Cryptography

Lecture 9

Announcements

- HW2 is due Monday, 2/26
- HW3 is up on Canvas and the Course webpage due Wednesday, 3/6

Agenda

- Last time:
  - PRF Class Exercise
  - Block Ciphers (K/L 3.5)
  - Modes of Operation (K/L 3.6)
- This time:
  - Introduction to MACs
  - Security Definition for MAC (K/L 4.2)
  - Constructing MAC from PRF (K/L 4.3)
  - Begin Discussing Domain Extension for MACs (K/L 4.4)
  - Class Exercise

Message Integrity

Authenticity/

- Secrecy vs. Integrity

- Encryption vs. Message Authentication

MAC

S

K

m

$t \leftarrow \mathrm{Mac}_K(m)$

$\xrightarrow{(m, t)}$

R

K

$\mathrm{Vrfy}_K(m, t) = 0/1$

1 looks good

0 it was modified.

Correctness: $\mathrm{Vrfy}_K(m, \mathrm{Mac}_K(m)) = 1$.

S $\qquad\qquad\qquad\qquad$ R

k $\qquad\qquad\qquad\qquad$ k

$$\xrightarrow{c = (c_1, c_2)}$$

m

$r \leftarrow \{0,1\}^n$

$c = (r, F_k(r) \oplus m)$ $\qquad$ Eve

Auth. any length $2n$ bitstring will decrypt

(c_1, c_2)

Dec will output $F_k(c_1) \oplus c_2$

Integrity: Flip the last message:

Just flip last bit of c_2: XOR'ing w/ $0^{n-1} \| 1$

Message Authentication Codes

Definition: A message authentication code (MAC) consists of three probabilistic polynomial-time algorithms $(Gen, Mac, Vrfy)$ such that:

1. The key-generation algorithm Gen takes as input the security parameter 1^n and outputs a key k with $|k| \geq n$.
2. The tag-generation algorithm Mac takes as input a key k and a message $m \in \{0,1\}^*$, and outputs a tag t. $t \leftarrow Mac_k(m)$.
3. The deterministic verification algorithm $Vrfy$ takes as input a key k, a message m, and a tag t. It outputs a bit b with $b = 1$ meaning valid and $b = 0$ meaning invalid. $b \coloneqq Vrfy_k(m, t)$.

It is required that for every n, every key k output by $Gen(1^n)$, and every $m \in \{0,1\}^*$, it holds that $Vrfy_k\big(m, Mac_k(m)\big) = 1$.

Unforgeability for MACs

Consider a message authentication code $\Pi = (Gen, Mac, Vrfy)$, any adversary A, and any value n for the security parameter.

Experiment $\underline{MACforge_{A,\Pi}(n)}$

Adversary $A(1^n)$

Challenger

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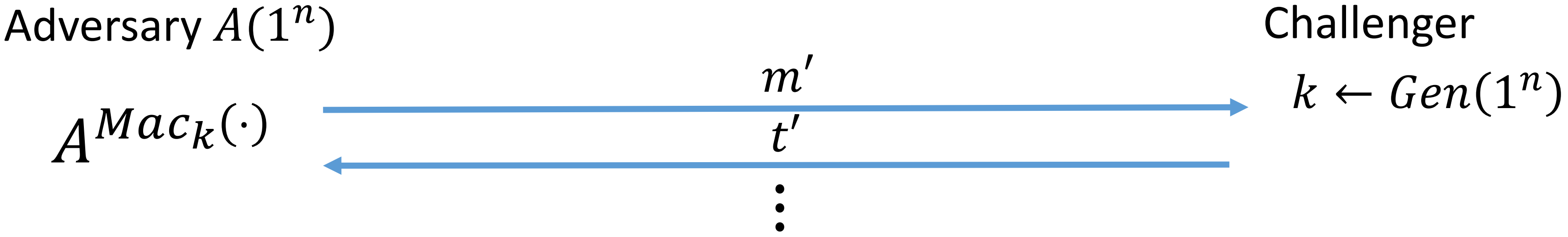

Adversary $A(1^n)$

Challenger

$k \leftarrow Gen(1^n)$

$A^{Mac_k(\cdot)}$

m'

t'

$\vdots$

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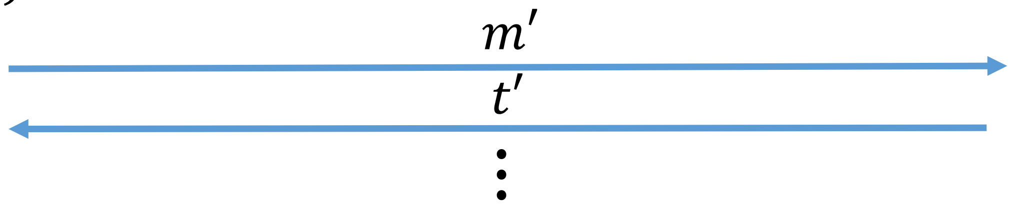

