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Chapter 2

Sets, Fcns, Seqs, Sums

coursenotes by Prof. J. L. Gross for Rosen 7th Edition

2.1   SETS

DEF: A *set* is a collection of objects. The objects are called *elements* or *members* of the set.

NOTATION: : $x \in S$

Example 2.1.1:

- $2 \in \{5, -7, \pi, \text{``algebra''}, 2, 2.718\}$

- $8 \notin \{p : p \text{ is a prime number}\}$

SOME STANDARD SETS of NUMBERS

$\mathbb{N}$ = the set of all non-negative integers
    (the "natural numbers")

$\mathbb{Z}$ = the set of all integers

$\mathbb{Z}^+$ = the set of all positive integers

$\mathbb{Q}$ = the rationals = $\left\{ \dfrac{p}{q} : p, q \in \mathbb{Z} \text{ and } q \neq 0 \right\}$

$\mathbb{R}$ = the real numbers

$\mathbb{C}$ = the complex numbers

ROSTERS for SETS

DEF: A **_roster_** specifies a finite set by enclosing in braces a list of representations of its elements. Repetitions and orderings are irrelevant to content.

Example 2.1.2: a roster

$$\{5, -7, \pi, \text{``algebra''}, 2, 2.718\}$$

Example 2.1.3: identical sets

$$\{1, 2, 3, 1, 1, 3\} = \{1, 2, 3\} = \{3, 1, 2\}$$

DEF: The **_empty set_** is the set { } that has no elements. NOTATION: $\emptyset$.

Remark: In mathematics, there is only one empty set. However, a computer programming language may have a different empty set for every datatype.

Example 2.1.4: The empty set of character strings is equal to the empty set of lions.

DEF: A **_singleton set_** is a set with one element.

Example 2.1.5: $\{x\}$ is a singleton set.

SPECIFICATION by PREDICATES

A predicate over a well-defined set can specify any subcollection within that set. (Warning: This "set-builder" method can lead to non-sets.)

Example 2.1.6: $\{x \in \mathcal{Z} : P(x)\}$ where $P(x)$ is TRUE if x is prime.

Example 2.1.7: $\{(x, y) : x, y \in \mathcal{R} \wedge x^2 + y^2 = 1\}$

OTHER WAYS to SPECIFY SETS

(1) By prose. (can also lead to non-sets).

Example 2.1.8:   The set of all palindromes.

(2) By operations on other sets.
Examples soon.

(3) By recursive construction.
Examples in §4.3.

SETS as ELEMENTS of SETS

Object x is not the same as the singleton set $\{x\}$. Moreover, $\{x\} \neq \{\{x\}\}$.

Analogy: Iterative pointers to a computer object creates new objects.

$$x \neq \&x \neq \&\&x$$

Analogy: Iterative enquotation of a character string creates new objects.

$$\text{““lion””} \neq \text{“lion”} \neq \text{lion}$$

RELATIONS on SETS

DEF: Set X is a **subset** of set Y if every element of X is also an element of Y. NOTATION: $X \subseteq Y$.

DEF: A subset X of a set Y is **proper** if Y has at least one element that is not in X.

DEF: Sets X and Y are **equal** if each set is a subset of the other. NOTATION: $X = Y$.

Example 2.1.9:

(1) $\emptyset$ is a proper subset of every set except itself.

(2) The integers are a subset of the real numbers.

DISAMBIGUATION: In computer languages, the integers and the reals are usually distinct **datatypes**. Integer-type values are represented differently from real-type whole numbers.

Remark: Whereas mathematics deals with objects, computation science deals with their representations.

POWER SET

DEF: The **power set** of a set S is the set of all subsets of S. NOTATION: 2^S or $P(S)$.

Example 2.1.10:

- $P(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$.
- $P(\emptyset) = \{\emptyset\}$.
- $P(P(\emptyset)) = \{\emptyset, \{\emptyset\}\}$.

Proposition 2.1.1. *If set S has n elements, then the power set $P(S)$ has 2^n elements.* $\diamondsuit$

CARTESIAN PRODUCT

DEF: The ***cartesian product*** of sets A and B is the set

$$\{(a, b) \mid a \in A \wedge b \in B\}$$

NOTATION: $A \times B$.

