

Cryptography ENEE/CMSC/MATH 456: Homework 7

Due by 2pm on 4/27/2020.

1. Compute $3^{1000} \bmod 100$ by hand.
2. Compute $[101^{4,800,000,023} \bmod 35]$ by hand.
3. Let $N = pq$ be a product of two distinct primes. Show that if $\phi(N)$ and N are known, then it is possible to compute p and q in polynomial time.

Hint: Derive a quadratic equation (over the integers) in the unknown p .

4. Let $N = pq$ be a product of two distinct primes. Show that if N and an integer $d \leq \phi(N)$ such that $3 \cdot d \equiv 1 \pmod{\phi(N)}$ are known, then it is possible to compute p and q in polynomial time.

Hint: Obtain a small list of possibilities for $\phi(N)$ and then use the previous exercise.

5. Fix N, e with $\gcd(e, \phi(N)) = 1$, and assume there is an adversary A running in time t for which

$$\Pr[A(x^e \bmod N) = x] = 0.01,$$

where the probability is taken over uniform choice of $x \in Z_N^*$. Show that it is possible to construct an adversary A' for which

$$\Pr[A'(x^e \bmod N) = x] = 0.99$$

for *all* x . The running time t' of A' should be polynomial in t and $\|N\|$ (the number of *bits* it takes to write down N).

Hint: Use the fact that $y^{1/e} \cdot r = (y \cdot r^e)^{1/e}$.