

Class Exercise

9/17/26

Shannon's Expansion Theorem:

Theorem 3.11

$$\begin{aligned}(a) \quad f(x_1, \dots, x_i, \dots, x_n) &= x_i \cdot f(x_1, \dots, 1, \dots, x_n) + \bar{x}_i \cdot f(x_1, \dots, 0, \dots, x_n) \\(b) \quad f(x_1, \dots, x_i, \dots, x_n) &= (\bar{x}_i + f(x_1, \dots, 1, \dots, x_n))(x_i + f(x_1, \dots, 0, \dots, x_n))\end{aligned}$$

Shannon's Reduction Theorems:

Theorem 3.12

$$\begin{aligned}(a) \quad x_i \cdot f(x_1, \dots, x_i, \dots, x_n) &= x_i \cdot f(x_1, \dots, 1, \dots, x_n) \\(b) \quad x_i + f(x_1, \dots, x_i, \dots, x_n) &= x_i + f(x_1, \dots, 0, \dots, x_n)\end{aligned}$$

Theorem 3.13

$$\begin{aligned}(a) \quad \bar{x}_i \cdot f(x_1, \dots, x_i, \dots, x_n) &= \bar{x}_i \cdot f(x_1, \dots, 0, \dots, x_n) \\(b) \quad \bar{x}_i + f(x_1, \dots, x_i, \dots, x_n) &= \bar{x}_i + f(x_1, \dots, 1, \dots, x_n)\end{aligned}$$

P2. Identity

There exist identity elements in B , denoted 0,1 relative to $(+)$ and $(\cdot)$, respectively.

For all $x \in B$, $0 + x = x + 0 = x$, $1 \cdot x = x \cdot 1 = x$.

P4. Distributivity

Each operation $(+)$, $(\cdot)$ is distributive over the other.

For all $x, y, z \in B$:

$$\begin{aligned}x + (y \cdot z) &= (x + y) \cdot (x + z) \\x \cdot (y + z) &= (x \cdot y) + (x \cdot z)\end{aligned}$$

P5. Complement

For every element $x \in B$ there exists an element $\bar{x} \in B$ called the complement of x such that:

$$\begin{aligned}x + \bar{x} &= 1 \\x \cdot \bar{x} &= 0\end{aligned}$$

Theorem 3.2: Null elements:

For each element x in a Boolean algebra:

$$\begin{aligned}x + 1 &= 1 \\x \cdot 0 &= 0.\end{aligned}$$

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For the following problems, simplify the expressions using the postulates and theorems on the previous page.

1. Simplify: $\overline{w} \overline{x} y z + w x y + \overline{w} \overline{y} + x \overline{y} + \overline{x} \overline{y}$

$$= x(wy + \overline{w} \overline{y} + \overline{y}) + \overline{x}(\overline{w} y z + \overline{w} \overline{y} + \overline{y}) \text{ (Th. 3.11 (a))}$$

$$= x(\overline{y} + w) + \overline{x}(\overline{w} z + \overline{y}) \text{ (Th. 3.13 (b))}$$

$$= x \overline{y} + xw + \overline{x} \overline{w} z + \overline{x} \overline{y} \text{ (P4)}$$

$$= \overline{y} + xw + \overline{x} \overline{w} z \text{ (P3, P4, P5, P2)}$$

2. Simplify: $(w + x + \overline{y} + z)(w + x + \overline{y} + \overline{z})(w + \overline{x} + \overline{y} + z)(w + \overline{x} + \overline{y} + \overline{z}) \cdot (\overline{w} + \overline{x} + \overline{y} + z)(\overline{w} + \overline{x} + \overline{y} + \overline{z})$

$$= z(w + x + \overline{y})(w + \overline{x} + \overline{y})(\overline{w} + \overline{x} + \overline{y}) + \overline{z}(w + x + \overline{y})(w + \overline{x} + \overline{y})(\overline{w} + \overline{x} + \overline{y}) \text{ (Th 3.11(a))}$$

$$= (w + x + \overline{y})(w + \overline{x} + \overline{y})(\overline{w} + \overline{x} + \overline{y}) \text{ (P4, P5, P2)}$$

$$= x(w + \overline{y})(\overline{w} + \overline{y}) + \overline{x}(w + \overline{y}) \text{ (Th 3.11(a))}$$

$$= x \overline{y} + \overline{x} w + \overline{x} \overline{y} \text{ (P4, P5, P2, 3.11)}$$

$$= \overline{y} + \overline{x} w \text{ (P3, P4, P5, P2)}$$

3. Simplify: $(x + z)(w + x)(\overline{y} + z)(w + \overline{y})$

$$= x(\overline{y} + z)(w + \overline{y}) + \overline{x} z w \text{ (Th. 3.11(a))}$$

$$= x(y z w + \overline{y}) + \overline{x} z w \text{ (Th. 3.11(a))}$$

$$= x(z w + \overline{y}) + \overline{x} z w \text{ (Th. 3.13(b))}$$

$$= x z w + x \overline{y} + \overline{x} z w \text{ (P4)}$$

$$= z w + x \overline{y} \text{ (P3, P4, P5, P2)}$$