Cryptography

Lecture 10

Announcements

- HW3 due on Wednesday, 3/6

Agenda

- Last time:
  - MACs (K/L 4.1, 4.2, 4.3)
- This time:
  - Domain Extension for MACs (K/L 4.4) and Class Exercise solutions
  - CCA security (K/L 3.7)
  - Authenticated Encryption (K/L 4.5)

Message Authentication Codes

Definition: A message authentication code (MAC) consists of three probabilistic polynomial-time algorithms $(Gen, Mac, Vrfy)$ such that:

1. The key-generation algorithm Gen takes as input the security parameter 1^n and outputs a key k with $|k| \geq n$.
2. The tag-generation algorithm Mac takes as input a key k and a message $m \in \{0,1\}^*$, and outputs a tag t.
   $t \leftarrow Mac_k(m)$.
3. The deterministic verification algorithm $Vrfy$ takes as input a key k, a message m, and a tag t. It outputs a bit b with $b = 1$ meaning valid and $b = 0$ meaning invalid.
   $b := Vrfy_k(m, t)$.

It is required that for every n, every key k output by $Gen(1^n)$, and every $m \in \{0,1\}^*$, it holds that $Vrfy_k\big(m, Mac_k(m)\big) = 1$.

Unforgeability for MACs

Consider a message authentication code $\Pi = (Gen, Mac, Vrfy)$, any adversary A, and any value n for the security parameter.

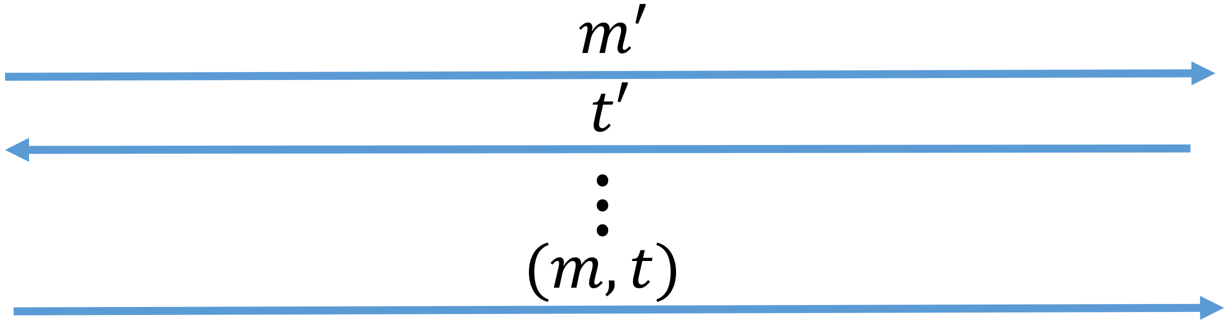
Experiment $MACforge_{A,\Pi}(n)$

Adversary $A(1^n)$

Challenger

$k \leftarrow Gen(1^n)$

$A^{Mac_k(\cdot)}$



Q is the set of all messages m' queried by A

$MACforge_{A,\Pi}(n) = 1$ if both of the following hold:

1. $m \notin Q$
2. $Vrfy_k(m, t) = 1$

Otherwise, $MACforge_{A,\Pi}(n) = 0$

Security of MACs

The message authentication experiment $MACforge_{A,\Pi}(n)$:

1. A key k is generated by running $Gen(1^n)$.
2. The adversary A is given input 1^n and oracle access to $Mac_k(\cdot)$. The adversary eventually outputs (m, t). Let Q denote the set of all queries that A asked its oracle.
3. A succeeds if and only if (1) $Vrfy_k(m, t) = 1$ and (2) $m \notin Q$. In that case, the output of the experiment is defined to be 1.

Security of MACs

Definition: A message authentication code $\Pi = (Gen, Mac, Vrfy)$ is existentially unforgeable under an adaptive chosen message attack if for all probabilistic polynomial-time adversaries A, there is a negligible function neg such that:

$$\Pr\left[MACforge_{A,\Pi}(n) = 1\right] \leq neg(n).$$

Strong Unforgeability for MACs

Consider a message authentication code $\Pi = (Gen, Mac, Vrfy)$, any adversary A, and any value n for the security parameter.

