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9.6.1 INTRODUCTION

This design example demonstrates the design of a three-span (110-120-110 ft) AASHTO-PCI bulb-tee beam bridge with no skew, as shown in Figure 9.6.1-1. This example illustrates in detail the design of a typical interior beam in the center span at the critical sections in positive flexure, negative flexure, shear, and deflection due to prestress, dead loads and live load. The superstructure consists of four beams spaced at 12'-0" centers as shown in Figure 9.6.1-2. Beams are designed to act compositely with the 8-in.-thick cast-in-place concrete deck slab to resist all superimposed dead loads, live loads and impact. A 1/2 in. wearing surface is considered to be an integral part of the 8-in. deck. Design live load is AASHTO LRFD HL-93. The design will be carried out in accordance with the AASHTO LRFD Bridge Design Specifications, 2nd Edition, 1998, and including through the 2003 Interim Revisions.
9.6.2 MATERIALS

Cast-in-place slab: Actual thickness, \( t_s = 8.0 \) in.
Structural thickness = 7.5 in.
Note that a 1/2 in. wearing surface is considered to be an integral part of the 8-in. deck.
Concrete strength at 28 days, \( f'_{c} = 4.0 \) ksi
Concrete unit weight, \( w_c = 0.150 \) kcf

Precast beams: AASHTO-PCI, BT-72 bulb-tee beam shown in Figure 9.6.2-1.
Concrete strength at transfer, \( f'_{ci} = 5.5 \) ksi
Concrete strength at 28 days, \( f'_{c} = 7.0 \) ksi
Concrete unit weight, \( w_c = 0.150 \) kcf

Overall beam length (Figure 9.6.1-1) = 110.0 ft (end spans) and 119.0 ft (center span)
Design spans (Figure 9.6.1-1):
For non-composite beam: 109.0 ft (end spans) and 118.0 ft (center span)
For composite beam: 110.0 ft (end spans) and 120.0 ft (center span)
Prestressing strands: 1/2 in. diameter, low-relaxation

Area of one strand = 0.153 in.²

Ultimate strength, \( f_{pu} = 270.0 \) ksi

Yield strength, \( f_{py} = 0.9f_{pu} = 243.0 \) ksi \[\text{[LRFD Table 5.4.4.1-1]}\]

Stress limits for prestressing strands:

- before transfer, \( f_{pi} \leq 0.75f_{pu} = 202.5 \) ksi
- at service limit state (after all losses), \( f_{pe} \leq 0.80f_{py} = 194.4 \) ksi \[\text{[LRFD Table 5.9.3-1]}\]

Modulus of elasticity, \( E_p = 28,500 \) ksi \[\text{[LRFD Art. 5.4.4.2]}\]

Reinforcing bars:

- Yield strength, \( f_y = 60 \) ksi
- Modulus of elasticity, \( E_s = 29,000 \) ksi \[\text{[LRFD Art. 5.4.3.2]}\]

Future wearing surface: additional 2 in. with unit weight equal to 0.150 kcf

New Jersey-type barrier: Unit weight = 0.300 kip/ft/side

### 9.6.3 CROSS-SECTION PROPERTIES FOR A TYPICAL INTERIOR BEAM

#### 9.6.3.1 Non-Composite Section

- \( A = \text{area of cross-section of beam} = 767 \text{ in.}^2 \)
- \( h = \text{overall depth of beam} = 72 \text{ in.} \)
- \( I = \text{moment of inertia about the centroid of the non-composite precast beam} = 545,894 \text{ in.}^4 \)
- \( y_b = \text{distance from centroid to extreme bottom fiber of the non-composite precast beam} = 36.60 \text{ in.} \)
- \( y_t = \text{distance from centroid to extreme top fiber of the non-composite precast beam} = 35.40 \text{ in.} \)
- \( S_b = \text{section modulus for the extreme bottom fiber of the non-composite precast beam} = I/y_b = 14,915 \text{ in.}^3 \)
- \( S_t = \text{section modulus for the extreme top fiber of the non-composite precast beam} = I/y_t = 15,421 \text{ in.}^3 \)
- \( W_t = 0.799 \text{ kip/ft} \)
- \( E_c = 33,000(W_c)^{1.5}\sqrt{f_c'} \) \[\text{[LRFD Eq. 5.4.2.4-1]}\]

where

- \( E_c = \text{modulus of elasticity of concrete, ksi} \)
- \( w_c = \text{unit weight of concrete} = 0.150 \text{ kcf} \)

The *LRFD Specifications*, commentary C5.4.2.4, indicates that the unit weight of normal weight concrete is 0.145 kcf. However, precast concrete mixes typically have a relatively low water/cementitious materials ratio and high density. Therefore, a unit weight of 0.150 kcf is used in this example. For high strength concrete, this value may need to be increased further based on test results.

