ENEE244-010x
Digital Logic Design

Lecture 5

Announcements

- HW 2 up on course webpage, due on Monday, Sept. 21.

Agenda

- Last time:
  - Boolean Algebra axioms and theorems (3.1,3.2)
  - Canonical Forms (3.5)
- This time:
  - Examples of Application of DeMorgan's Law (3.6)
  - Simplification of Boolean Formulas (3.6)
  - Canonical Forms (3.5)

Applications of DeMorgan’s Law

Equation Complementation

- For every Boolean function f there is an associated complementary function $\overline{f}$ in which $\overline{f}(x_1, \ldots, x_n) = 1$ iff $f(x_1, \ldots, x_n) = 0$.
- Example:

$$f = \overline{w}x\overline{z} + w(x + \overline{y}z)$$
$$\overline{f} = \overline{\overline{w}x\overline{z} + w(x + \overline{y}z)}$$

Equation Complementation

- Use DeMorgan's Law to simplify:

$$\overline{f} = \overline{\overline{w}x\overline{z} + w(x + \overline{y}z)}$$

$$= \overline{(\overline{w}x\overline{z})}\ \overline{[w(x + \overline{y}z)]}$$

$$= (w + \overline{x} + z)\overline{[w(x + \overline{y}z)]}$$

$$= (w + \overline{x} + z)[\overline{w} + \overline{(x + \overline{y}z)}]$$

$$= (w + \overline{x} + z)[\overline{w} + \overline{x}\ (y + \overline{z})\ ]$$

A Two-Valued Boolean Algebra

- $B = \{0,1\}$
- (+) = OR, (·) = AND
- $\overline{1} = 0, \overline{0} = 1$

Can verify that all the postulates hold by "perfect induction."
[Checking that equality holds for all possible variable settings]

OR

x

		0	1
y	0	0	1
	1	1	1

AND

x

		0	1
y	0	0	0
	1	0	1

Example:

Distributivity: $x + (y \cdot z) = (x + y) \cdot (x + z)$

1. Show that if the left hand side is true then the right hand side is true

2. If the right hand side is true then the left hand side is true.

Theorems Specific to Two-Valued Boolean Algebras

Shannon's Expansion Theorem

Theorem 3.11

$(a) f(x_1, \dots, x_i, \dots, x_n) =$
$x_i \cdot f(x_1, \dots, 1, \dots, x_n) + \overline{x}_i \cdot f(x_1, \dots, 0, \dots, x_n)$

$(b) f(x_1, \dots, x_i, \dots, x_n) =$
$(\overline{x}_i + f(x_1, \dots, 1, \dots, x_n))(x_i + f(x_1, \dots, 0, \dots, x_n))$

Example of Equation Simplification (Using Expansion about a Variable)

$$f(x, y, z) = (x + xy)(\overline{x} + y) + yz$$
$$= x(1 + 1\ \cdot y)(0 + y) + \overline{x}(0 + 0 \cdot y)(1 + y) + yz$$
$$xy + yz$$

Shannon's Reduction Theorems

Theorem 3.12

(a) $x_i \cdot f(x_1, \ldots, x_i, \ldots, x_n) = x_i \cdot f(x_1, \ldots, 1, \ldots, x_n)$

(b) $x_i + f(x_1, \ldots, x_i, \ldots, x_n) = x_i + f(x_1, \ldots, 0, \ldots, x_n)$

Theorem 3.13

(a) $\overline{x}_i \cdot f(x_1, \ldots, x_i, \ldots, x_n) = \overline{x}_i \cdot f(x_1, \ldots, 0, \ldots, x_n)$

(b) $\overline{x}_i + f(x_1, \ldots, x_i, \ldots, x_n) = \overline{x}_i + f(x_1, \ldots, 1, \ldots, x_n)$

Example of Equation Simplification (Using Shannon's Reduction Theorems)

$$
\begin{aligned}
f(w,x,y,z) &= x + \boxed{\overline{x}\,\overline{y} + \overline{w}\,\overline{x}(w+z)(y+\overline{w}z)} \\
&= x \boxed{+ \overline{y} + \overline{w}\boxed{(w+z)(y+\overline{w}z)}} \\
&= x + \overline{y} + \overline{w}\boxed{z\,(y+z)} \\
&= \overline{y} + \boxed{x + \overline{w}z\,(y+z)} \\
&= \overline{y} + \boxed{x + \overline{w}z}
\end{aligned}
$$

Simplification In-Class Exercise

Boolean Formulas and Functions

- Example: $f(x, y, z) = (\overline{x} + y)z$
- Can be specified via a truth table.

X	Y	Z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Normal Forms

- Consider the function:

$$f(w,x,y,z) = \overline{x} + w\overline{y} + \overline{w}\,\overline{y}z$$

- A literal is an occurrence of a complemented or uncomplemented variable in a formula.
- A product term is either a literal or a product (conjunction) of literals.
- Disjunctive normal form: A Boolean formula written as a single product term or as a sum (disjunction) of product terms.

Normal Forms

- Consider the function:

$$f(w,x,y,z) = z(x+\overline{y})(w+\overline{x}+\overline{y})$$

- A sum term is either a literal or a sum (disjunction) of literals.
- Conjunctive normal form: A Boolean formula written as a single sum term or as a product (conjunction) of sum terms.

Canonical Formulas

- How to obtain a Boolean formula given a truth table?

X	Y	Z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

Minterm Canonical Formula

X	Y	Z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

$\overline{x}\,\overline{y}\,z$

$\overline{x}\,y\,z$

$x\,\overline{y}\,\overline{z}$

$$f(x,y,z) = \overline{x}\,\overline{y}\,z + \overline{x}y\,z + x\,\overline{y}\,\overline{z}$$

m-Notation

X	Y	Z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

$\bar{x}\,\bar{y}\,z$

$\bar{x}\,y\,z$

$x\,\bar{y}\,\bar{z}$

- $f(x,y,z)$ can be written as $f(x,y,z) = m_1 + m_3 + m_4$
- $f(x,y,z) = \Sigma m(1,3,4)$