Experiment $MACsforge_{A,\Pi}(n)$

Adversary $A(1^n)$

Challenger

$k \leftarrow Gen(1^n)$

$A^{Mac_k(\cdot)}$

m' →

← t'

⋮

(m, t) →

Q is the set of all message, tag pairs (m', t') queried/received by A

$MACsforge_{A,\Pi}(n) = 1$ if both of the following hold:

1. $m \notin Q$
2. $Vrfy_k(m, t) = 1$

Otherwise, $MACsforge_{A,\Pi}(n) = 0$

Strong MACs

The strong message authentication experiment $MACsforge_{A,\Pi}(n)$:

1. A key k is generated by running $Gen(1^n)$.
2. The adversary A is given input 1^n and oracle access to $Mac_k(\cdot)$. The adversary eventually outputs (m, t). Let Q denote the set of all pairs (m, t) that A asked its oracle.
3. A succeeds if and only if (1) $Vrfy_k(m, t) = 1$ and (2) $(m, t) \notin Q$. In that case, the output of the experiment is defined to be 1.

Strong MACs

Definition: A message authentication code $\Pi = (Gen, Mac, Vrfy)$ is a strong MAC if for all probabilistic polynomial-time adversaries A, there is a negligible function neg such that:

$$\Pr\left[MACsforge_{A,\Pi}(n) = 1\right] \leq neg(n)\,.$$

Domain Extension for MACs

CBC-MAC

Let F be a pseudorandom function, and fix a length function ℓ. The basic CBC-MAC construction is as follows:

- Mac: on input a key $k \in \{0,1\}^n$ and a message m of length $\ell(n) \cdot n$, do the following:
  1. Parse m as $m = m_1, \ldots, m_\ell$ where each m_i is of length n.
  2. Set $t_0 := 0^n$. Then, for $i = 1\ to\ \ell$:

     Set $t_i := F_k(t_{i-1} \oplus m_i)$.

  Output t_ℓ as the tag.
- $Vrfy$: on input a key $k \in \{0,1\}^n$, a message m, and a tag t, do: If m is not of length $\ell(n) \cdot n$ then output 0. Otherwise, output 1 if and only if $t = Mac_k(m)$.

CBC-MAC

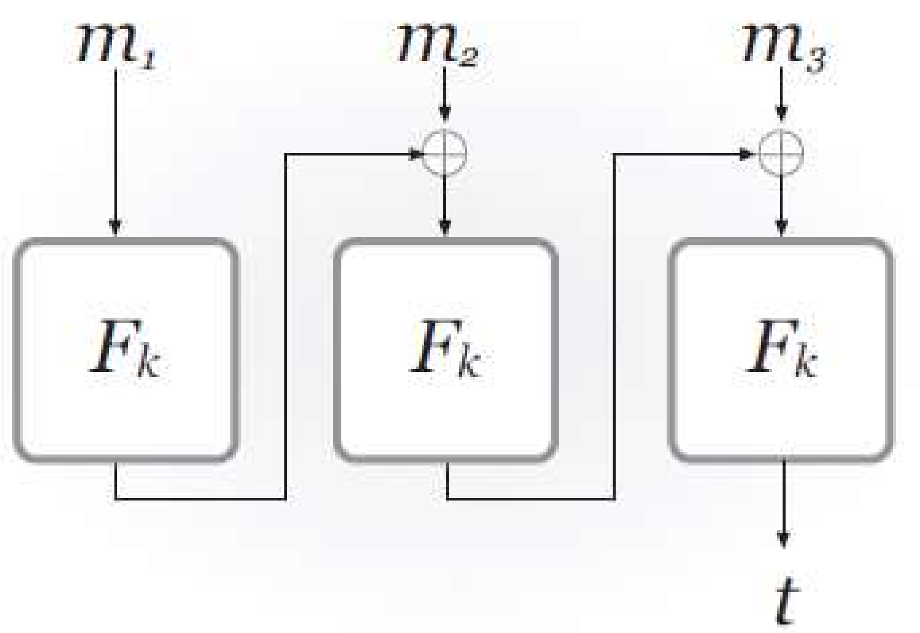


FIGURE 4.1: Basic CBC-MAC (for fixed-length messages).

