ENEE244-02xx
Digital Logic Design

Lecture 7

Announcements

- Homework 3 due on Wednesday, Oct. 30.
- Midterm on Wednesday, Oct. 8.
- List of topics for Midterm up on course webpage.
  - Review sheet will be posted by end of week.

Agenda

- Last time:
  - Gates and Combinational Networks (3.7)
  - Incomplete Boolean Functions and Don’t Care Conditions (3.8)
  - Universal Gates (3.9.3)

- This time:
  - NAND/NOR Gate Realizations (3.9.4-3.9.6)
  - Some examples of Synthesis Procedure
  - The simplification problem (4.1)
  - Prime Implicants (4.2)
  - Prime Implicates (4.3)

NAND-Gate Realizations

- Naïve approach: build network out of AND/OR/NOT gates, use the universal property above to replace each one with several NAND gates.
- A better approach: manipulate Boolean expression into the form NAND(A, B, . . .,C)

NAND-Gate Realizations

- Basic idea:
  - Use the property: $x + y = \overline{(\overline{x})(\overline{y})}$
  - i.e. $x + y = NAND(\overline{x}, \overline{y})$
- Keep doing this recursively.

NAND-Gate Realizations

- Example:

$$f(w,x,y,z) = \overline{w}z + w\overline{z}(x+\overline{y})$$
$$= \overline{\overline{(\overline{w}z)}\ \overline{[w\,\overline{z}(x+\overline{y})]}}$$

  - $\overline{(\overline{w}z)}$--already in correct form, equivalent to $NAND(\overline{w},z)$
  - $\overline{w\overline{z}(x+\overline{y})} = \mathrm{NAND}\big(\mathrm{w},\overline{z},(x+\overline{y})\big)$
  - $x+\overline{y} = \overline{\overline{x}y}$
  - $f(w,x,y,z) = NAND(NAND(\overline{w},z),NAND(w,\overline{z},(NAND(\overline{x},y))))$

NAND-Gate Realizations

- Only works if highest-order operation is an or-operation.
- Highest-order operation is the last operation that is performed when the expression is evaluated.
- What to do otherwise? Negate and repeat the procedure for $\overline{f}$. Then note that $f(x_1, \ldots, x_n) = NAND(1, \overline{f}(x_1, \ldots, x_{n)}))$

NAND-Gate Realization

- Example:

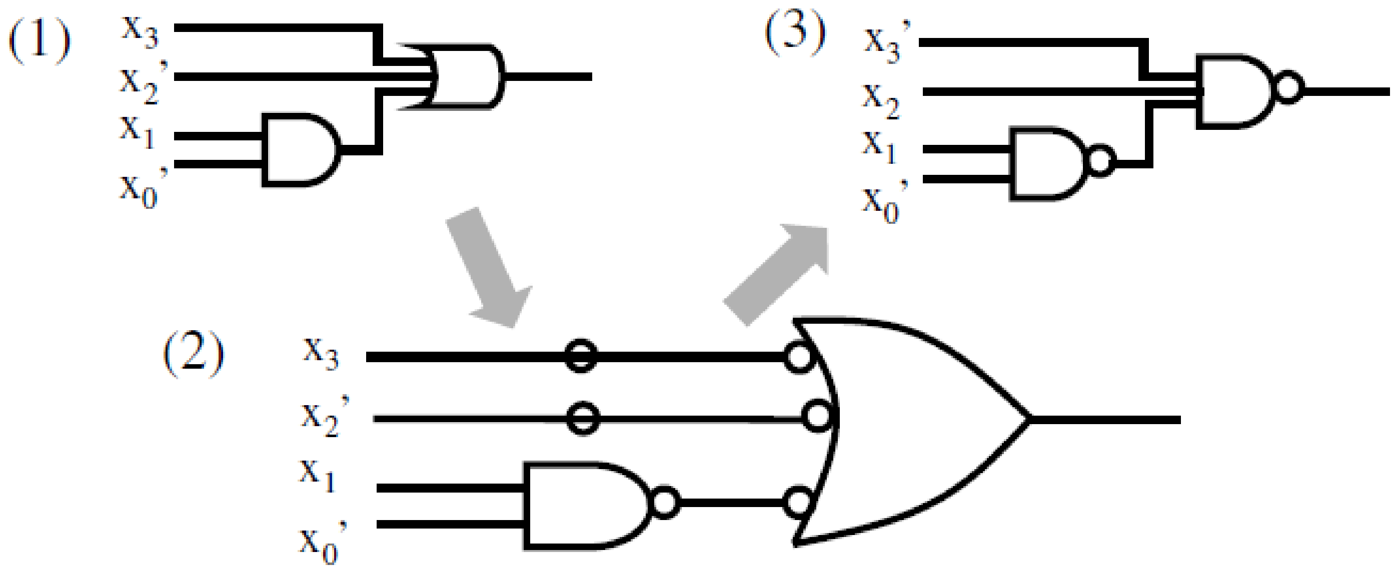
$$f(x,y,z) = (x+y)(\overline{y} \ + z)$$