Example 2.1.11: $\quad A = \{1, 2\} \quad B = \{a, b, c\}$
Then

$$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$$

Proposition 2.1.2. *The cartesian product $A \times B$ is empty iff either A or B is empty.* $\qquad \diamondsuit$

2.2 SET OPERATIONS

Geometric figures were used by John Venn (1834-1923) to illustrate the effect of various operations on sets called the **universal set** or the **domain of discourse**.

Remark: Not all set operations can be represented by Venn diagrams.

VENN DIAGRAMS

DEF: In a generic **Venn diagram** for a subset S of a fixed universal set U, the universal set is represented by a rectangular region in the plane and the set S is represented by a subregion.
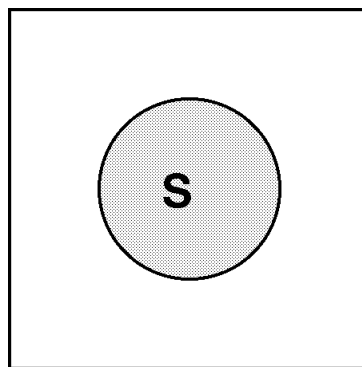

Fig 2.2.1   Set S is shaded.

DYADIC SET OPERATIONS

DEF: The **union** of sets S and T is the set containing every object that is either in S or in T. NOTATION: $S \cup T$.
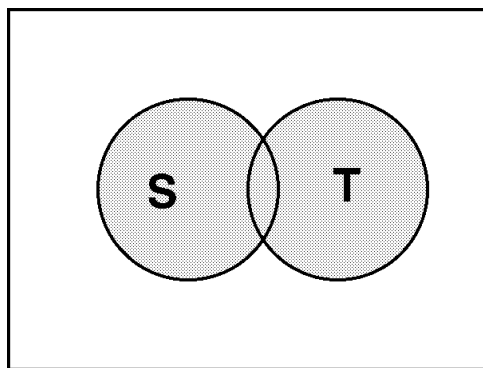

Fig 2.2.2   Union $S \cup T$ is shaded.

DEF: The **intersection** of sets S and T is the set containing every object that is in both S and T. NOTATION: $S \cap T$.
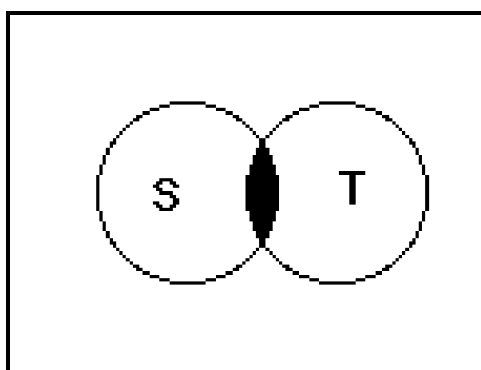

Fig 2.2.3   Intersection $S \cap T$ is shaded.

DEF: The **difference** of sets S and T is the set containing every object that is in S but not in T.
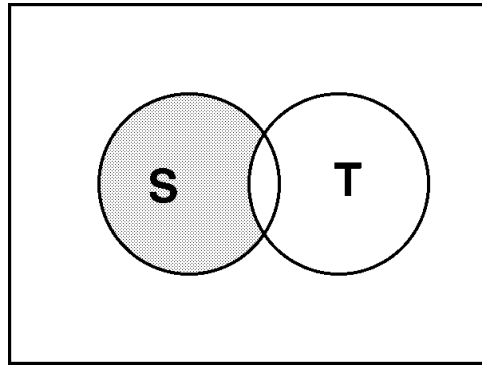
NOTATION: $S - T$.

Fig 2.2.4   Difference $S - T$ is shaded.

Example 2.2.1:

$S = \{1, 3, 5, 7, 9\}$ $T = \{2, 3, 5, 7\}$. Then

$S \cup T = \{1, 2, 3, 5, 7, 9\}$.

$S \cap T = \{3, 5, 7\}$.

$S - T = \{1, 9\}$.

Example 2.2.2:   Cartesian product (which is dyadic) is not representable by a Venn diagram.