- \( f_c' = \text{specified strength of concrete, ksi} \)

Therefore, the modulus of elasticity for the cast-in-place concrete deck is:

\[
E_c = 33,000(0.150)^{1.5}\sqrt{4.0} = 3,834 \text{ ksi}
\]
for the precast beam at transfer, \( E_{ci} = 33,000(0.150)^{1.5} \sqrt{5.5} = 4,496 \text{ ksi} \)

for the precast beam at service loads, \( E_c = 33,000(0.150)^{1.5} \sqrt{7.0} = 5,072 \text{ ksi} \)

The effective flange width is the lesser of:

\[
(1/4) \text{ span length: } (120 \times 12/4) = 360 \text{ in.}
\]

12t, plus greater of web thickness or 1/2 beam top flange width
\[
= (12 \times 7.5 + 0.5 \times 42) = 111 \text{ in.}; \text{ or,}
\]

average spacing between beams = (12 x 12) = 144 in.

Therefore, the effective flange width is 111 in.

Modular ratio between slab and beam concrete, \( n = \frac{E_c(\text{slab})}{E_c(\text{beam})} = \frac{3,834}{5,072} = 0.7559 \)

Transformed flange width = \( n \) (effective flange width) = \( 0.7559 \times 111 = 83.91 \text{ in.} \)

Transformed flange area = \( n \) (effective flange width)(t_s) = \( 0.7559 \times 111 \times 7.5 = 629.29 \text{ in.}^2 \)

Note that only the structural thickness of the deck, 7.5 in., is considered.

Due to camber of the precast, prestressed beam, a minimum haunch thickness of 1/2 in., at midspan, is considered in the structural properties of the composite section. Also, the width of haunch must be transformed.

Transformed haunch width = \( 0.7559 \times 42 = 31.75 \text{ in.} \)

Transformed area of haunch = \( 0.7559 \times 42 \times 0.5 = 15.87 \text{ in.}^2 \)

Figure 9.6.3.2.3-1 shows the dimensions of the composite section.
Note that the haunch should only be considered to contribute to section properties if it is required to be provided in the completed structure. Some designers neglect its contribution to the section properties.

\[ \text{Ac} = \text{total area of composite section} = 1,412 \text{ in.}^2 \]

\[ \text{hc} = \text{overall depth of the composite section} = 80 \text{ in.} \]

\[ \text{Ic} = \text{moment of inertia of the composite section} = 1,097,252 \text{ in.}^4 \]

\[ y_{bc} = \text{distance from the centroid of the composite section to the extreme bottom fiber of the precast beam} = \frac{77,202}{1,412} = 54.67 \text{ in.} \]

\[ y_{tg} = \text{distance from the centroid of the composite section to the extreme top fiber of the precast beam} = 72 - 54.67 = 17.33 \text{ in.} \]

\[ y_{tc} = \text{distance from the centroid of the composite section to the extreme top fiber of the slab} = 80 - 54.67 = 25.33 \text{ in.} \]

\[ S_{bc} = \text{composite section modulus for the extreme bottom fiber of the precast beam} = \frac{1,097,252}{54.67} = 20,070 \text{ in.}^3 \]

\[ S_{tg} = \text{composite section modulus for the top fiber of the precast beam} = \frac{1,097,252}{17.33} = 63,315 \text{ in.}^3 \]

\[ S_{tc} = \text{composite section modulus for extreme top fiber of the deck slab} = \left( \frac{1}{0.7559} \right) \left( \frac{1,097,252}{25.33} \right) = 57,307 \text{ in.}^3 \]

The self-weight of the beam and the weight of the slab and haunch act on the non-composite, simple-span structure, while the weight of barriers, future wearing surface, and live loads with impact act on the composite, continuous structure. Refer to Table 9.6.4-1 which follows for a summary of unfactored values, calculated below:

<table>
<thead>
<tr>
<th></th>
<th>Transformed Area, in.²</th>
<th>( y_b ) in.</th>
<th>( A_{y_b} ) in.³</th>
<th>( A(y_{bc} - y_b)^2 ) in.⁴</th>
<th>( I ) in.⁴</th>
<th>( I + A(y_{bc} - y_b)^2 ) in.⁴</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>767.00</td>
<td>36.60</td>
<td>28,072.20</td>
<td>250,444.60</td>
<td>545,894.00</td>
<td>796,338</td>
</tr>
<tr>
<td>Haunch</td>
<td>15.87</td>
<td>72.25</td>
<td>1,146.61</td>
<td>4,904.73</td>
<td>0.33</td>
<td>4,905</td>
</tr>
<tr>
<td>Deck</td>
<td>629.29</td>
<td>76.25</td>
<td>47,983.36</td>
<td>293,058.09</td>
<td>2,949.61</td>
<td>296,007</td>
</tr>
<tr>
<td>( \sum )</td>
<td>1,412.16</td>
<td>77,202.17</td>
<td></td>
<td></td>
<td></td>
<td>1,097,251</td>
</tr>
</tbody>
</table>

**9.6.4 SHEAR FORCES AND BENDING MOMENTS**

**9.6.4.1 Shear Forces and Bending Moments Due to Dead Loads**

**9.6.4.1.1 Dead Loads**

\[ DC = \text{Dead load of structural components and non-structural attachments} \]

Dead loads acting on the simple-span structure, non-composite section:

- Beam self-weight = 0.799 kip/ft
8-in. deck weight = (8/12 ft)(12 ft)(0.150 kcf) = 1.200 kip/ft
1/2 in. haunch weight = (0.5)(42/144)(0.150) = 0.022 kip/ft

Notes:
1. Actual slab thickness (8 in.) is used for computing dead load.
2. A 1/2 in. minimum haunch thickness is assumed in the computations of dead load. If a deeper haunch will be used because of final beam camber, the weight of the actual haunch should be used.
3. The weight of cross-diaphragms is ignored since most agencies are moving away from cast-in-place concrete diaphragms to lightweight steel diaphragms.

