)$$

$$(a) \ x'_i \cdot f(x_1, \dots, x_i, \dots, x_n) = x'_i \cdot f(x_1, \dots, 0, \dots, x_n)$$

$$(b) \ x'_i + f(x_1, \dots, x_i, \dots, x_n) = x'_i + f(x_1, \dots, 1, \dots, x_n)$$

$$g(\vec{x}) = x_i \cdot g(\vec{x})_{|x_i=1} + x'_i \cdot g(\vec{x})_{|x_i=0}$$

$$x_i \cdot (x_i \cdot f(\vec{x}))_{|x_i=1} + x'_i \cdot (x_i \cdot f(\vec{x}))_{|x_i=0}$$

Example of Equation Simplification (Using Shannon's Reduction Theorems)

$$\begin{aligned} f(w, x, y, z) &= x + x'y' + w'x'(w + z)(y + w'z) \\ &= x + y' + w'(w + z)(y + w'z) \\ &= x + y' + w'z(y + z) \\ &= y' + x + w'z(y + z) \\ &= y' + x + w'z \end{aligned}$$

Simplification In-Class Exercise

Boolean Formulas and Functions

- Example: $f(x, y, z) = (x' + y)z$
- Can be specified via a **truth table**.

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Normal Forms

- Consider the function: $(x+y)(z+w)$
$$f(w, x, y, z) = x' + wy' + w' y' z$$
- A **literal** is an occurrence of a complemented or uncomplemented variable in a formula.
- A **product term** is either a literal or a product (conjunction) of literals.
AND
- Disjunctive normal form**: A Boolean formula written as a single product term or as a sum (disjunction) of product terms.
OR of ANDs

Normal Forms

- Consider the function:

$$f(w, x, y, z) = z(x + y')(w + x' + y') + x' + wy' + w'y'z$$

- A **sum term** is either a literal or a sum (disjunction) of literals.
- Conjunctive normal form**: A Boolean formula written as a single sum term or as a product (conjunction) of sum terms.

AND of OR's

Canonical Formulas

- How to obtain a Boolean formula given a truth table?

X	Y	Z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

Minterm Canonical Formula

	X	Y	Z	f	
$x'y'z'$	0	0	0	0	$x'y'z$
$x'y'z$	0	0	1	1	
	0	1	0	0	$x'y z$
	0	1	1	1	
	1	0	0	1	$x y' z'$
	1	0	1	0	
	1	1	0	0	
	1	1	1	0	

$$f(x, y, z) = x' y' z + x' y z + x y' z'$$

m-Notation

X	Y	Z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

$x' y' z$

$x' y z$

$x y' z'$

- $f(x, y, z)$ can be written as $f(x, y, z) = m_1 + m_3 + m_4$
- $f(x, y, z) = \Sigma m(1,3,4)$

Maxterm Canonical Formula

X	Y	Z	f	
0	0	0	0	$x + y + z$
0	0	1	1	
0	1	0	0	$x + y' + z$
0	1	1	1	
1	0	0	1	$x' + y + z'$
1	0	1	0	
1	1	0	0	$x' + y' + z$
1	1	1	0	$x' + y' + z'$

$$f(x, y, z) = (x + y + z)(x + \bar{y} + z)(\bar{x} + y + \bar{z})(\bar{x} + \bar{y} + z)(\bar{x} + \bar{y} + \bar{z})$$

M-Notation

x	y	z	f	
0	0	0	0	$x + y + z$
0	0	1	1	
0	1	0	0	$x + y' + z$
0	1	1	1	
1	0	0	1	$x' + y + z'$
1	0	1	0	
1	1	0	0	$x' + y' + z$
1	1	1	0	$x' + y' + z'$

- $f(x, y, z)$ can be written as $f(x, y, z) = M_0 M_2 M_5 M_6 M_7$
- $f(x, y, z) = \Pi M(0, 2, 5, 6, 7)$

Table 2.3
Minterms and Maxterms for Three Binary Variables.