MONADIC SET OPERATIONS

DEF: The **complement** of a set S is the set $U - S$, where U is the universal set. NOTATION: $\overline{S}$.
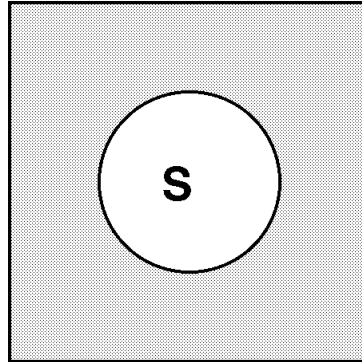

Fig 2.2.5 Complement $\overline{S}$ is shaded.

Example 2.2.3: These monadic operations are not readily representable by Venn diagrams.

$S \to \{S\}$ (**enbracement**)

$S \to P(S)$ (**empowerment**)

TERMINOLOGY NOTE: These two (original) names have excellent mnemonicity. The mildly frivolous character may deter their widespread adoption.

SET IDENTITIES

Various set equivalences have earned the honorific appelation *i*dentity. Many of them are analogous to the logical equivalences of §1.2.

Example 2.2.4: The ***Double Negation Law***

$$\neg\neg p \Leftrightarrow p$$

has the following set-theoretic analogy:

DEF: ***Double Complementation Law:***

$$\overline{\overline{S}} = S$$

Example 2.2.5: The tautology $p \vee \neg p$ is called the ***Law of the Excluded Middle***. It converts to the equivalence

$$p \vee \neg p \Leftrightarrow T$$

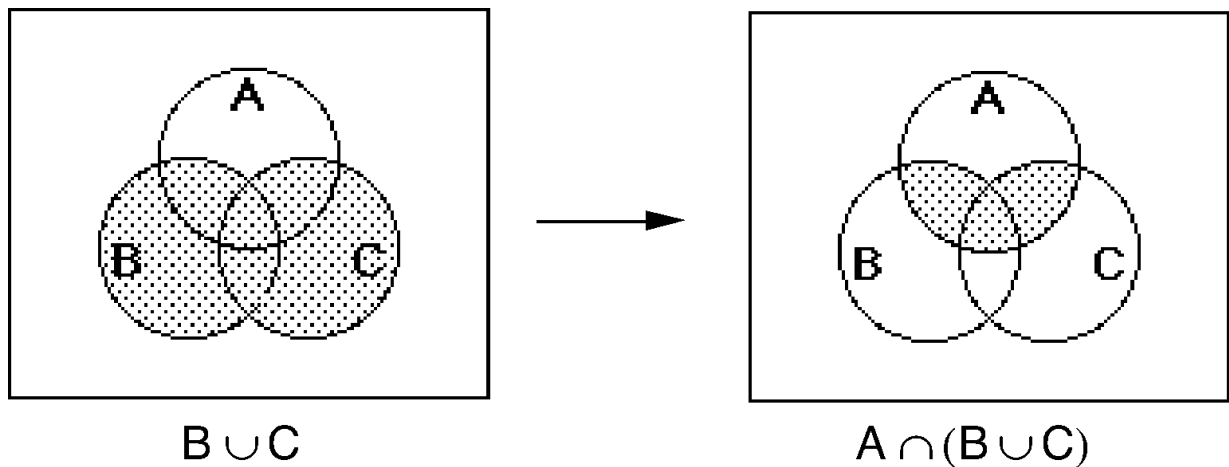which has the following set-theoretic analogy:

$$S \cup \overline{S} = U$$

AVOIDING BOREDOM

Example 2.2.6: Table 1 of §2.2 (de Morgan, associativity, etc.) is good for self-study, but not for exhaustive classroom presentation.

CONFIRMING IDENTITIES with VENN DIAGRAMS

Example 2.2.7: $\cap$ distributes over $\cup$.

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

B $\cup$ C   A $\cap$ (B $\cup$ C)

Example 2.2.8: $\cup$ distributes over $\cap$.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

B $\cap$ C   A $\cup$ (B $\cap$ C)