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$\vdots$

Challenger

$k \leftarrow Gen(1^n)$

Q is the set of all messages m' queried by A

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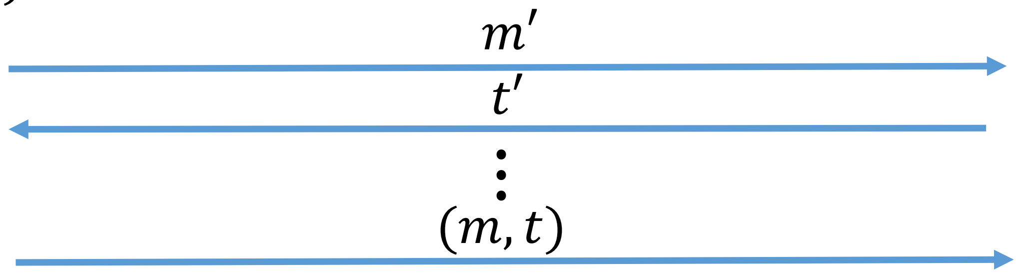

m'

t'

⋮

(m, t)

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Unforgeability for MACs

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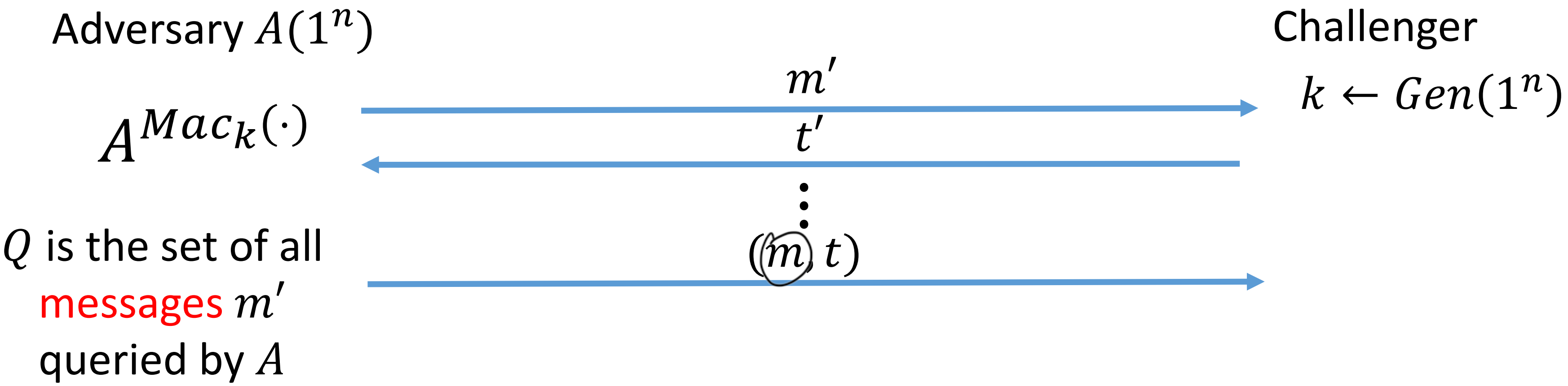

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$k \leftarrow Gen(1^n)$

$A^{Mac_k(\cdot)}$

m'

t'

$\vdots$

(m, t)

Q is the set of all messages m' queried by A

$MACforge_{A,\Pi}(n) = 1$ if both of the following hold:

1. $m \notin Q$
2. $Vrfy_k(m, t) = 1$

Otherwise, $MACforge_{A,\Pi}(n) = 0$

Security of MACs

The message authentication experiment $MACforge_{A,\Pi}(n)$:

1. A key k is generated by running $Gen(1^n)$.
2. The adversary A is given input 1^n and oracle access to $Mac_k(\cdot)$. The adversary eventually outputs (m, t). Let Q denote the set of all queries that A asked its oracle.
3. A succeeds if and only if (1) $Vrfy_k(m, t) = 1$ and (2) $m \notin Q$. In that case, the output of the experiment is defined to be 1.