$$f(x,y,z) = NAND(1, \overline{f}(x,y,z))$$

  - $\overline{f}(x,y,z) = \overline{(x+y)(\overline{y}+z)} = NAND(x+y, \overline{y}+z)$
  - $x + y = \overline{\overline{x}\ \overline{y}} = NAND(\overline{x}, \overline{y})$
  - $\overline{y} + z = \overline{y\ \overline{z}} \ = NAND(y, \overline{z})$
  - $f(x,y,z) =$ $NAND(1, NAND(NAND(\overline{x}, \overline{y}), NAND(y, \overline{z})))$

NAND Implementation Example

$$f(x_0, x_1, x_2, x_3) = x_3 + \overline{x}_2 + x_1\overline{x}_0$$

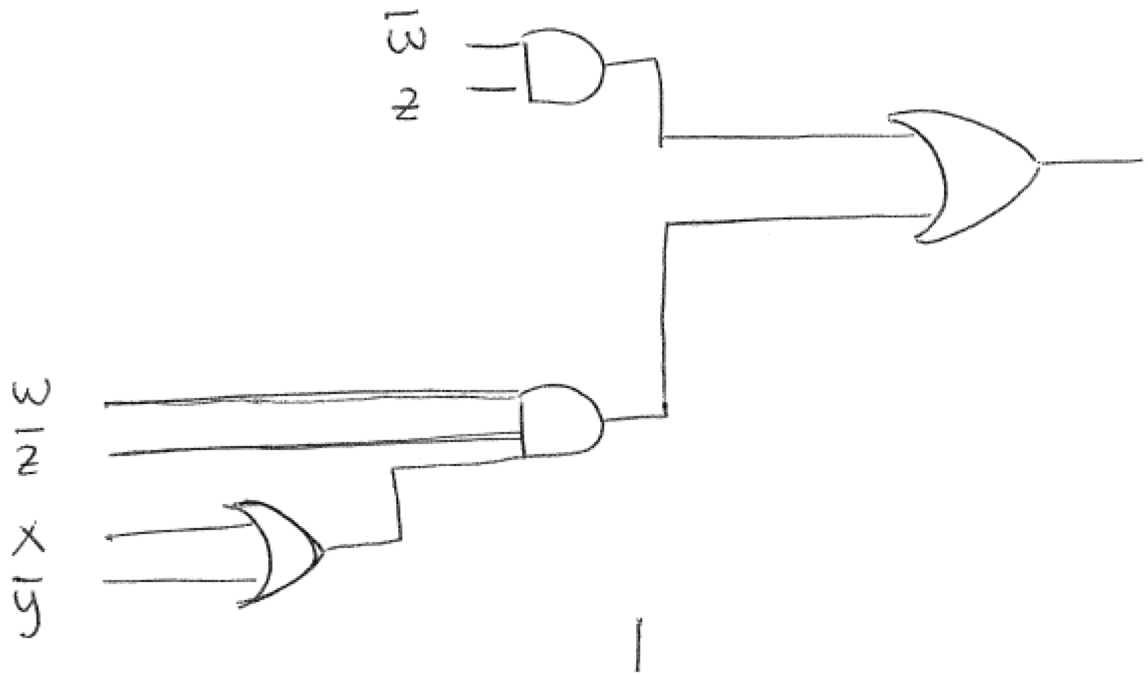


General NAND Implementation

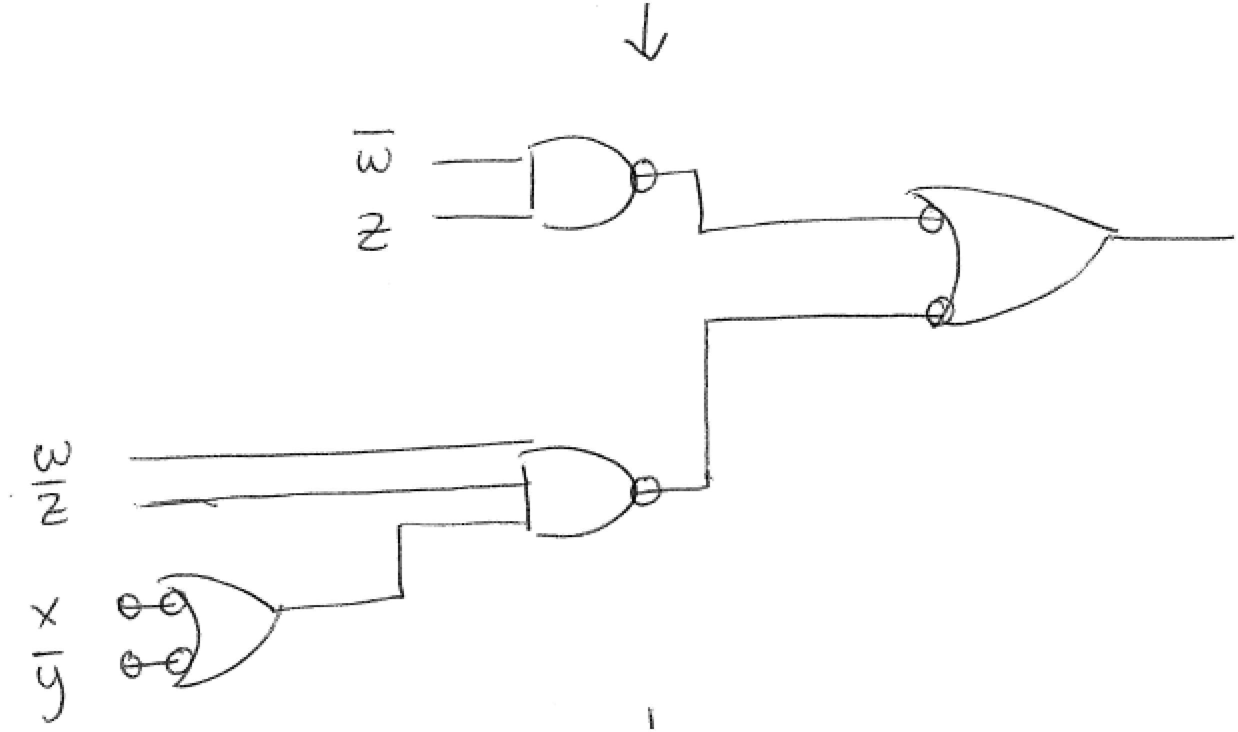
- Draw the {AND,OR,NOT} implementation
- AND ⇒ AND-NOT-NOT ⇒ NAND-NOT
- OR ⇒ NOT-NOT-OR ⇒ NOT-NAND
- Delete "NOT-NOT"
- NOT ⇒ NAND