Dead loads placed on the continuous structure, composite section:
LRFD Article 4.6.2.2.1 states that permanent loads (curbs and future wearing surface) may be distributed uniformly among all beams if the following conditions are met:
• Width of the deck is constant O.K.
• Number of beams, N_b, is not less than four (N_b = 4) O.K.
• Roadway part of the overhang, d_e = 3.0 ft

\[ d_e = 3.0 - 1.25 - 0.5 \left( \frac{6}{12} \right) = 1.5 \text{ ft} \quad \text{O.K.} \]

• Curvature in plan is less than 4° (curvature = 0.0) O.K.
• Cross-section of the bridge is consistent with one of the cross-sections given in LRFD Table 4.6.2.2.1-1 O.K.

Since these criteria are satisfied, the barrier and wearing surface loads are equally distributed among the 4 beams.

Barrier weight = (2 barriers)(0.300 kip/ft)/(4 beams) = 0.150 kip/ft
DW = Dead load of future wearing surface = (2/12)(0.15) = 0.250 ksf = (0.025 ksf)(42.0 ft)/(4 beams) = 0.263 kip/ft

For a simply supported beam with a span (L) loaded with a uniformly distributed load (w), the shear force (V_x) and bending moment (M_x) at any distance (x) from the support are given by:

\[ V_x = w(0.5L - x) \quad \text{(Eq. 9.6.4.1.2-1)} \]
\[ M_x = 0.5wx(L - x) \quad \text{(Eq. 9.6.4.1.2-2)} \]

Using the above equations, values of shear forces and bending moments for a typical interior beam, under self-weight of beam and weight of slab and haunch are computed and given in Table 9.6.4-1 that is found at the end of Section 9.6.4. The span length for each span to be considered depends on the construction stage:
• overall length immediately after prestress release
• centerline-to-centerline distance between beam bearings at the time of deck placement
• centerline-to-centerline distance between supports after beams are made continuous
Shear forces and bending moments due to barrier weight and future wearing surface are calculated based on the continuous span lengths, 110, 120 and 110 ft. The three-span structure was analyzed using a continuous beam program. The shear forces and bending moments are given in Table 9.6.4-1.

Design live load is HL-93, which consists of a combination of: [LRFD Art. 3.6.1.2.1]

1. Design truck or design tandem with dynamic allowance. [LRFD Art. 3.6.1.2.1]
The design truck is the same as the HS20 design truck specified by the Standard Specifications [STD Art. 3.6.1.2.2]. The design tandem consists of a pair of 25.0-kip axles spaced 4.0 ft apart. [LRFD Art. 3.6.1.2.3]. Spans in the range used in this example are much larger than those controlled by the tandem loading. For this reason, tandem loading effects are not included.

2. Design lane load of 0.64 kips/ft without dynamic allowance [LRFD Art. 3.6.1.2.4]

Art. 3.6.1.3.1 in the LRFD Specifications requires that for negative moment between points of dead load contraflexure, and, for reactions at interior piers only, 90% of the effect of two design trucks spaced at a minimum of 50.0 ft between the lead axle of one truck and the rear axle of the other truck, combined with 90% of the effect of the design lane load be considered. The distance between the 32 kip axles of each truck should be taken as 14 ft.

This three-span structure was analyzed using a continuous beam program that has the ability to generate live load shear force, and bending moment envelopes in accordance with the LRFD Specifications on a per-lane basis. The span lengths used are the continuous span lengths, 110, 120 and 110 ft.

The live load bending moments and shear forces are determined by using the simplified distribution factor formulas [LRFD Art. 4.6.2.2]. To use the simplified live load distribution factor formulas, the following conditions must be met. [LRFD Art. 4.6.2.2.1]

Width of deck is constant O.K.

Number of beams, \( N_b \geq 4 \) (\( N_b = 4 \)) O.K.

Beams are parallel and approximately of the same stiffness O.K.

Roadway part of overhang, \( d_c \leq 3.0 \text{ ft} \) \( d_c = 3.0 - 1.25 - 0.5 \left( \frac{6}{12} \right) = 1.50 \text{ ft} \) O.K.

Curvature is less than 4° [LRFD Table. 4.6.1.2.1-1] (Curvature = 0.0°) O.K.