Security of MACs

"Secure MAC"

Definition: A message authentication code $\Pi = (Gen, Mac, Vrfy)$ is existentially unforgeable under an adaptive chosen message attack if for all probabilistic polynomial-time adversaries A, there is a negligible function neg such that:

$$\Pr\left[MACforge_{A,\Pi}(n) = 1\right] \le neg(n).$$

"Strongly Secure"

Strong Unforgeability for MACs

Consider a message authentication code $\Pi = (Gen, Mac, Vrfy)$, any adversary A, and any value n for the security parameter.

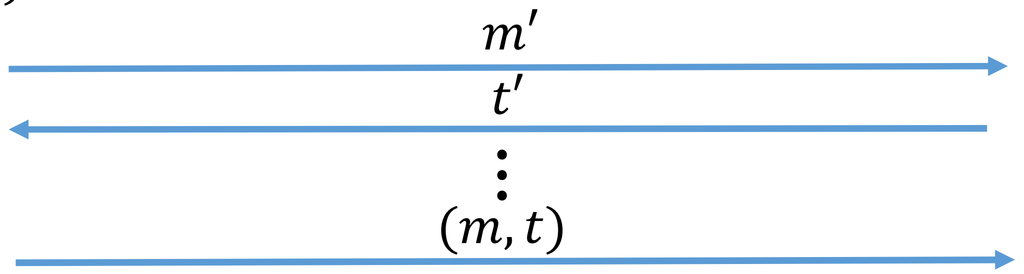
Experiment $MACsforge_{A,\Pi}(n)$

Adversary $A(1^n)$

Challenger

$k \leftarrow Gen(1^n)$

$A^{Mac_k(\cdot)}$



Q is the set of all message, tag pairs (m', t') queried/received by A

$MACsforge_{A,\Pi}(n) = 1$ if both of the following hold:

1. $(m, t) \notin Q$
2. $Vrfy_k(m, t) = 1$

Otherwise, $MACsforge_{A,\Pi}(n) = 0$

Strong MACs

The strong message authentication experiment $MACsforge_{A,\Pi}(n)$:

1. A key k is generated by running $Gen(1^n)$.
2. The adversary A is given input 1^n and oracle access to $Mac_k(\cdot)$. The adversary eventually outputs (m,t). Let Q denote the set of all pairs (m,t) that A asked its oracle.
3. A succeeds if and only if (1) $Vrfy_k(m,t) = 1$ and (2) $(m,t) \notin Q$. In that case, the output of the experiment is defined to be 1.

Strong MACs

Definition: A message authentication code $\Pi = (Gen, Mac, Vrfy)$ is a strong MAC if for all probabilistic polynomial-time adversaries A, there is a negligible function neg such that:

$$\Pr\left[MACsforge_{A,\Pi}(n) = 1\right] \leq neg(n)\,.$$

Constructing Secure Message Authentication Codes

$k \xleftarrow{R} \{0,1\}^n$

$\mathrm{Mac}_k(m) = \underbrace{F_k(m)}_{\text{tag}}$

$\mathrm{Vrfy}_k(m, t)$: Compute $F_k(m)$

Check $F_k(m) \overset{?}{=} t$

If yes, output 1

If no, output 0

A Fixed-Length MAC

Let F be a pseudorandom function. Define a fixed-length MAC for messages of length n as follows:

- Mac: on input a key $k \in \{0,1\}^n$ and a message $m \in \{0,1\}^n$, output the tag $t \coloneqq F_k(m)$.
- $Vrfy$: on input a key $k \in \{0,1\}^n$, a message $m \in \{0,1\}^n$, and a tag $t \in \{0,1\}^n$, output 1 if and only if $t = F_k(m)$.

Security Analysis

Theorem: If F is a pseudorandom function, then the construction above is a secure fixed-length MAC for messages of length n.

Proof:

$$\left.\begin{array}{l}\text{Assume MAC is insecure}\\ \text{Prove PRF is insecure}\end{array}\right\}\text{Contrapositive}$$

Pseudorandom Function

Definition: Let $F: \{0,1\}^* \times \{0,1\}^* \to \{0,1\}^*$ be an efficient, length-preserving, keyed function. We say that F is a pseudorandom function if for all ppt distinguishers D, there exists a negligible function $negl$ such that:

$$\left|\Pr\left[D^{F_k(\cdot)}(1^n) = 1\right] - \Pr\left[D^{f(\cdot)}(1^n) = 1\right]\right| \\ \leq negl(n).$$

where $k \leftarrow \{0,1\}^n$ is chosen uniformly at random and f is chosen uniformly at random from the set of all functions mapping n-bit strings to n-bit strings.