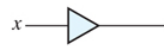
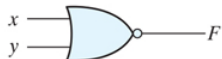
			Minterms		Maxterms	
<i>x</i>	<i>y</i>	<i>z</i>	Term	Designation	Term	Designation
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

Canonical Forms Conversion

- Minterm to Maxterm:
 - $f(x, y, z) = \Sigma m(2, 5, 6, 7)$
 - $f'(x, y, z) = \Sigma m(0, 1, 3, 4)$
 - $f(x, y, z) = \Pi M(0, 1, 3, 4)$
- Maxterm to Minterm:
 - $f(x, y, z) = \Pi M(1, 2, 3, 5, 7)$
 - $f'(x, y, z) = \Pi M(0, 4, 6)$
 - $f(x, y, z) = \Sigma m(0, 4, 6)$

Gates and Combinational Networks

Digital Logic Gates

Name	Graphic symbol	Algebraic function	Truth table		
AND		$F = x \cdot y$	x	y	F
			0	0	0
			0	1	0
			1	0	0
			1	1	1
OR		$F = x + y$	x	y	F
			0	0	0
			0	1	1
			1	0	1
			1	1	1
Inverter		$F = x'$	x	F	
			0	1	
Buffer		$F = x$	x	F	
			0	0	
NAND		$F = (xy)'$	x	y	F
			0	0	1
			0	1	1
			1	0	1
			1	1	0
NOR		$F = (x + y)'$	x	y	F
			0	0	1
			0	1	0
			1	0	0
			1	1	0
Exclusive-OR (XOR)		$F = xy' + x'y$ $= x \oplus y$	x	y	F
			0	0	0
			0	1	1
			1	0	1
			1	1	0
Exclusive-NOR or equivalence		$F = xy + x'y'$ $= (x \oplus y)'$	x	y	F
			0	0	1
			0	1	0
			1	0	0
			1	1	1

Gates and Combinational Networks

- Synthesis Procedure
- Example: Truth table for parity function on three variables

X	Y	Z	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

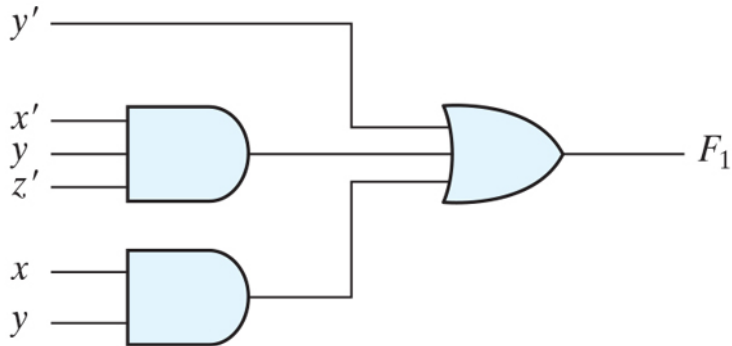
Synthesis Procedure

X	Y	Z	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Minterm Canonical Form:

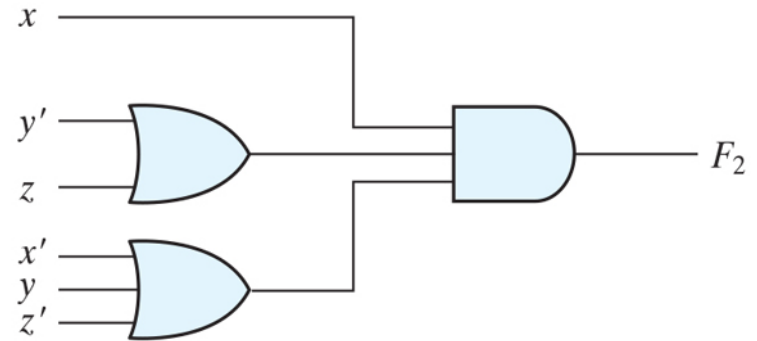
$$f(x, y, z) = x'y'z + x'y z' + xy'z' + xyz$$

Two-level Implementation



(a) Sum of Products

DNF



(b) Product of Sums

CNF

(Normal)

Standard Forms

*What is the difference between canonical
and standard forms?*