Chosen Ciphertext Security

CCA Security*

Consider a private-key encryption scheme $\Pi = (Gen, Enc, Dec)$, any adversary A, and any value n for the security parameter.

Experiment $PrivK_{A,\Pi}^{cca}(n)$

Adversary $A(1^n)$

Challenger

CCA Security*

Consider a private-key encryption scheme $\Pi = (Gen, Enc, Dec)$, any adversary A, and any value n for the security parameter.

Experiment $PrivK^{cca}_{A,\Pi}(n)$

Adversary $A(1^n)$

Challenger

$k \leftarrow Gen(1^n)$

CCA Security*

Consider a private-key encryption scheme $\Pi = (Gen, Enc, Dec)$, any adversary A, and any value n for the security parameter.

Experiment $PrivK^{cca}_{A,\Pi}(n)$

Adversary $A(1^n)$

Challenger

$k \leftarrow Gen(1^n)$

$A^{Enc_k(\cdot),Dec_k(\cdot)}$

m'/c'

c'/m'

$\vdots$

CCA Security*

Consider a private-key encryption scheme $\Pi = (Gen, Enc, Dec)$, any adversary A, and any value n for the security parameter.

Experiment $PrivK_{A,\Pi}^{cca}(n)$

Adversary $A(1^n)$

Challenger

$k \leftarrow Gen(1^n)$

$A^{Enc_k(\cdot),Dec_k(\cdot)}$

m'/c' →

← c'/m'

⋮

m_0, m_1 →

CCA Security*

Consider a private-key encryption scheme $\Pi = (Gen, Enc, Dec)$, any adversary A, and any value n for the security parameter.

Experiment $PrivK^{cca}_{A,\Pi}(n)$

Adversary $A(1^n)$

Challenger

m'/c'

$A^{Enc_k(\cdot),Dec_k(\cdot)}$

$k \leftarrow Gen(1^n)$

c'/m'