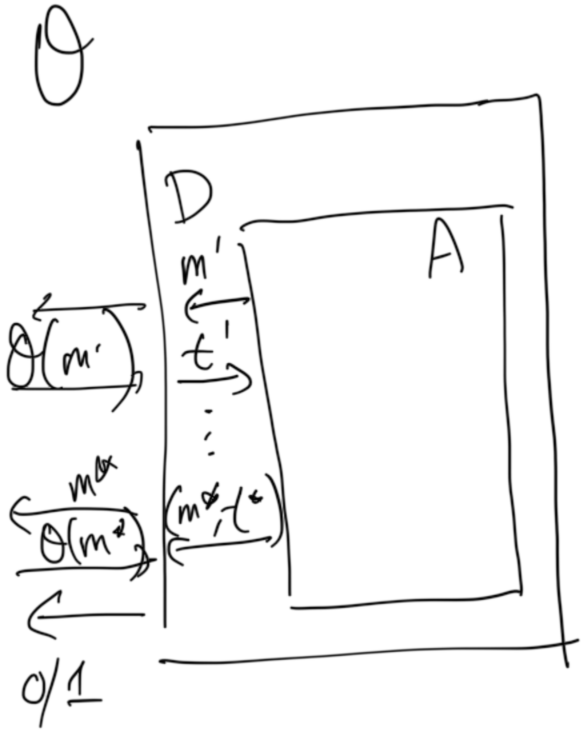
$$\exists \text{ ppt } D \text{ s.t. } \left|\Pr\left[D^{F_k(\cdot)}(1^n) = 1\right] - \Pr\left[D^{f(\cdot)}(1^n) = 1\right]\right| \geq \underbrace{p'(n)}_{\text{non-negl.}}$$

Security of MACs

Definition: A message authentication code $\Pi = (Gen, Mac, Vrfy)$ is existentially unforgeable under an adaptive chosen message attack if for all probabilistic polynomial-time adversaries A, there is a negligible function neg such that:

$$\Pr\left[MACforge_{A,\Pi}(n) = 1\right] \leq neg(n).$$

$$\exists \text{ ppt } A \text{ s.t. } \Pr\left[MACforge_{A,\Pi}(n) = 1\right] \geq \overbrace{p(n)}^{\text{non-negl.}}$$



1. How does D respond to m'
   (a) Send m' to oracle get back $\mathcal{O}(m')$
   (b) return $t = \mathcal{O}(m')$ to A

2. Given (m^*, t^*) how to decide an 0/1 output?

$$\text{If } m^* \notin Q \text{ AND } \underbrace{\mathcal{O}(m^*)} = t^*$$

$\longrightarrow$ Output 1

o/w Output 0

Case 1: $\mathcal{O} = F_K$

by assumption

$$\Pr\left[D^{F_K(\cdot)}(1^n) = 1\right] = \Pr\left[\text{MACforge}_{A,\Pi}(n) = 1\right] \geq \rho(n)$$

Case 2: $\mathcal{O} = f$

$$\Pr\left[D^{f(\cdot)}(1^n) = 1\right] = \frac{1}{2^n}$$

Diff in prob: $\underbrace{\rho'(n)}_{\text{Show it is non-negl.}} = \rho(n) - \frac{1}{2^n}$

☑

Security Analysis

Let A be a ppt adversary trying to break the security of the construction. We construct a distinguisher D that uses A as a subroutine to break the security of the PRF.

Distinguisher D:

D gets oracle access to oracle O, which is either F_k, where F is pseudorandom or f which is truly random.

1. Instantiate $A^{Mac_k(\cdot)}(1^n)$.
2. When A queries its oracle with message m, output $O(m)$.
3. Eventually, A outputs (m^*, t^*) where $m^*, t^* \in \{0,1\}^n$.
4. If $m^* \in Q$, output 0.
5. If $m^* \notin Q$, query $O(m^*)$ to obtain output z^*.
6. If $t^* = z^*$ output 1. Otherwise, output 0.

Security Analysis

Consider the probability D outputs 1 in the case that O is truly random function f vs. O is a pseudorandom function F_k.

- When O is pseudorandom, D outputs 1 with probability $\Pr\left[MACforge_{A,\Pi}(n) = 1\right] = \rho(n)$, where ρ is non-negligible.
- When O is random, D outputs 1 with probability at most $\frac{1}{2^n}$. Why?

Security Analysis

D's distinguishing probability is:

$$\left|\frac{1}{2^n} - \rho(n)\right| = \rho(n) - \frac{1}{2^n}.$$

Since, $\frac{1}{2^n}$ is negligible and $\rho(n)$ is non-negligible, $\rho(n) - \frac{1}{2^n}$ is non-negligible.

This is a contradiction to the security of the PRF.

Domain Extension for MACs