For precast concrete I- or bulb-tee beams with cast-in-place concrete deck slab, the bridge type is (k) [LRFD Table 4.6.2.2.1-1]

The number of design lanes:

Number of design lanes = The integer part of the ratio of \((w/12)\), where \((w)\) is the clear roadway width, in ft, between the curbs. [Art. 3.6.1.1.1]

From Figure 9.6.1-2, \( w = 42 \text{ ft} \)

Number of design lanes = integer part of \((42/12) = 3 \) lanes
For all limit states except for fatigue limit state

For two or more lanes loaded:

\[
\text{DFM} = 0.075 + \left( \frac{S}{9.5} \right)^{0.6} \left( \frac{S}{L} \right)^{0.2} \left( \frac{K_g}{12.0L^3} \right)^{0.1} \quad [\text{LRFD Table 4.6.2.2.2b-1}]
\]

Provided that:

\[
\begin{align*}
3.5 & \leq S \leq 16 \quad S = 12.0 \text{ ft} \quad \text{O.K.} \\
4.5 & \leq t_s \leq 12 \quad t_s = 7.5 \text{ in.} \quad \text{O.K.} \\
20 & \leq L \leq 240 \quad L = 120 \text{ ft} \quad \text{O.K.} \\
N_b & \geq 4 \quad N_b = 4 \quad \text{O.K.} \\
10,000 & \leq K_g \leq 7,000,000 \quad \text{O.K. (see below)}
\end{align*}
\]

where

\[
\begin{align*}
\text{DFM} &= \text{distribution factor for moment for interior beam} \\
S &= \text{beams spacing, ft} \\
L &= \text{beam span, ft} \\
t_s &= \text{depth of concrete slab, in.} \\
K_g &= \text{longitudinal stiffness parameter, in.}^4, = n(I + A e_g^2) \quad [\text{LRFD Eq. 4.6.2.2.1-1}]
\end{align*}
\]

\[
n = \frac{E_\text{(beam)}}{E_\text{(slab)}} = \frac{5,072}{3,834} = 1.3229
\]

\[
A = \text{cross-section area of the beam (non-composite section), in.}^2
\]

\[
I = \text{moment of inertia of the beam (non-composite section), in.}^4
\]

\[
e_g = \text{distance between the centers of gravity of the beam and deck, in.} \\
= (7.5/2 + 0.5 + 35.4) = 39.65 \text{ in.}
\]

Therefore,

\[
K_g = (1.3229)[545,894 + 767(39.65)^2] = 2,317,339.75 \text{ in.}^4
\]

At center span:

\[
\text{DFM} = 0.075 + \left( \frac{12}{9.5} \right)^{0.6} \left( \frac{12}{120} \right)^{0.2} \left( \frac{2,317,339.75}{(12.0)(120)(7.5)^3} \right)^{0.1}
\]

\[
= 0.075 + (1.150)(0.631)(1.143) = 0.905 \text{ lanes/beam}
\]

For one design lane loaded:

\[
\text{DFM} = 0.06 + \left( \frac{S}{14} \right)^{0.4} \left( \frac{S}{L} \right)^{0.3} \left( \frac{K_g}{12.0L^3} \right)^{0.1} \quad [\text{LRFD Table 4.6.2.2.2b-1}]
\]

\[
= 0.06 + (0.40)(0.501)(1.138) = 0.596 \text{ lanes/beam}
\]

Thus, the case of the two design lanes loaded controls, \( \text{DFM} = 0.905 \text{ lanes/beam} \).
For Fatigue Limit State:

LRFD Article 3.4.1 states that for fatigue limit state, the single design truck should be used. However, live load distribution factors given in LRFD Article 4.6.2.2 take into consideration the multiple presence factor, m. LRFD Article 3.6.1.1.2 states that the multiple presence factor, m, for one design lane loaded is 1.2. Therefore, the distribution factor for one design lane loaded, with the multiple presence factor removed, should be used. The distribution factor for fatigue limit state is: $0.596/1.2 = 0.497$ lanes/beam.

Fatigue limit state is not checked in this example. However, the live load moment that is used to compute the fatigue stress range is a moment due to a truck load with a constant spacing of 30 ft between the 32.0 kip axles.

For two or more lanes loaded:

\[
DFV = 0.2 + \left( \frac{S}{12} \right) - \left( \frac{S}{35} \right)^2
\]

[LRFD Table 4.6.2.3a-1]

Provided that: $3.5 \leq S \leq 16$ $S = 12.0$ ft O.K.
$20 \leq L \leq 240$ $L = 120$ ft O.K.
$4.5 \leq t_s \leq 12$ $t_s = 7.5$ in. O.K.
$N_b \geq 4$ $N_b = 4$ O.K.

where
\[
DFV = \text{Distribution factor for shear for interior beam}
\]
\[
S = \text{Beam spacing, ft}
\]

Therefore, the distribution factor for shear force for both end spans and center span is:

\[
DFV = 0.2 + \left( \frac{12}{12} \right) - \left( \frac{12}{35} \right)^2 = 1.082 \text{ lanes/beam}
\]

For one design lane loaded:

\[
DFV = 0.36 + \left( \frac{S}{25.0} \right) = 0.36 + \left( \frac{12}{25.0} \right) = 0.840 \text{ lanes/beam}
\]

Thus, the case of two or more lanes loaded controls, $DFV = 1.082$ lanes/beam.

\[
IM = 33\% \quad \text{[LRFD Table 3.6.2.1-1]}
\]

where $IM = \text{dynamic load allowance, applied only to truck load}$
For all limit states except for fatigue limit state:
Unfactored shear forces and bending moments due to HL-93, per beam, are:

\[ \begin{align*}
V_{LT} &= (\text{shear force per lane})(DFV)(1 + \text{IM}) = (\text{shear force per lane})(1.082)(1 + 0.33) \\
&= (\text{shear force per lane})(1.439) \text{ kips} \\
M_{LT} &= (\text{bending moment per lane})(DFM)(1 + \text{IM}) = (\text{bending moment per lane}) (0.905)(1+0.33) = (\text{bending moment per lane})(1.204) \text{ ft-kips}
\end{align*} \]

Values of \( V_{LT} \) and \( M_{LT} \) at different points are given in Table 9.6.4-1.

