Cryptography

Lecture 5

Announcements

- NEW: HW2 due Wednesday 2/15
- Canvas quizzes due on 2/10 at 11:59pm

Agenda

- Last time:
  - Limitations of Perfect Secrecy (K/L 2.3)
  - Shannon's Theorem (K/L 2.4)
  - The Computational Approach (K/L 3.1)
- This time:
  - The Computational Approach (K/L 3.1)
  - Defining computationally secure SKE (K/L 3.2)

The Computational Approach

Two main relaxations:

1. Security is only guaranteed against efficient adversaries that run for some feasible amount of time.
2. Adversaries can potentially succeed with some very small probability.

poly-time

negl.

Security Parameter

- Integer valued security parameter denoted by n that parameterizes both the cryptographic schemes as well as all involved parties.
- When honest parties initialize a scheme, they choose some value n for the security parameter.

n=128

- Can think of security parameter as corresponding to the length of the key.
- Security parameter is assumed to be known to any adversary attacking the scheme.
- View run time of the adversary and its success probability as functions of the security parameter.

Polynomial Time

all about the quantifiers

- Efficient adversaries = Polynomial time adversaries

  ∀ poly-time adv

  - There is some polynomial p such that the adversary runs for time at most $p(n)$ when the security parameter is n.
  - Honest parties also run in polynomial time.

    ∃ a polynomial n^2

  - The adversary may be much more powerful than the honest parties.

Negligible

$\log(n^{\log n})$ super poly.

$\log n \cdot \log n > c \cdot \log n$

- Small probability of success = negligible probability
  - A function f is negligible if for every polynomial p and all sufficiently large values of n it holds that $f(n) < \frac{1}{p(n)}$.
  - Intuition, $f(n) < n^{-c}$ for every constant c, as n goes to infinity.

$f(n) := \frac{1}{n^2}$ non-negl.

$\frac{1}{2^n}$ negl

$\frac{1}{n^{\log n}}$ negl.

Practical Implications of Computational Security

- For key size n, any adversary running in time $2^{n/2}$ breaks the scheme with probability $1/2^{n/2}$.
- Meanwhile, Gen, Enc, Dec each take time n^2.
- If $n = 128$ then:
  - Gen, Enc, Dec take time 16,384
  - Adversarial run time is $2^{64} \approx 10^{18}$
- If n = 256 then:
  - Gen, Enc, Dec quadruples--takes time 65,536
  - Adversary run time is multiplied by 2^{64}. Becomes $2^{128} \approx 10^{38}$

Defining Computationally Secure Encryption

Syntax

A private-key encryption scheme is a tuple of probabilistic polynomial-time algorithms (Gen, Enc, Dec) such that:

Gen(n)

1. The key-generation algorithm Gen takes as input security parameter 1^n and outputs a key k denoted $k \leftarrow Gen(1^n)$. We assume WLOG that $|k| \geq n$.
2. The encryption algorithm Enc takes as input a key k and a message $m \in \{0,1\}^*$, and outputs a ciphertext c denoted $c \leftarrow Enc_k(m)$. Enc can be prob.
3. The decryption algorithm Dec takes as input a key k and ciphertext c and outputs a message m denoted by $m \coloneqq Dec_k(c)$. always deterministic.

Correctness: For every n, every key $k \leftarrow Gen(1^n)$, and every $m \in \{0,1\}^*$, it holds that $Dec_k\big(Enc_k(m)\big) = m$.

same.

Indistinguishability in the presence of an eavesdropper

Consider a private-key encryption scheme $\Pi = (Gen, Enc, Dec)$, any adversary A, and any value n for the security parameter.

Experiment $PrivK^{eav}_{A,\Pi}(n)$

Adversary $A(1^n)$

Challenger

the honest parties

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m_0, m_1 →

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m_0, m_1 →

$k \leftarrow Gen(1^n)$

$b \leftarrow \{0,1\}$

$c \leftarrow Enc_k(m_b)$

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← c

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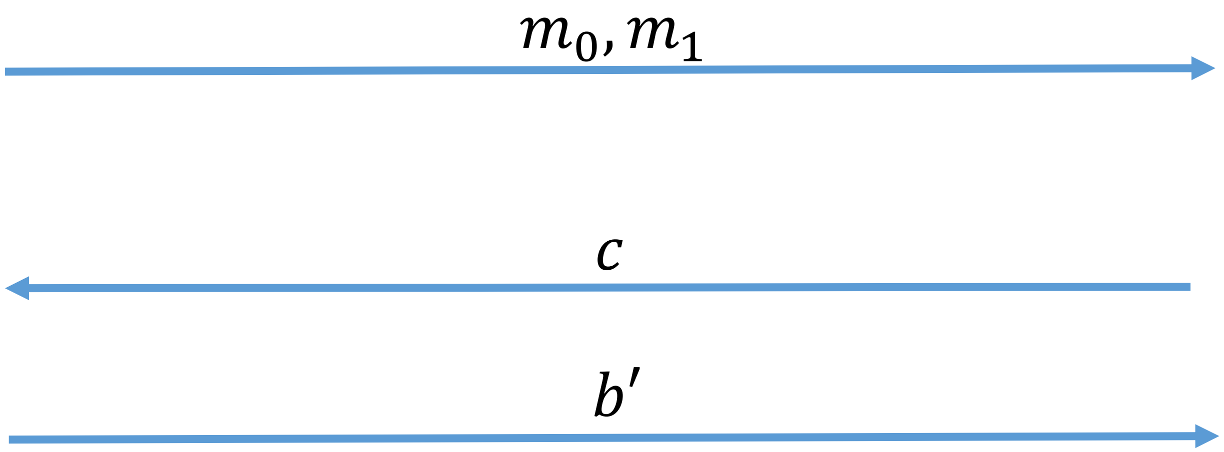