⋮

m_0, m_1

$b \leftarrow \{0,1\}$

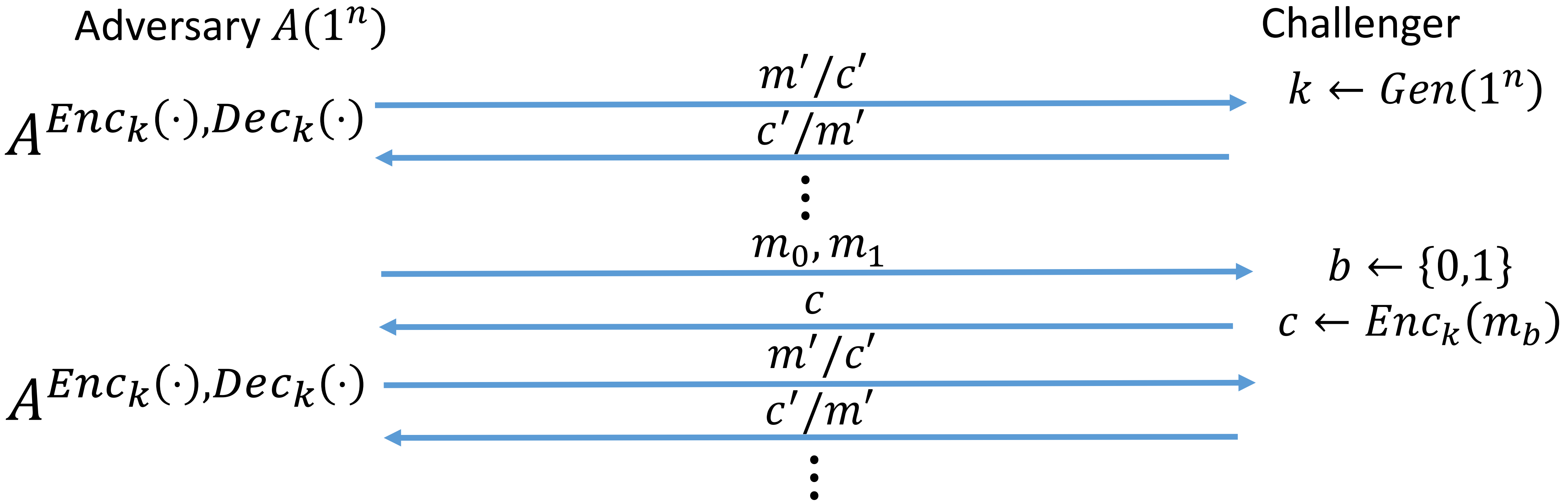
c

$c \leftarrow Enc_k(m_b)$

CCA Security*

Consider a private-key encryption scheme $\Pi = (Gen, Enc, Dec)$, any adversary A, and any value n for the security parameter.

Experiment $PrivK^{cca}_{A,\Pi}(n)$



CCA Security*

Consider a private-key encryption scheme $\Pi = (Gen, Enc, Dec)$, any adversary A, and any value n for the security parameter.

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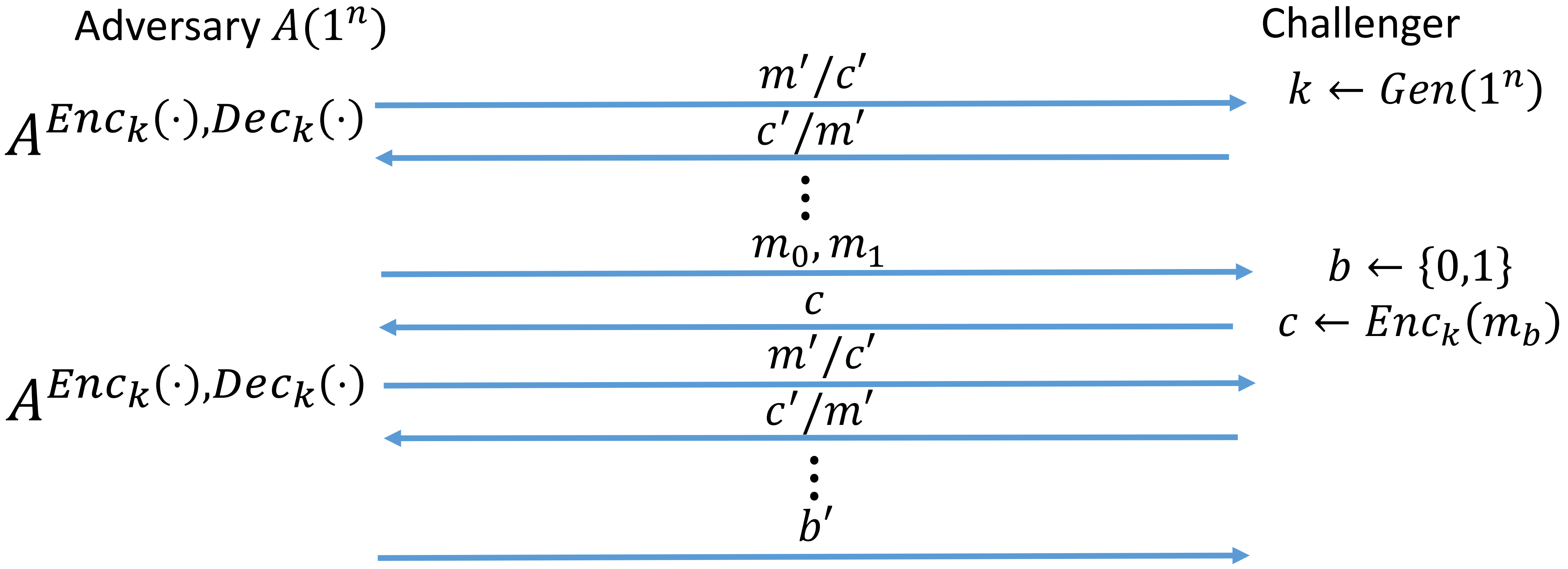

CCA Security*

Consider a private-key encryption scheme $\Pi = (Gen, Enc, Dec)$, any adversary A, and any value n for the security parameter.