NAND-Gate Realizations

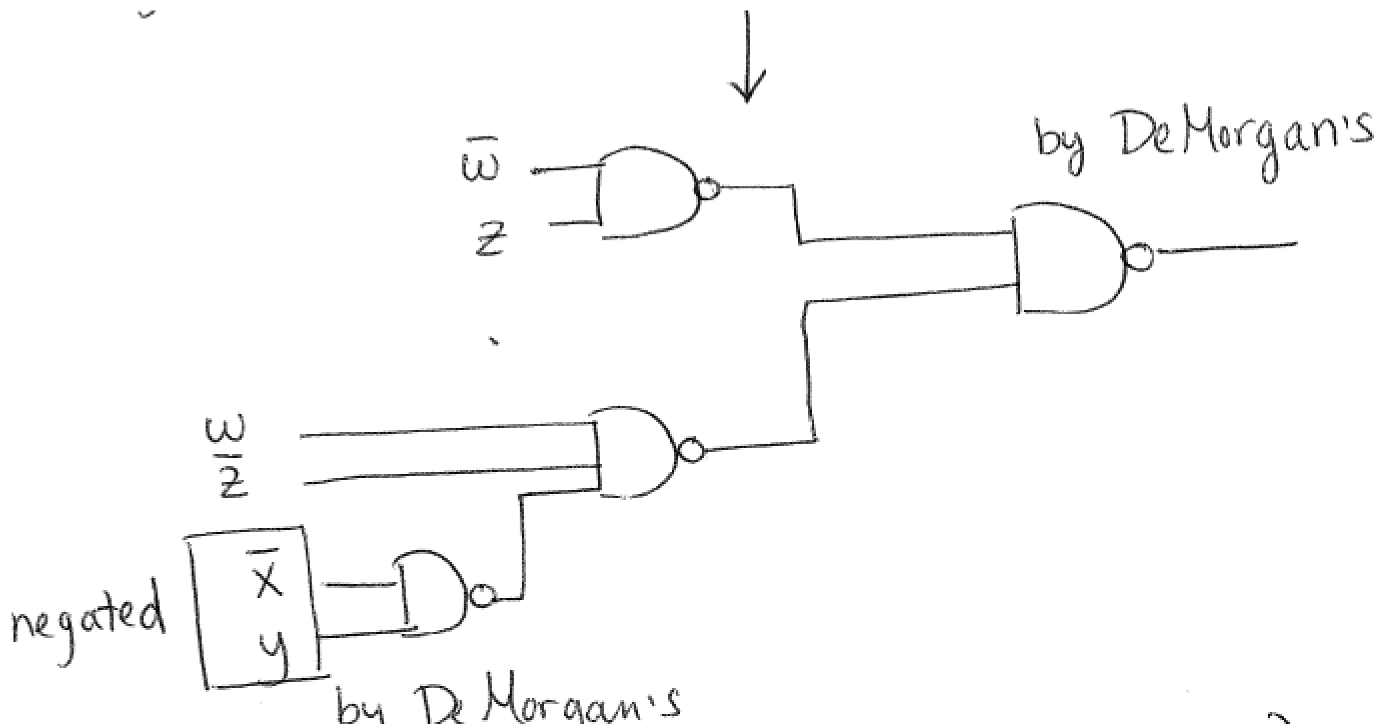
① $f(w,x,y,z) = \bar{w}z + w\bar{z}(x+\bar{y})$