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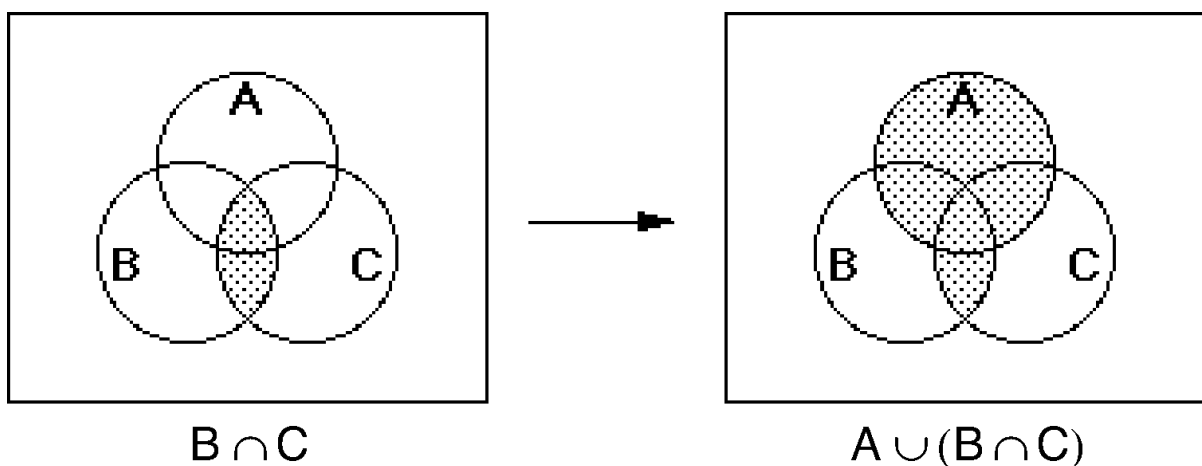
2.3   FUNCTIONS

DEF: Let A and B be sets. A ***function*** $f : A \rightarrow B$ (often abbreviated as f) is a rule that assigns to each element $a \in A$ exactly one element $f(a) \in B$, called the ***value of the function*** f at a.

- We also say that $f : A \rightarrow B$ is a ***mapping*** from ***domain*** A to ***codomain*** B.

- $f(a)$ is called the ***image set of the element*** a, and the element a is called a ***preimage*** of $f(a)$.

- The set $\{a \mid f(a) = b\}$ is called the ***preimage set*** of b. NOTATION: $f^{-1}(b)$.

DEF: The set $\{b \in B \mid (\exists a \in A)[f(a) = b]\}$ is called the ***image of the function*** $f : A \rightarrow B$.

DEF: The word ***range*** is commonly used to mean the image set.

DEF: A function is called ***discrete*** if its domain and codomain are both finite or countable (indexed by $\mathbb{Z}$).

Example 2.3.1:   Some functions from $\mathbb{R}$ to $\mathbb{Z}$.

(1) ***floor*** $\lfloor x \rfloor = \max\{k \in \mathbb{Z} \mid k \leq x\}$ image $= \mathbb{Z}$

(2) ***ceiling*** $\lceil x \rceil = \min\{k \in \mathbb{Z} \mid k \geq x\}$ im $= \mathbb{Z}$

(3) ***sign*** $\sigma(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ +1 & x > 0 \end{cases}$

    $\text{image}(\sigma) = \{-1, 0, +1\}$

Example 2.3.2:   Seq of functions from $\mathbb{R}$ to $\mathbb{R}$.

   falling powers $x^{\underline{n}} = x(x-1)\cdots(x-n+1)$.

$$7^{\underline{3}} = 7\cdot 6\cdot 5 = 210 \qquad \left(\frac{3}{2}\right)^{\underline{3}} = \frac{3}{2}\cdot\frac{1}{2}\cdot\left(\frac{-1}{2}\right) = \frac{-3}{8}$$

Example 2.3.3:   Functions in computation.

(1) *C compiler* maps the set of ASCII strings to the boolean set.

(2) The *halting function* maps the set of C programs to the boolean set, assigns TRUE iff this program will always halt eventually, no matter what input is supplied at run time.