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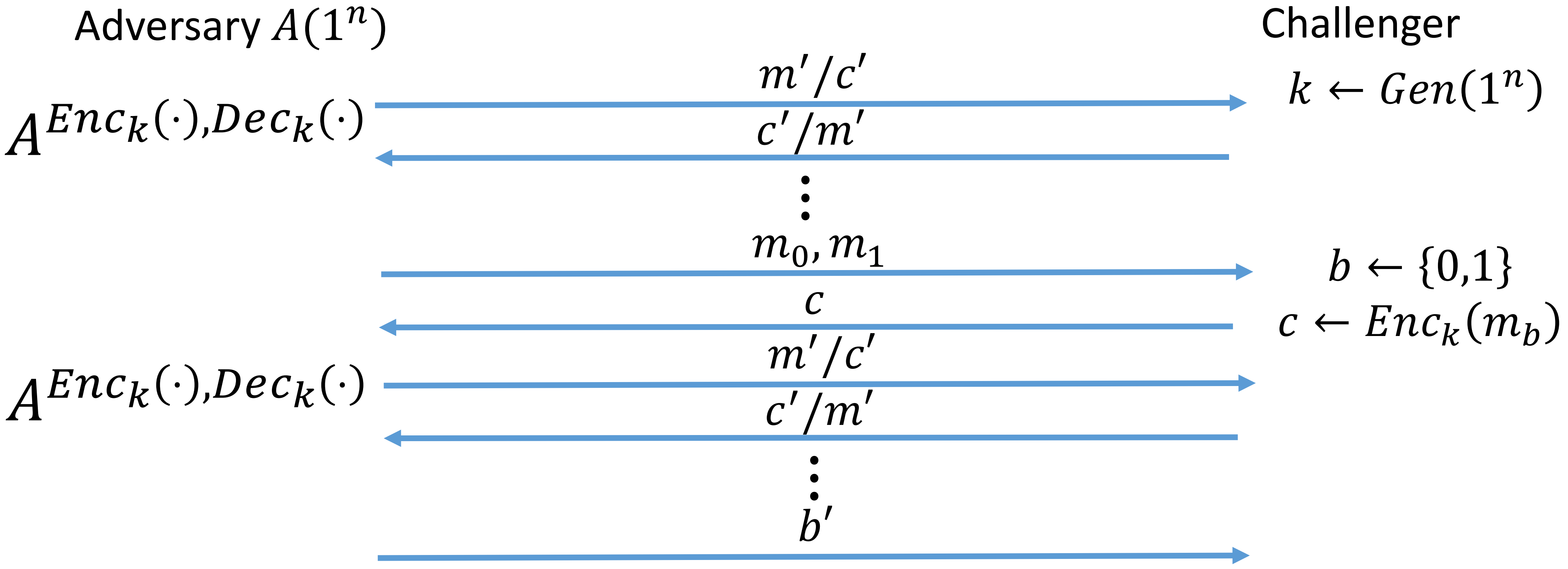

$PrivK^{cca}_{A,\Pi}(n) = 1$ if $b' = b$ and $PrivK^{cca}_{A,\Pi}(n) = 0$ if $b' \neq b$.

CCA Security

Consider a private-key encryption scheme $\Pi = (Gen, Enc, Dec)$, any adversary A, and any value n for the security parameter.

Experiment $PrivK^{cca}_{A,\Pi}(n)$

Adversary $A(1^n)$		Challenger
$A^{Enc_k(\cdot), Dec_k(\cdot)}$	m'/c' →	$k \leftarrow Gen(1^n)$
	← c'/m'	
	⋮	
	m_0, m_1 →	$b \leftarrow \{0,1\}$
	← c	$c \leftarrow Enc_k(m_b)$
$A^{Enc_k(\cdot), Dec^*_k(\cdot)}$	m'/c' →	
	← c'/m'	
	⋮	
	b' →	

A may not query challenge ciphertext c to the decryption oracle

$PrivK^{cca}_{A,\Pi}(n) = 1$ if $b' = b$ and $PrivK^{cca}_{A,\Pi}(n) = 0$ if $b' \neq b$.