NAND-Gate Realizations



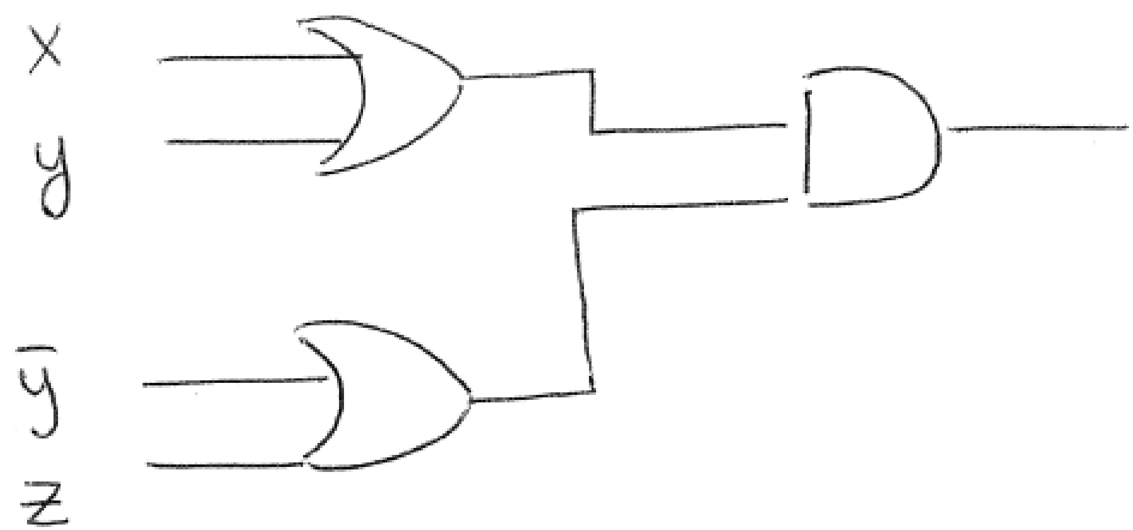
NAND-Gate Realizations



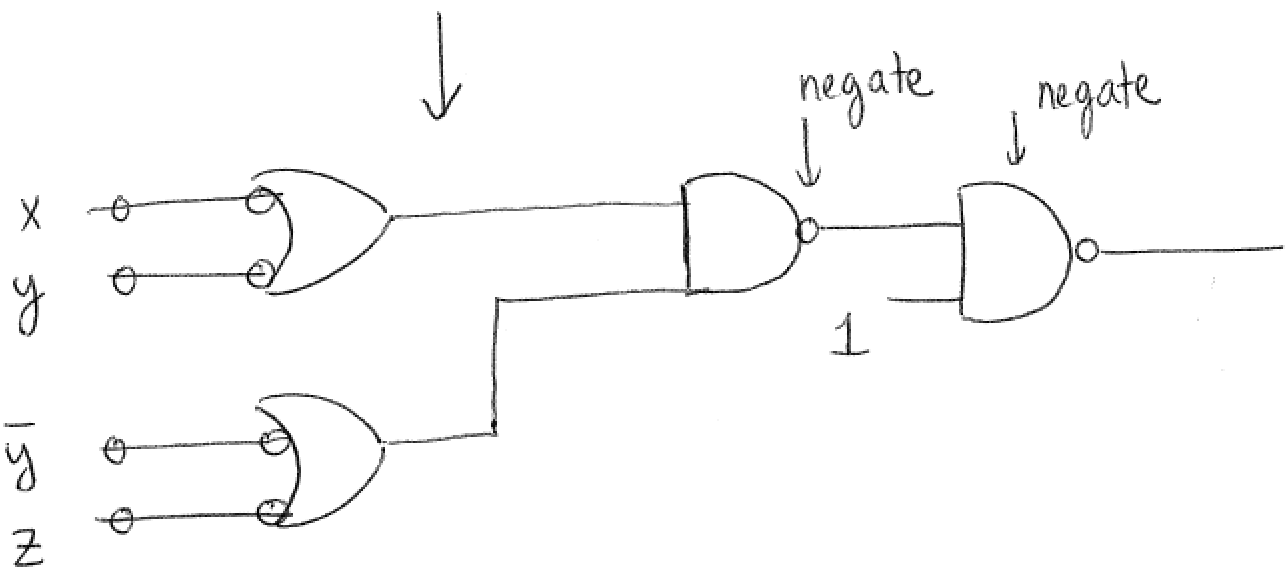
$$\mathrm{NAND}(\mathrm{NAND}(\bar{w}, z), \mathrm{NAND}(w, \bar{z}, \mathrm{NAND}(\bar{x}, y)))$$

