m-Notation, M-Notation

Lecture 5 supplement

Boolean Formulas and Functions

- Example: $f(x, y, z) = (\overline{x} + y)z$
- Can be specified via a truth table.

X	Y	Z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Normal Forms

- Consider the function:

$$f(w,x,y,z) = \overline{x} + w\overline{y} + \overline{w}\,\overline{y}z$$

- A literal is an occurrence of a complemented or uncomplemented variable in a formula.
- A product term is either a literal or a product (conjunction) of literals.
- Disjunctive normal form: A Boolean formula written as a single product term or as a sum (disjunction) of product terms.

Normal Forms

- Consider the function:

$$f(w, x, y, z) = z(x + \overline{y})(w + \overline{x} + \overline{y})$$

- A sum term is either a literal or a sum (disjunction) of literals.
- Conjunctive normal form: A Boolean formula written as a single sum term or as a product (conjunction) of sum terms.

Canonical Formulas

- How to obtain a Boolean formula given a truth table?

X	Y	Z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

Minterm Canonical Formula

X	Y	Z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

$\overline{x}\,\overline{y}\,z$

$\overline{x}\,y\,z$

$x\,\overline{y}\,\overline{z}$

$$f(x, y, z) = \overline{x}\,\overline{y}\,z + \overline{x}y\,z + x\,\overline{y}\,\overline{z}$$

m-Notation

X	Y	Z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

$\bar{x}\,\bar{y}\,z$

$\bar{x}\,y\,z$

$x\,\bar{y}\,\bar{z}$

- $f(x,y,z)$ can be written as $f(x,y,z) = m_1 + m_3 + m_4$
- $f(x,y,z) = \Sigma m(1,3,4)$

Maxterm Canonical Formula

X	Y	Z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

$x + y + z$

$x + \overline{y} + z$

$\overline{x} + y + \overline{z}$

$\overline{x} + \overline{y} + z$

$\overline{x} + \overline{y} + \overline{z}$

$$f(x, y, z) = (x + y + z)(x + \overline{y} + z)$$
$$(\overline{x} + y + \overline{z})(\overline{x} + \overline{y} + z)(\overline{x} + \overline{y} + \overline{z})$$

M-Notation

X	Y	Z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

$x + y + z$

$x + \overline{y} + z$

$\overline{x} + y + \overline{z}$

$\overline{x} + \overline{y} + z$

$\overline{x} + \overline{y} + \overline{z}$

- $f(x, y, z)$ can be written as $f(x, y, z) = M_0 M_2 M_5 M_6 M_7$
- $f(x, y, z) = \Pi M(0,2,5,6,7)$