Cryptography—ENEE/CMSC/MATH 456
Class Exercise 2/4/19

1. Prove or refute: An encryption scheme with message space $\boldsymbol{M}$ is perfectly secret if and only if for every probability distribution over $\boldsymbol{M}$ and every $c_0, c_1 \in \boldsymbol{C}$ we have $Pr[C = c_0] = Pr[C = c_1]$. False.

Given encryption scheme (Gen, Enc, Dec), construct scheme (Gen, Enc', Dec'). This is exactly the same except Enc appends a 0 to its output with prob. 1/4 and a 1 with prob 3/4.
Dec' ignores the final bit.
Note that if (Gen, Enc, Dec) is perfectly secret, so is (Gen, Enc', Dec').
But now choose any $c \in \mathcal{C}$ (where $\mathcal{C}$ is ciphertext space of (Gen, Enc, Dec)).
Then we have $Pr[C = c\|0] < Pr[C = c\|1]$.

2. Prove or refute: An encryption scheme with message space $\boldsymbol{M}$ is perfectly secret if and only if for every probability distribution over $\boldsymbol{M}$, every $m, m' \in \boldsymbol{M}$ and every $c \in \boldsymbol{C}$ we have $Pr[M = m \mid C = c] = Pr[M = m' | C = c]$. False.

Given any perfectly secret encryption scheme, we will choose a distribution over $\mathcal{M}$ and $m, m', c$ s.t. $Pr[M=m \mid C=c] \neq Pr[M=m' \mid C=c]$. This refutes the above.

Let's choose a distribution over $\mathcal{M}$ that sets $Pr[M=m] > Pr[M=m']$ for some $m, m'$.
Now by Def 1 of perfect secrecy, $\forall c$
$Pr[M=m \mid C=c] = Pr[M=m]$ and $Pr[M=m' \mid C=c] = Pr[M=m']$
So $Pr[M=m \mid C=c] = Pr[M=m] > Pr[M=m'] = Pr[M=m' \mid C=c]$.
So $Pr[M=m \mid C=c] \neq Pr[M=m' \mid C=c]$.