Theorem 2.3.1. *The halting function cannot be represented by a C program.*   ◇ *(CS W3261)*
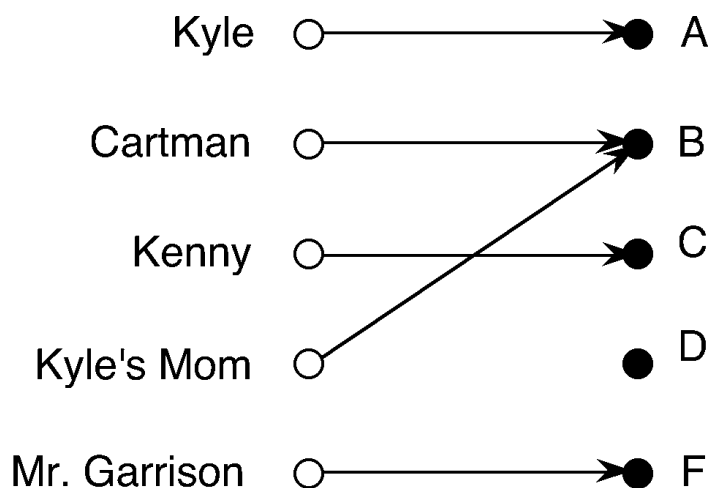
REPRESENTATION of DISCRETE FUNCTIONS

DEF: The $n \times 2$ *array representation* of a discrete function is a table with two columns. The left column contains every element of the domain. The second entry in each row is the image of the first entry.

DEF: The *(full) digraphic representation* of a discrete function is a diagram with two columns of dots. The left column contains a dot for every element of the domain, and the right entry contains a dot for every element of the codomain. From each domain dot an arrow is drawn to the codomain dot representing its image.

Example 2.3.4:   Representing a function.

student	grade
Kyle	A
Cartman	B
Kenny	C
Kyle's Mom	B
Mr. Garrison	F

ONE-TO-ONE and ONTO FUNCTIONS

DEF: A function $f : A \to B$ is **one-to-one** if for every $b \in B$, there is at most one $a \in A$ such that $f(a) = b$.

Prop 2.3.2. *A discrete function is one-to-one iff in its digraphic representation, no codomain dot is at the head of more than one arrow.* ◇

DEF: A function $f : A \to B$ is **onto** if

for every $b \in B$, there is at least one $a \in A$ such that $f(a) = b$.

Prop 2.3.3. *A discrete function is onto iff in its digraphic representation, every codomain dot is at the head of at least one arrow.* ◇

Example 2.3.5:   The grading function of Example 2.3.4 is neither one-to-one nor onto.

BIJECTIONS

DEF: A **bijection** is a function that is one-to-one and onto.

DEF: Let $f : A \to B$ be a bijection. The **inverse function**

$$f^{-1} : B \to A$$

is the rule that assigns to each $b \in B$ the unique element $a \in A$ such that $f(a) = b$.

Example 2.3.6:   The function

$$\{1 \mapsto b, \ 2 \mapsto c, \ 3 \mapsto a\}$$

is a bijection. Its inverse is the function

$$\{a \mapsto 3, \ b \mapsto 1, \ c \mapsto 2\}$$

DEF: A **permutation** is a bijection whose domain and codomain are the same set.

Example 2.3.7:   The function

$$\{1 \mapsto 2, \ 2 \mapsto 3, \ 3 \mapsto 1\}$$

is a permutation. Its inverse is the permutation

$$\{1 \mapsto 3, \ 2 \mapsto 1, \ 3 \mapsto 2\}$$

2.4   SEQUENCES AND SUMS

DEF: A **sequence in a set** A is a function f from a subset of the integers (usually $\{0, 1, 2, \ldots\}$ or $\{1, 2, 3, \ldots\}$) to A. The values of a sequence are also called **terms** or **entries**.

NOTATION: The value $f(n)$ is usually denoted a_n. A sequence is often written

$$a_0, \quad a_1, \quad a_2, \quad \ldots$$

Example 2.4.1:   Two sequences.

$$a_n = \frac{1}{n} \qquad 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$$
$$b_n = (-1)^n \quad 1, -1, 1, -1, \ldots$$

Example 2.4.2:   Five ubiquitous sequences.

$$n^2 \qquad 0, 1, 4, 9, 16, 25, 36, 49, \ldots$$

$$n^3 \qquad 0, 1, 8, 27, 64, 125, 216, 343, \ldots$$

$$2^n \qquad 1, 2, 4, 8, 16, 32, 64, 128, \ldots$$

$$3^n \qquad 1, 3, 9, 27, 81, 243, 729, 2187, \ldots$$

$$n! \qquad 1, 1, 2, 6, 24, 120, 720, 5040, \ldots$$

STRINGS

DEF: A set of characters is called an **alphabet.**

Example 2.4.3:   Some common alphabets:

- $\{0, 1\}$ the binary alphabet

- $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ the decimal digits

- $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$
    the hexadecimal digits

- $\{A, B, C, D, \ldots, X, Y, Z\}$ English uppercase

- ASCII

DEF: A **string** is a sequence in an alphabet.