$k \leftarrow Gen(1^n)$

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Indistinguishability in the presence of an eavesdropper

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Experiment $PrivK^{eav}_{A,\Pi}(n)$

Adversary $A(1^n)$ | Challenger

Adversary → Challenger: m_0, m_1

Challenger:
$k \leftarrow Gen(1^n)$
$b \leftarrow \{0,1\}$
$c \leftarrow Enc_k(m_b)$

Challenger → Adversary: c

Adversary → Challenger: b'

$PrivK^{eav}_{A,\Pi}(n) = 1$ if $b' = b$ and $PrivK^{eav}_{A,\Pi}(n) = 0$ if $b' \neq b$.

perfect sec.

any adv guessing right w/ pr exactly 1/2

Reminder $\Pr[C=c \mid M=m_0] = \Pr[C=c \mid M=m_1]$.

Indistinguishability in the presence of an eavesdropper

Consider a private-key encryption scheme $\Pi = (Gen, Enc, Dec)$, any adversary A, and any value n for the security parameter.

The eavesdropping indistinguishability experiment $PrivK^{eav}{}_{A,\Pi}(n)$:

1. The adversary A is given input 1^n, and outputs a pair of messages m_0, m_1 of the same length.
2. A key k is generated by running $Gen(1^n)$, and a random bit $b \leftarrow \{0,1\}$ is chosen. A challenge ciphertext $c \leftarrow Enc_k(m_b)$ is computed and given to A.
3. Adversary A outputs a bit b'.
4. The output of the experiment is defined to be 1 if $b' = b$, and 0 otherwise. If $PrivK^{eav}{}_{A,\Pi}(n) = 1$, we say that A succeeded.

Indistinguishability in the presence of an eavesdropper

Definition: A private key encryption scheme $\Pi = (Gen, Enc, Dec)$ has indistinguishable encryptions in the presence of an eavesdropper if for all probabilistic polynomial-time adversaries A there exists a negligible function $negl$ such that

$$\Pr\left[PrivK^{eav}{}_{A,\Pi}(n) = 1\right] \leq \frac{1}{2} + negl(n),$$

Where the prob. Is taken over the random coins used by A, as well as the random coins used in the experiment.

Coming up with the right definition

Third Attempt:

"An encryption scheme is secure if no adversary learns meaningful information about the plaintext after seeing the ciphertext"

How do you formalize learns meaningful information?

Semantic Security

Definition: A private key encryption scheme $\Pi = (Gen, Enc, Dec)$ is semantically secure in the presence of an eavesdropper if for every ppt adversary A there exists a ppt algorithm A' such that for all efficiently sampleable distributions $X = (X_1, \dots,)$ and all poly time computable functions f, h, there exists a negligible function $negl$ such that

$$|\Pr[A(1^n, Enc_k(m), h(m)) = f(m)] \\ - \Pr[A'(1^n, h(m)) = f(m)]| \leq negl(n),$$

where m is chosen according to distribution X_n, and the probabilities are taken over choice of m and the key k, and any random coins used by A, A', and the encryption process.

Semantic Security

- The full definition of semantic security is even more general.
- Consider arbitrary distributions over plaintext messages and arbitrary external information about the plaintext.

Equivalence of Definitions

Theorem: A private-key encryption scheme has indistinguishable encryptions in the presence of an eavesdropper if and only if it is semantically secure in the presence of an eavesdropper.

For us: we can work w/ the game based definition

Pseudorandom Generator

- Functionality
  - Deterministic algorithm G
  - Takes as input a short random seed s
  - Ouputs a long string $G(s)$
- Security
  - No efficient algorithm can "distinguish" $G(s)$ from a truly random string r.
  - i.e. passes all "statistical tests."
- Intuition:
  - Stretches a small amount of true randomness to a larger amount of pseudorandomness.
- Why is this useful?
  - We will see that pseudorandom generators will allow us to beat the Shannon bound of $|\boldsymbol{K}| \geq |\boldsymbol{M}|$.
  - I.e. we will build a computationally secure encryption scheme with $|\boldsymbol{K}| < |\boldsymbol{M}|$

Pseudorandom Generators

Definition: Let $\ell(\cdot)$ be a polynomial and let G be a deterministic poly-time algorithm such that for any input $s \in \{0,1\}^n$, algorithm G outputs a string of length $\ell(n)$. We say that G is a pseudorandom generator if the following two conditions hold:

1. (Expansion:) For every n it holds that $\ell(n) > n$.
2. (Pseudorandomness:) For all ppt distinguishers D, there exists a negligible function $negl$ such that:

$$\left|\Pr[D(r) = 1] - \Pr\left[D\left(G(s)\right) = 1\right]\right| \leq negl(n),$$

where r is chosen uniformly at random from $\{0,1\}^{\ell(n)}$, the seed s is chosen uniformly at random from $\{0,1\}^n$, and the probabilities are taken over the random coins used by D and the choice of r and s.

The function $\ell(\cdot)$ is called the expansion factor of G.