Investigating different limit states given in LRFD Article 3.4.1, the following limit states are applicable:

Service I: check compressive stresses in prestressed concrete components:
\[ Q = 1.00(DC + DW) + 1.00(LL + IM) \]  
This load combination is the general combination for service limit state stress checks and applies to all conditions other than Service III.

Service III: Check tensile stresses in prestressed concrete components:
\[ Q = 1.00(DC + DW) + 0.80(LL + IM) \]  
This load combination is a special combination for the service limit state stress check that applies only to tension in prestressed concrete structures to control cracks.

Strength I: Check ultimate strength:
Maximum \( Q = 1.25(DC) + 1.50(DW) + 1.75(LL + IM) \)  
Minimum \( Q = 0.90(DC) + 0.65(DW) + 1.75(LL + IM) \)  
This load combination is the general load combination for strength limit state design. The minimum load factors for dead load (DC) and future wearing surface (DW) are used when dead load and future wearing surface stresses are of an opposite sign to that of the live load.

Fatigue: Check stress range in strands:
\[ Q = 0.75(LL + IM) \]  
This is a special load combination to check the tensile stress range in the strands due to live load and dynamic allowance.
### Table 9.6.4-1

Unfactored Shear Forces and Bending Moments for a Typical Interior Beam

<table>
<thead>
<tr>
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<tbody>
<tr>
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<td>Shear Moment</td>
<td>Shear Moment</td>
<td>Shear Moment</td>
<td>Shear Moment</td>
<td>Shear Moment</td>
</tr>
<tr>
<td></td>
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<td>M&lt;sub&gt;x&lt;/sub&gt;</td>
<td>M&lt;sub&gt;b&lt;/sub&gt;</td>
<td>M&lt;sub&gt;b&lt;/sub&gt;</td>
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<td>kips</td>
<td>ft-kips</td>
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<td>0.0</td>
<td>66.6</td>
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<td>60.0</td>
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<td>0.0</td>
<td>1,390.7</td>
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<td>2,126.9</td>
</tr>
</tbody>
</table>

---

<sup>[1]</sup> Centerline of bearing<br>
<sup>[2]</sup> Centerline of pier<br>
<sup>[3]</sup> Critical section in shear

* Determined using linear interpolation

Note: Shear values shown are absolute values.
Note: The live load used in the above equation results only from a single design truck with a 30-ft constant spacing between the 32.0 kip axles with the special dynamic allowance, (IM) for fatigue.

The required number of strands is usually governed by concrete tensile stresses at the bottom fiber for the load combination at Service III at the section of maximum moment or at the harp points. For estimating the number of strands, only the stresses at midspan are considered.

Bottom tensile stress due to applied dead and live loads using load combination Service III, is:

$$f_b = \frac{M_g + M_i}{S_b} + \frac{M_b + M_{ws} + (0.8)(M_{LL+I})}{S_{bc}}$$

where

- $f_b$ = bottom tensile stresses, ksi
- $M_g$ = unfactored bending moment due to beam self-weight, ft-kips
- $M_i$ = unfactored bending moment due to slab and haunch weights, ft-kips
- $M_b$ = unfactored bending moment due to barrier weight, ft-kips
- $M_{ws}$ = unfactored bending moment due to weight of future wearing surface, ft-kips
- $M_{LL+I}$ = unfactored bending moment due to design vehicular live load including impact, ft-kips

Using values of bending moments from Table 9.6.4-1, the bottom tensile stress at midspan of the center span (point 0.5, center span), is:

$$f_b = \frac{(1,390.7 + 2,126.9)(12)}{14,915} + \frac{(73 + 128 + 0.8 \times 2,115)(12)}{20,070}$$

$$= 2.830 + 1.132 = 3.962 \text{ ksi}$$

The tensile stress limit at service loads $= 0.19 f'_c$ [LRFD Art. 5.9.4.2.b] where $f'_c$ = specified 28-day concrete strength, ksi

Therefore, the tensile stress limit in concrete $= 0.19 \sqrt{7.0} = -0.503 \text{ ksi}$

The required precompressive stress at the bottom fiber of the beam is the difference between bottom tensile stress due to the applied loads and the concrete tensile stress limit:

$$f_{pb} = (3.962 - 0.503) = 3.459 \text{ ksi.}$$

The location of the strand center of gravity at midspan, ranges from 5 to 15% of the beam depth, measured from the bottom of the beam. A value of 5% is appropriate for newer efficient sections like the bulb-tee beams and 15% for less efficient AASHTO standard shapes.
Assume the distance from the center of gravity of strands to the bottom fiber of the beam, $y_{bs}$, is equal to 7% of the beam depth.