NOTATION: Usually a string is written without commas, so that consecutive characters are juxtaposed.

Example 2.4.4:   If

$$f(0) = M, \; f(1) = A, \; f(2) = T, \;\; \text{and} \;\; f(3) = H$$

then write

$$``MATH"$$

SPECIFYING a RULE

Problem: Given some initial terms

$$a_0, a_1, ..., a_k$$

of a sequence, try to construct a rule that is consistent with those initial terms.

Approaches: There are two standard kinds of rule for calculating a generic term a_n.

DEF: A **recursion** for a_n is a function whose arguments are earlier terms in the sequence.

DEF: A **closed form** for a_n is a formula whose argument is the subscript n.

Example 2.4.5: $1, 3, 5, 7, 9, 11, \ldots$

recursion: $a_0 = 1;$ $a_n = a_{n-1} + 2$ for $n \geq 1$

closed form: $a_n = 2n + 1$

The differences between consecutive terms often suggest a recursion. Finding a recursion is usually easier than finding a closed formula.

Example 2.4.6: $1, 3, 7, 13, 21, 31, 43, \ldots$

recursion: $b_0 = 1;$ $b_n = b_{n-1} + 2n$ for $n \geq 1$

closed form: $b_n = n^2 + n + 1$

Constructing a closed formula is sometimes much harder than constructing a recursion.

Example 2.4.7: $1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$

recursion: $c_0 = 1, c_1 = 1$;
$\quad c_n = c_{n-1} + c_{n-2}$ for $n \geq 1$

closed form: $c_n = \dfrac{1}{\sqrt{5}} \left[G^{n+1} - g^{n+1} \right]$

$\quad$ where $G = \dfrac{1 + \sqrt{5}}{2}$ and $g = \dfrac{1 - \sqrt{5}}{2}$

INFERRING a RULE

The ESSENCE of science is inferring rules from partial data.

Example 2.4.8:   Sit under apple tree.
Infer gravity.

Example 2.4.9:   Watch starlight move 0.15 arcseconds in total eclipse. Infer relativity.

Example 2.4.10:   Observe biological species.
Infer DNA.

Important life skill: Given a difficult general problem, start with special cases you can solve.

Example 2.4.11: Find a recursion and a closed form for the arithmetic progression:

$$c, c + d, c + 2d, c + 3d, \ldots$$

recursion: $a_0 = c; \quad a_n = a_{n-1} + d$

closed form: $a_n = c + nd$.

Q: How would you decide that a given sequence is an arithmetic progression?

A: Calculate differences betw consec terms.

DEF: The **difference sequence** for a sequence a_n is the sequence $a_n' = a_n - a_{n-1}$ for $n \geq 1$.

Example 2.4.5 redux:
$$
\begin{array}{lcccccc}
a_n : & 1 & 3 & 5 & 7 & 9 & 11 \\
a_n' : & 2 & 2 & 2 & 2 & 2 &
\end{array}
$$

Analysis: Since a_n' is constant, the sequence is specified by this recursion:

$a_0 = 1; \quad a_n = a_{n-1} + 2$ for $n \geq 1$.

Moreover, it has this closed form:

$$a_n = a_0 + a_1' + a_2' + \cdots + a_n'$$
$$= a_0 + 2 + 2 + \cdots + 2 \quad = 1 + 2n$$

If you don't get a constant sequence on the first difference, then try reiterating.