CCA Security

The CCA Indistinguishability Experiment $PrivK^{cca}{}_{A,\Pi}(n)$:

1. A key k is generated by running $Gen(1^n)$.
2. The adversary A is given input 1^n and oracle access to $Enc_k(\cdot)$ and $Dec_k(\cdot)$, and outputs a pair of messages m_0, m_1 of the same length.
3. A random bit $b \leftarrow \{0,1\}$ is chosen, and then a challenge ciphertext $c \leftarrow Enc_k(m_b)$ is computed and given to A.
4. The adversary A continues to have oracle access to $Enc_k(\cdot)$ and $Dec_k(\cdot)$, but is not allowed to query the latter on the challenge ciphertext itself. Eventually, A outputs a bit b'.
5. The output of the experiment is defined to be 1 if $b' = b$, and 0 otherwise.

CCA Security

A private-key encryption scheme $\Pi = (Gen, Enc, Dec)$ has indistinguishable encryptions under a chosen-ciphertext attack if for all ppt adversaries A there exists a negligible function $negl$ such that

$$\Pr\left[PrivK^{cca}{}_{A,\Pi}(n) = 1\right] \leq \frac{1}{2} + negl(n),$$

where the probability is taken over the random coins used by A, as well as the random coins used in the experiment.

Authenticated Encryption

The unforgeable encryption experiment $EncForge_{A,\Pi}(n)$:

1. Run $Gen(1^n)$ to obtain key k.
2. The adversary A is given input 1^n and access to an encryption oracle $Enc_k(\cdot)$. The adversary outputs a ciphertext c.
3. Let $m := Dec_k(c)$, and let Q denote the set of all queries that A asked its encryption oracle. The output of the experiment is 1 if and only if (1) $m \neq \perp$ and (2) $m \notin Q$.

Authenticated Encryption

Definition: A private-key encryption scheme Π is unforgeable if for all ppt adversaries A, there is a negligible funcion neg such that:

$$\Pr\left[EncForge_{A,\Pi}(n) = 1\right] \leq neg(n).$$

Definition: A private-key encryption scheme is an authenticated encryption scheme if it is CCA-secure and unforgeable.

Generic Constructions

Encrypt-and-authenticate

Encryption and message authentication are computed independently in parallel.

$$c \leftarrow Enc_{k_E}(m) \qquad t \leftarrow Mac_{k_M}(m)$$

$$\langle c, t \rangle$$

Is this secure?

Encrypt-and-authenticate

Encryption and message authentication are computed independently in parallel.

$$c \leftarrow Enc_{k_E}(m) \qquad t \leftarrow Mac_{k_M}(m)$$

$$\langle c, t \rangle$$

Is this secure? NO! Tag can leak info on m

Authenticate-then-encrypt

Here a MAC tag t is first computed, and then the message and tag are encrypted together.

$$t \leftarrow Mac_{k_M}(m) \qquad c \leftarrow Enc_{k_E}(m||t)$$

c is sent

Is this secure?

Authenticate-then-encrypt

Here a MAC tag t is first computed, and then the message and tag are encrypted together.

$$t \leftarrow Mac_{k_M}(m) \qquad c \leftarrow Enc_{k_E}(m||t)$$

c is sent

Is this secure? NO! Encryption scheme may not be CCA-secure.

Encrypt-then-authenticate

The message m is first encrypted and then a MAC tag is computed over the result

$$c \leftarrow Enc_{k_E}(m) \quad t \leftarrow Mac_{k_M}(c)$$

$$\langle c, t \rangle$$

Is this secure?

Encrypt-then-authenticate

The message m is first encrypted and then a MAC tag is computed over the result

$$c \leftarrow Enc_{k_E}(m) \quad t \leftarrow Mac_{k_M}(c)$$

$$\langle c, t \rangle$$

Is this secure? YES! As long as the MAC is strongly secure.