$$y_{bs} = 0.07h = 0.07(72) = 5.04\text{ in.}$$

Then, the strand eccentricity at midspan, $e_c$, is $= (y_b - y_{bs}) = (36.60 - 5.04) = 31.56\text{ in.}$

If $P_{pe}$ is the total prestressing force, the stress at the bottom fiber due to prestress is:

$$f_{pb} = \frac{P_{pe} e_c}{A S_b}$$

or, setting the required precompression (3.459 ksi) equal to the bottom fiber stress due to prestress, solve for the minimum required $P_{pe}$:

$$3.459 = \frac{P_{pe}}{767} + \frac{P_{pe} (31.56)}{14,915}$$

Solving for $P_{pe}$, the required $P_{pe} = 1,011.5$ kips

Final Prestress force per strand = (area of strand)($f_{pi}$)(1 − losses, %)

where $f_{pi}$ = initial prestressing stress before transfer, ksi.

Assuming final loss of 25% of $f_{pi}$, the prestress force per strand after all losses $= (0.153)(202.5)(1 - 0.25) = 23.2$ kips

Number of strands required $= (1,011.5/23.2) = 43.6$ strands

Try (44) 1/2 in. diameter, 270 ksi, low-relaxation strands

9.6.5.4 Strand Pattern

The assumed strand pattern for the 44 strands at midspan is shown in Figure 9.6.5.4-1. Each available position was filled beginning with the bottom row.
The distance between the center of gravity of strands and the bottom concrete fiber of the beam, \( y_{bs} \), is:
\[
y_{bs} = \frac{[(12(2) + 12(4) + 8(6) + 4(8) + 2(10) + 2(12) + 2(14) + 2(16))/44] = 5.82 \text{ in.}}
\]
Strand eccentricity at midspan, \( e_c = y_b - y_{bs} = 36.60 - 5.82 = 30.78 \text{ in.} \)

9.6.6 PRESTRESS LOSSES

Total prestress losses:
\[
\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR2}
\]
where
\[
\Delta f_{pES} = \text{loss due to elastic shortening, ksi}
\]
\[
\Delta f_{pSR} = \text{loss due to shrinkage, ksi}
\]
\[
\Delta f_{pCR} = \text{loss due to creep, ksi}
\]
\[
\Delta f_{pR2} = \text{loss due to relaxation of steel after transfer, ksi}
\]

9.6.6.1 Elastic Shortening

\[
\Delta f_{pES} = \frac{E_p}{E_c} f_{cgp}
\]
where
\[
E_p = \text{modulus of elasticity of prestressing reinforcement} = 28,500 \text{ ksi}
\]
\[
E_c = \text{modulus of elasticity of beam at release} = 4,496 \text{ ksi}
\]
\[
f_{cgp} = \text{sum of concrete stresses at the center of gravity of prestressing tendons due to prestressing force at transfer and the self-weight of the member at sections of maximum moment}
\]
\[
= \frac{P_i}{A} + \frac{P_i e_c^2}{I} - \frac{M_{gc} e_c}{I}
\]

The LRFD Specifications, Art. 5.9.5.2.3a, states that \( f_{cgp} \) can be calculated on the basis of a prestressing steel stress assumed to be \( 0.7 f_{pu} \) for low-relaxation strands. However, common practice assumes the initial losses as a percentage of initial prestressing stress before release, \( f_{pi} \). In both procedures, assumed initial losses should be checked and if different from the assumed value, a second iteration should be carried out. In this example, a 9% \( f_{pi} \) initial loss is used.

Force per strand at transfer = (area of strand) (prestress stress at release)
\[
= 0.153(202.5)(1 - 0.09) = 28.2 \text{ kips}
\]
\( P_i \) = total prestressing force at release = (44 strands)(28.2) = 1,240.8 kips

\( M_{gc} \) should be calculated based on the overall beam length of 119 ft. However, since the elastic shortening losses will be a part of the total losses, \( f_{cgp} \) will be conservatively computed based on \( M_{gc} \) using the design span length of 118 ft.

\[
f_{cgp} = \frac{1,240.8}{767} + \frac{(1,240.8)(30.78)^2}{545,894} - \frac{(1,390.7)(12)(30.78)}{545,894}
\]
\[
= 1.618 + 2.153 - 0.941 = 2.830 \text{ ksi}
\]
Therefore, the loss due to elastic shortening is:

\[ \Delta f_{pES} = \frac{28,500}{4,496} (2.830) = 17.9 \text{ ksi} \]

The percent of actual losses due to elastic shortening = \( \frac{17.9}{202.5} \times 100 = 8.8\% \). Since calculated loss of 8.8% is approximately equal to the initial assumption of 9%, a second iteration is not necessary. Note that this loss is equivalent to a stress after initial losses of 0.68 \( f_{pu} \). This stress is lower than the estimate of 0.70 \( f_{pu} \) provided in Article 5.9.5.2.3a. If the elastic shortening loss was calculated using a stress of 0.70 \( f_{pu} \), a second iteration would be required to arrive at a steel stress of 0.68 \( f_{pu} \).

