

An Introduction to Lattice-Based Cryptography II

Dana Dachman-Soled
University of Maryland
danadach@umd.edu

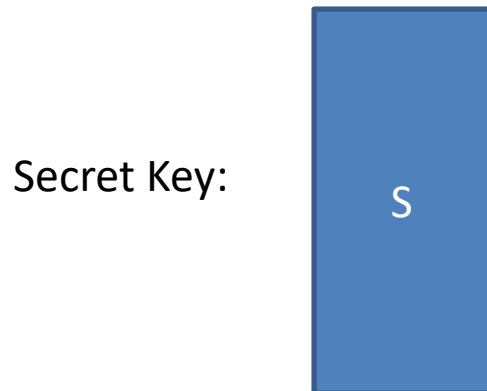
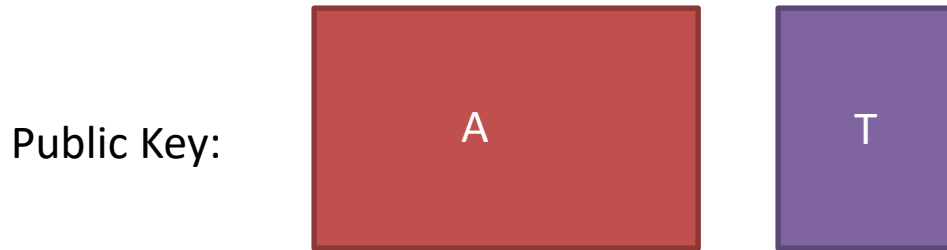
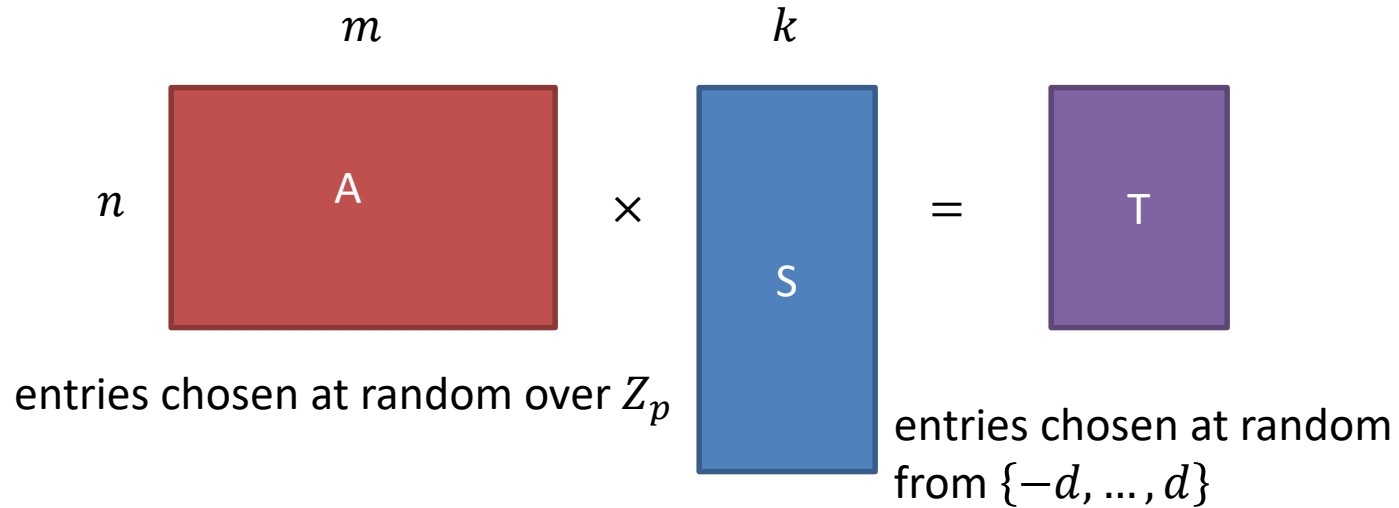
Rejection Sampling

- Problem: Sample from a distribution D_f with probability density function $f(x)$ given draws from a distribution D_g with probability density function $g(x)$.
- Assuming $\forall x, f(x) \leq M \cdot g(x)$:
 - Sample from $x \leftarrow D_g$
 - Accept x with probability $\frac{f(x)}{M \cdot g(x)}$.
- If condition holds then $\forall x, \frac{f(x)}{M} \cdot g(x) \leq 1$
- Probability of outputting x is $\Pr[\text{sampling } x]$.
$$\Pr[\text{sample is accepted}] = g(x) \cdot \frac{f(x)}{M \cdot g(x)} = \frac{f(x)}{M}.$$
- Normalizing, we get the correct probability distribution
- Expected number of draws from $g(x)$ before a sample is accepted is M .