Revisit Example 2.4.6: $1, 3, 7, 13, 21, 31, 43, \ldots$

$$
\begin{array}{llllllll}
b_n : & 1 & 3 & 7 & 13 & 21 & 31 & 43 \\
b_n' : & 2 & 4 & 6 & 8 & 10 & 12 \\
b_n'' : & 2 & 2 & 2 & 2 & 2
\end{array}
$$

Analysis: Since b_n'' is constant, we have

$$
b_n' = 2 + 2n
$$

Therefore,

$$
b_n = b_0 + b_1' + b_2' + \cdots + b_n'
$$

$$
= b_0 + 2 \sum_{j=1}^{n} j = 1 + (n^2 + n) = n^2 + n + 1
$$

Consolation Prize: Without knowing about finite sums, you can still extend the sequence:

$$
\begin{array}{lllllllll}
b_n : & 1 & 3 & 7 & 13 & 21 & 31 & 43 & \underline{57} \\
b_n' : & 2 & 4 & 6 & 8 & 10 & 12 & \underline{14} \\
b_n'' : & 2 & 2 & 2 & 2 & 2 & \underline{2}
\end{array}
$$

SUMMATIONS

DEF: Let a_n be a sequence. Then the **big-sigma** notation

$$\sum_{j=m}^{n} a_j$$

means the sum

$$a_m + a_{m+1} + a_{m+2} + \cdots + a_{n-1} + a_n$$

TERMINOLOGY: j is the **index of summation**

TERMINOLOGY: m is the **lower limit**

TERMINOLOGY: n is the **upper limit**

TERMINOLOGY: a_j is the **summand**

Theorem 2.4.1. *These formulas for summing falling powers are provable by induction (see §5.1):*

$$\sum_{j=1}^{n} j^{\underline{1}} = \frac{1}{2}(n+1)^{\underline{2}} \qquad \sum_{j=1}^{n} j^{\underline{2}} = \frac{1}{3}(n+1)^{\underline{3}}$$

$$\sum_{j=1}^{n} j^{\underline{3}} = \frac{1}{4}(n+1)^{\underline{4}} \qquad \sum_{j=1}^{n} j^{\underline{k}} = \frac{1}{k+1}(n+1)^{\underline{k+1}}$$

Example 2.4.12:   True Love and Thm 2.4.1

On the jth day ... True Love gave me

$$j + (j - 1) + \cdots + 1 \;=\; \frac{(j+1)^2}{2} \text{ gifts.}$$

$$= \frac{1}{2}\sum_{j=2}^{13} j^2 \;=\; \frac{1}{2}\left[2^2 + \cdots + 13^2\right]$$

$$= \frac{1}{2}\left[2 + 6 + \cdots + 78\right] \;=\; 364 \quad \text{slow}$$

$$= \frac{1}{2}\cdot\frac{14^3}{3} \;=\; 364 \quad \text{fast}$$

Cor 2.4.2. *High-powered look-ahead to formulas for summing* $j^k : j = 0, 1, ..., n.$

$$\sum_{j=1}^{n} j^2 \;=\; \sum_{j=1}^{n}(j^2 + j^1) \;=\; \frac{1}{3}(n+1)^3 + \frac{1}{2}(n+1)^2$$

$$\sum_{j=1}^{n} j^3 \;=\; \sum_{j=1}^{n}(j^3 + 3j^2 + j^1) \;=\; \cdots$$

2.5   CARDINALITY OF SETS

POTLATCH RULES for CARDINALITY

DEF: ***nondominating cardinality:***
Let A and B be sets. Then $|A| \leq |B|$ means that $\exists$ one-to-one function $f : A \to B$.

DEF: Set A and B have **equal cardinality**, and we write $|A| = |B|$, if $\exists$ bijection $f : A \to B$, which obviously implies that $|A| \leq |B|$ and $|B| \leq |A|$.

DEF: ***strictly dominating cardinality:***
Let A and B be sets. Then $|A| < |B|$ means that $|A| \leq |B|$ and $|A| \neq |B|$.

DEF: The **cardinality** of a set A is

$$|A| \;=\; \begin{cases} n & \text{if } |A| = |\{1, 2, \ldots, n\}| \\ 0 & \text{if } A = \emptyset \end{cases}$$

Such cardinalities are called **finite**.

