

Hamming Codes

Lecture 3

9/8/26

Recall the following parity check matrix for Hamming Codes:

$$H = \begin{pmatrix} \underline{1} & \underline{0} & \underline{1} & \underline{1} & \underline{0} & \underline{0} & \underline{0} \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\underline{7} = 2^{\underline{3}} - 1$$

encode 4 bits

$$\underline{3} = 2^{\underline{2}} - 1$$

encodes 1 bit.

1. Encode message $\vec{m} = 1011 = 0110011$

$$\underline{15} = 2^{\underline{4}} - 1$$

encode 11 bits

2. Decode $\vec{s} = 1011000 = 1010$

What if we use the following parity check matrix to construct a Hamming Code:

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

1. Encode message $\vec{m} = 1011$

2. Apply error correction and decode $\vec{s} = 0111100$

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Discussion:

- What happens if we want to encode a 3-bit message? A 5-bit message?

An 11 bit message?

- What is the rate of the Hamming code?
- How many errors can be detected with the Hamming code?

$$\frac{|m|}{|c|} = \frac{2^k - 1 - k}{2^k - 1} = 1 - \frac{k}{2^k - 1}$$

k parity bits

1

Challenge: Construct a Hamming Code for encoding messages of length 5

- What is the parity check matrix?
- Show how to encode the message $\vec{m} = 10101$
- Apply error correction and decode $\vec{s} = 110010001$