NAND-Gate Realizations

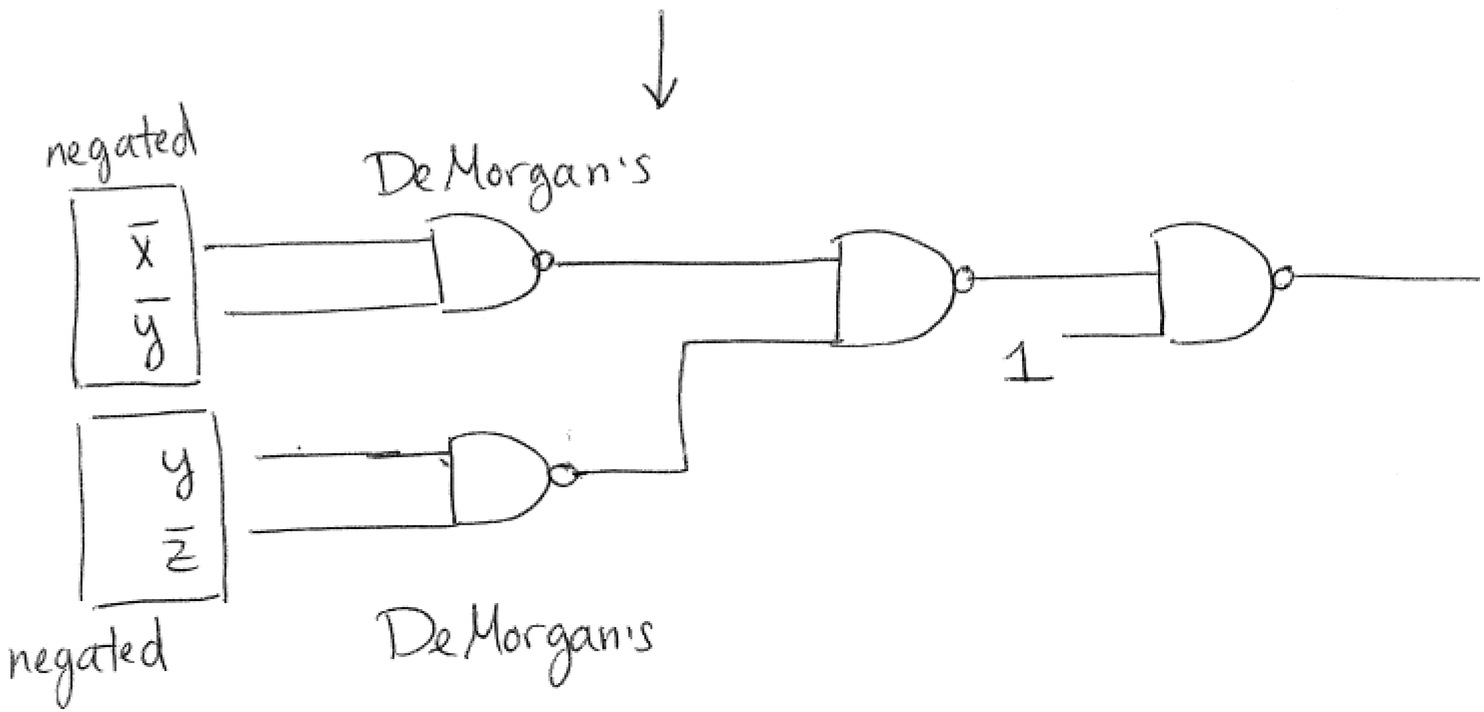
② $f(x,y,z) = (x+y)(\bar{y}+z)$



NAND-Gate Realizations



NAND-Gate Realizations



$$\text{NAND}(1,\ \text{NAND}(\text{NAND}(\bar{x}, \bar{y}),\ \text{NAND}(y, \bar{z})))$$

NOR-Gate Realizations

- Essentially the same procedure. See Section 3.9.5 in the textbook.