DEF: The **cardinality** of $\mathbb{N}$ is ω ("omega"), or alternatively, $\aleph_0$ ("aleph null").
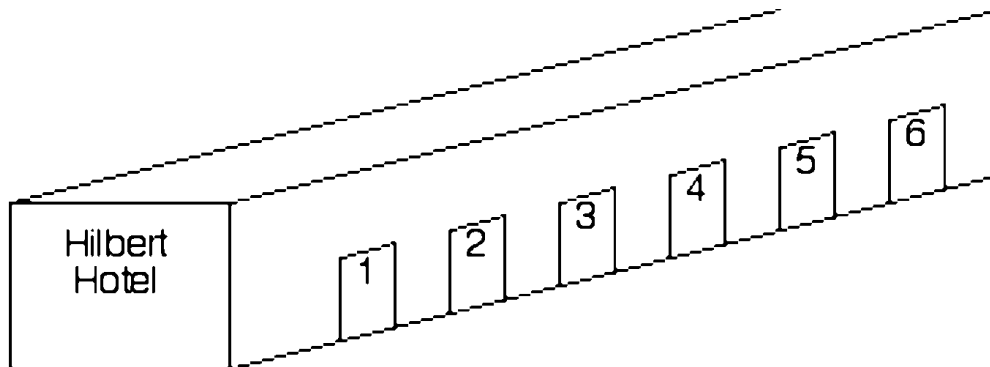
DEF: A set is **countable** if it is finite or ω.

Remark: $\aleph_0$ is the smallest infinite cardinality. The set $\mathbb{R}$ has cardinality $\aleph_1$ ("aleph one"), which is larger than $\aleph_0$, for reasons to be given.

INFINITE CARDINALITIES

Example 2.5.1:   There is always a room at the HILBERT HOTEL.

Proposition 2.5.1. *There are as many even non-negative numbers as non-negative numbers.*

Pf:　$f(2n) = n$ is a bijection.　　　　　　　　　　$\Diamond$

Theorem 2.5.2. *There are as many positive integers as rational fractions.*

$$
\begin{array}{cccccc}
\dfrac{1}{1} & \dfrac{1}{2} & \dfrac{1}{3} & \dfrac{1}{4} & \dfrac{1}{5} & \dfrac{1}{6} & \cdots \\[2mm]
\dfrac{2}{1} & \dfrac{2}{2} & \dfrac{2}{3} & \dfrac{2}{4} & \dfrac{2}{5} & \dfrac{2}{6} & \cdots \\[2mm]
\dfrac{3}{1} & \dfrac{3}{2} & \dfrac{3}{3} & \dfrac{3}{4} & \dfrac{3}{5} & \dfrac{3}{6} & \cdots \\[2mm]
\dfrac{4}{1} & \dfrac{4}{2} & \dfrac{4}{3} & \dfrac{4}{4} & \dfrac{4}{5} & \dfrac{4}{6} & \cdots \\[2mm]
\dfrac{5}{1} & \dfrac{5}{2} & \dfrac{5}{3} & \dfrac{5}{4} & \dfrac{5}{5} & \dfrac{5}{6} & \cdots \\[2mm]
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}
$$

Pf:　$f\left(\dfrac{p}{q}\right) = \dfrac{(p+q-1)(p+q-2)}{2} + p$　　　$\Diamond$

Example 2.5.2:　$f\left(\dfrac{2}{3}\right) = \dfrac{4 \cdot 3}{2} + 2 = 8$

Thm 2.5.3. *(G. Cantor) There are more positive real numbers than positive integers.*

Semi-proof: A putative bijection $[0,1] \to \mathbb{Z}^+$ would induce a sequence x_j such that $\{x_j\} = [0,1]$. Suppose we write each real number as an infinite decimal fraction. (We exclude any decimal expansion that ends in all 9's, since it has an equivalent form that ends in all 0's.)

$$x_1 = .\underline{8}841752032669031\ldots \mapsto 1$$
$$x_2 = .1\underline{4}15926531424450\ldots \mapsto 2$$
$$x_3 = .32\underline{0}2313932614203\ldots \mapsto 3$$
$$x_4 = .167\underline{9}888138381728\ldots \mapsto 4$$
$$x_5 = .0452\underline{9}98136712310\ldots \mapsto 5$$
$$\vdots$$

The jth digit of x_j is underscored. Consider the number $.45444\ldots$

whose j^{th} decimal digit is 4 if the jth digit of x_j is not 4, or 5 if it is 4, which implies that

$$.45444\ldots \not\mapsto j \qquad (\forall j \in \mathbb{Z}+)$$

Accordingly, this putative bijection is not even a function, since it fails to assign an element of the codomain $\mathbb{Z}^+$ to some number of its domain. $\diamondsuit$