Hamming Codes

Homework 2 Supplement

Due on 9/21/15

Recall the following parity check matrix for Hamming Codes:

$$H = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

1. Encode message $\vec{m} = 1001$

2. Decode $\vec{s} = 0100001$

Now consider the Hamming Code corresponding to the following parity check matrix:

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

1. Encode message $\vec{m} = 1001$

2. Apply error correction and decode $\vec{s} = 0001001$

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Extra Credit I: (Up to 5 points)

Consider the Hamming Code corresponding to the following parity check matrix:

$$H = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

In class, we always put parity check bits in positions corresponding to the columns of H that have exactly one 1 and put message bits in positions corresponding to the columns of H that have at least two 1's. Is this necessary? Does it matter which positions correspond to parity check bits and which positions correspond to message bits? Justify your answer.

Extra Credit II: (Up to 5 points)

The Hamming Code is a linear code, which means that the encoding process can be done by multiplying the message $\vec{m} = (m_1, m_2, m_3, m_4)$ by a matrix C of dimension 4×7. For the Hamming Code defined by parity check matrix

$$H = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

what is the corresponding matrix C? What is the relationship between C and H?