**9.6.6.2 Shrinkage**

\[ \Delta f_{pSR} = (17 - 0.15H) \]

where \( H = \) relative humidity (assume 70%)

Relative humidity varies significantly throughout the country. See LRFD Figure 5.4.2.3.3-1.

\[ \Delta f_{pSR} = 17 - 0.15(70) = 6.5 \text{ ksi} \]

**9.6.6.3 Creep of Concrete**

\[ \Delta f_{pCR} = 12f_{egp} - 7\Delta f_{cdp} \]

where

\[ \Delta f_{cdp} = \text{Change of stresses at center of gravity of prestressing due to permanent loads except the loads acting at time of applying prestressing force, calculated at the same section as } f_{egp} \]

\[ = \frac{M_{i} e_{c}}{I} + \frac{(M_{w} + M_{b})(y_{bc} - y_{bs})}{I_{c}} \]

\[ = \frac{(2,126.9)(12)(30.78)}{545,894} + \frac{(73 + 128)(12)(54.67 - 5.82)}{1,097,252} \]

\[ = 1.439 + 0.107 = 1.546 \text{ ksi} \]

Therefore, the loss due to creep is:

\[ \Delta f_{pCR} = 12(2.830) - 7(1.546) = 23.1 \text{ ksi} \]

**9.6.6.4 Relaxation of Prestressing Strands**

**9.6.6.4.1 Relaxation before Transfer**

Initial loss due to relaxation of prestressing steel is accounted for in the beam fabrication process. Therefore, loss due to relaxation of the prestressing steel prior to transfer will not be computed, i.e. \( \Delta f_{pri} = 0 \). Recognizing this for pretensioned members, LRFD Article 5.9.5.1 excludes the portion of the relaxation loss that occurs prior to transfer from the final loss.
For low-relaxation strands, loss due to strand relaxation after transfer is:
\[ \Delta f_{pR2} = 30\% [20.0 - 0.4(\Delta f_{pES}) - 0.2(\Delta f_{pSR} + \Delta f_{pCR})] \]
\[ = 0.30[20.0 - 0.4(17.9) - 0.2(6.5 + 23.1)] = 2.1 \text{ ksi} \]

\[ \Delta f_{pi} = \Delta f_{pES} = 17.9 \text{ ksi} \]

\[ \text{Stress in tendons after transfer, } f_{pt} = f_{pi} - \Delta f_{pi} = (202.5 - 17.9) = 184.6 \text{ ksi} \]

\[ \text{Force per strand } = (f_{pt})(\text{strand area}) = 184.6(0.153) = 28.2 \text{ kips} \]

Therefore, the total prestressing force after transfer, \( P_t = 28.2 \times 44 = 1,240.8 \text{ kips} \)

Initial loss, \( \% = \frac{(\text{total losses at transfer})}{(f_{pi})} = 17.9/(202.5) = 8.9\% \)

The first estimation of loss at transfer, 9%, is very close to the actual computed initial loss of 8.9%. Thus, there is no need for a second iteration to refine the initial losses.

Total loss of prestress at service loads is:
\[ \Delta f_{pT} = \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR2} = 17.9 + 6.5 + 23.1 + 2.1 = 49.6 \text{ ksi} \]

\[ \text{Stress in tendon after all losses, } f_{pe} = f_{pi} - \Delta f_{pT} = 202.5 - 49.6 = 152.9 \text{ ksi} \]

Check prestressing stress limit at service limit state:
\[ f_{pe} \leq 0.8f_{py} = 0.8(243) = 194.4 \text{ ksi} > 152.9 \text{ ksi} \]

Therefore, \( P_{pe} = 23.4(44) = 1,029.6 \text{ kips} \)

Final losses, \( \% = \frac{\Delta f_{pT}}{f_{pi}} = \frac{49.6}{202.5}(100) = 24.5\% \)

\[ \text{Force per strand after initial losses } = 28.2 \text{ kips} \]

Therefore, the total prestressing force after transfer is, \( P_i = 1,240.8 \text{ kips} \)

Compression: \( 0.6f'_{ci} = 0.6(5.5) = +3.300 \text{ ksi} \)

Tension:
- without bonded auxiliary reinforcement:
  \[ -0.0948 \sqrt{f'_{ci}} \leq -0.200 \text{ ksi}; -0.0948 \sqrt{5.5} = -0.222 \text{ ksi} \]
  Therefore, \( -0.200 \text{ ksi} \)  (Controls)
- with bonded auxiliary reinforcement which is sufficient to resist 120% of the tension force in the cracked concrete:
  \[ -0.22 \sqrt{f'_{ci}} = -0.22 \sqrt{5.5} = -0.516 \text{ ksi} \]

Stresses at this location need only be checked at release since this stage almost always governs. Also, losses with time will reduce the concrete stresses making them less critical.

\[ \text{Transfer length } = 60(\text{strand diameter}) = 60(0.5) = 30 \text{ in.} = 2.5 \text{ ft} \]
Due to the camber of the beam at release, the beam self-weight acts on the overall beam length (119 ft). Therefore, values of bending moment given in Table 9.6.4-1 cannot be used since they are based on the span between centerlines of bearings (118 ft). Using Equation Eq. 9.6.4.1.2-2 given previously, the bending moment at a distance 2.5 ft from the end of the beam is calculated due to beam self-weight:

\[ M_g = (0.5)(0.799)(2.5)(119 - 2.5) = 116.4 \text{ ft-kips} \]

Compute top stress at the top fiber of the beam:

\[ f_t = \frac{P - P_e}{A} + \frac{M_g}{S_b} = \frac{1,240.8}{767} - \frac{(1,240.8)(30.78)}{14,915} + \frac{(116.4)(12)}{14,915} = 1.618 - 2.477 + 0.091 = -0.768 \text{ ksi} \]

Tensile stress limit for concrete with bonded reinforcement: -0.516 ksi N.G.