In-Class Exercise

See handout (lec_7.pdf)

Synthesis Procedure Examples

Synthesis Procedure

- High-level description: A function with finite domain and range.
- Binary-level: All input-output variables are binary.

Example 1

A system takes one decimal digit and outputs 0 on even number and 1 on odd number.

- High-level:
  - Input: $x \in \{0,1,2,3,4,5,6,7,8,9\}$
  - Output: $z \in \{0,1\}$
  - Function:

$$z = F(x) = \begin{cases} 0 & x=0,2,4,6,8 \\ 1 & x=1,3,5,7,9 \end{cases}$$

x	0	1	2	3	4	5	6	7	8	9
z	0	1	0	1	0	1	0	1	0	1

Example 1 (cont'd)

- Binary-level (using BCD)
  - Input variables: x_3, x_2, x_1, x_0
  - Output variables: z
  - Functions:

    $z(x_3,x_2,x_1,x_0) = x_0$

x	$x_3x_2x_1x_0$	z
0	0 0 0 0	0
1	0 0 0 1	1
2	0 0 1 0	0
3	0 0 1 1	1
4	0 1 0 0	0
5	0 1 0 1	1
6	0 1 1 0	0
7	0 1 1 1	1
8	1 0 0 0	0
9	1 0 0 1	1

Example 2

A system takes one decimal digit and outputs its 9's complement.

- High-level:
  - Input: $x \in \{0,1,2,3,4,5,6,7,8,9\}$
  - Output: $z \in \{0,1,2,3,4,5,6,7,8,9\}$
  - Function: $z = F(x) = 9 - x$

x	0	1	2	3	4	5	6	7	8	9
z	9	8	7	6	5	4	3	2	1	0

Example 2 (cont'd)

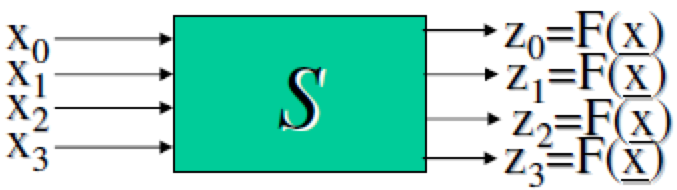
- Binary-level (using BCD)
  - Input variables: x_0, x_1, x_2, x_3
  - Output variables: z_0, z_1, z_2, z_3
  - Functions: (sum of minterms)

$$z_3(x_3,x_2,x_1,x_0) = \sum m(0,1)$$

$$z_2(x_3,x_2,x_1,x_0) = \sum m(2,3,4,5)$$

$$z_1(x_3,x_2,x_1,x_0) = \sum m(2,3,6,7)$$

$$z_0(x_3,x_2,x_1,x_0) = \sum m(0,2,4,6,8)$$



x	$x_3x_2x_1x_0$	$z_3z_2z_1z_0$	z
0	0 0 0 0	1 0 0 1	9
1	0 0 0 1	1 0 0 0	8
2	0 0 1 0	0 1 1 1	7
3	0 0 1 1	0 1 1 0	6
4	0 1 0 0	0 1 0 1	5
5	0 1 0 1	0 1 0 0	4
6	0 1 1 0	0 0 1 1	3
7	0 1 1 1	0 0 1 0	2
8	1 0 0 0	0 0 0 1	1
9	1 0 0 1	0 0 0 0	0