Lattice-Based Signatures

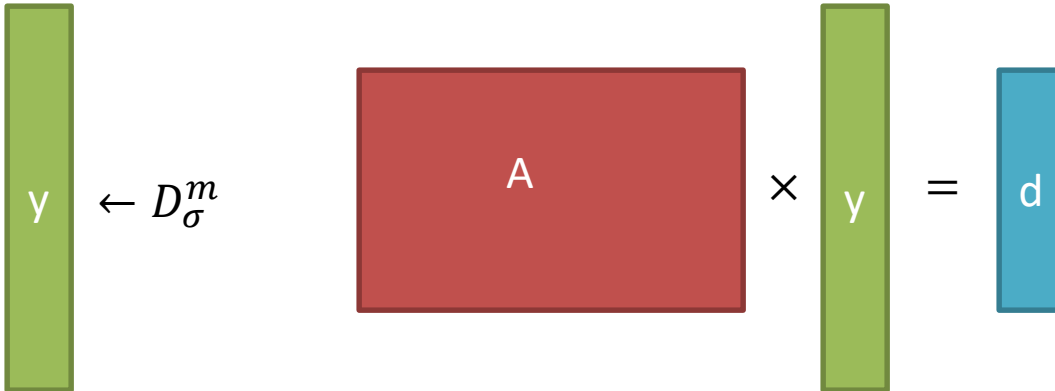
Lyubashevsky 2011

Key Generation



Sign—Attempt 1

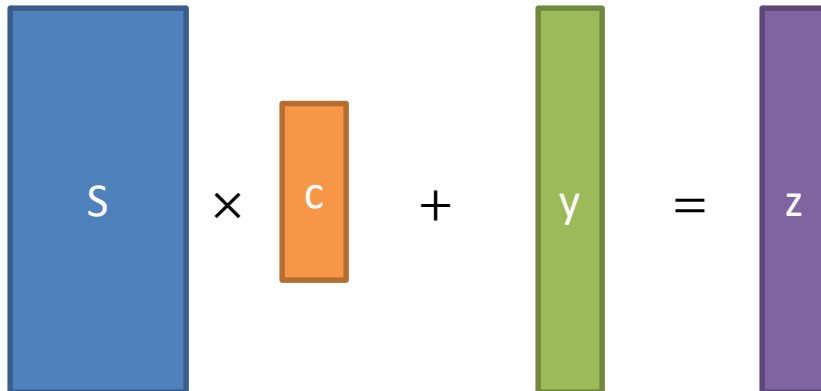
(1) $y \leftarrow D_{\sigma}^m$ $A \times y = d$



(2) $c = H(d||m)$ c Output (c,z)



(3) $S \times c + y = z$



Verify

Given public key (A, T) , message m and signature (\tilde{c}, \tilde{z}) :

$$A \times \tilde{z} - T \times \tilde{c} = \tilde{d}$$

Check that $\tilde{c} = H(\tilde{d}||m)$ and \tilde{z} is short.

Security

- If adversary has not seen any signatures, can show (using RO methodology) that it is possible to extract the following from a forging adversary:
 - z_1 s.t. $Az_1 - Tc_1 = Ay$
 - z_2 s.t. $Az_2 - Tc_2 = Ay$
 - Subtracting and recalling that $T = AS$ we obtain:
$$A(z_1 - z_2) - T(c_1 - c_2) = 0$$
$$A(z_1 - z_2) - A(S(c_1 - c_2)) = 0$$
- Finding such z_1, z_2 was shown to be as hard as SIS.
- But what if adversary gets to see signatures? Is this still hard?

Sign

(1) $y \leftarrow D_{\sigma}^m$ \times A $=$ d

(2) $c = H(d||m)$ c

(3) S \times c $+$ y $=$ z

Output (c, z) with probability
 $\frac{D_{\sigma}^m(z)}{M \cdot D_{\sigma, Sc}^m(z)}$
Rejection sampling step