Compute bottom stress at the bottom fiber of the beam:

\[ f_b = \frac{P + P_e}{A} - \frac{M_g}{S_b} = \frac{1,240.8}{767} + \frac{(1,240.8)(30.78)}{14,915} - \frac{(116.4)(12)}{14,915} = 1.618 + 2.561 - 0.094 = +4.085 \text{ ksi} \]

Compressive stress limit for concrete: +3.300 ksi N.G.

Since the top and bottom concrete stresses exceed the stress limits, harp strands to make stresses fall within the specified limits. Harp 12 strands at the 0.3L points, as shown in Figures 9.6.7.2-1 and 9.6.7.2-2. This harp location is more appropriate for the end spans of multi-span continuous bridges because the maximum positive moment is closer to the abutment than in the interior spans. For simple spans, it is more common to use a harp point at least 0.4L from the ends.

**Figure 9.6.7.2-1**

*Strand Pattern*
Compute the center of gravity of the prestressing strands at the transfer length using the harped pattern.

The distance between the center of gravity of the 12 harped strands at the end of the beam and the top fiber of the precast beam is:

\[
\frac{2(2) + 2(4) + 2(6) + 2(8) + 2(10) + 2(12)}{12} = 7.00 \text{ in.}
\]

The distance between the center of gravity of the 12 harped stands at the harp point and the bottom fiber of the beam is:

\[
\frac{2(6) + 2(8) + 2(10) + 2(12) + 2(14) + 2(16)}{12} = 11.0 \text{ in.}
\]

The distance between the center of gravity of the 12 harped strands and the top fiber of the beam at the transfer length section is:

\[
7 \text{ in.} + \frac{(72 - 11 - 7) \text{ in.}}{35.5 \text{ ft}} (2.5) \text{ ft} = 10.80 \text{ in.}
\]

The distance between the center of gravity of the 32 straight bottom strands and the extreme bottom fiber of the beam is:

\[
\frac{12(2) + 12(4) + 6(6) + 2(8)}{32} = 3.88 \text{ in.}
\]

Therefore, the distance between the center of gravity of the total number of the strands and the bottom fiber of the precast beam at transfer length is:

\[
\frac{12(72 - 10.80) + 32(3.88)}{44} = 19.51 \text{ in.}
\]

Eccentricity of the strand group at transfer length, \(e\), is: 36.60 − 19.51 = 17.09 in.

The distance between the center of gravity of the total number of the strands and the bottom fiber of the precast beam at the end of the beam is:

\[
\frac{12(72 - 7) + 32(3.88)}{44} = 20.55 \text{ in.}
\]

and the eccentricity at the end of the beam, \(e_e\), is: 36.60 − 20.55 = 16.05 in.

Recompute top and bottom stresses at the transfer length section using the harped pattern:
Concrete stress at top fiber of the beam:

\[ f_t = \frac{1,240.8}{767} - \frac{(1,240.8)(17.09)}{15,421} + \frac{(116.4)(12)}{15,421} = 1.618 - 1.375 + 0.091 = +0.334 \text{ ksi} \]

Compressive stress limit for concrete: +3.300 ksi  O.K.

Concrete stress at bottom fiber of the beam,

\[ f_b = \frac{1,240.8}{767} + \frac{(1,240.8)(17.09)}{14,915} - \frac{(116.4)(12)}{14,915} = 1.618 + 1.422 - 0.094 = +2.946 \text{ ksi} \]

Compressive stress limit for concrete: +3.300 ksi  O.K.

### 9.6.7.3 Stresses at the Harp Points

The strand eccentricity at the harp points is the same as at midspan, \( e_c = 30.78 \text{ in} \).
The bending moment due to beam self-weight at a distance 35.5 ft (0.3L) from the end of the beam is:

\[ M_g = (0.5)(0.799)(35.5)(119 - 35.5) = 1,184.2 \text{ ft-kips} \]

Concrete stress at top fiber of the beam,

\[ f_t = \frac{1,240.8}{767} - \frac{(1,240.8)(30.78)}{15,421} + \frac{(1,184.2)(12)}{15,421} = 1.618 - 2.477 + 0.921 = +0.062 \text{ ksi} \]

Compressive stress limit for concrete: +3.300 ksi  O.K.

Concrete stress at bottom fiber of the beam:

\[ f_b = \frac{1,240.8}{767} + \frac{(1,240.8)(30.78)}{14,915} - \frac{(1,184.2)(12)}{14,915} = 1.618 + 2.561 - 0.953 = +3.226 \text{ ksi} \]

Compressive stress limit for concrete: +3.300 ksi  O.K.

### 9.6.7.4 Stresses at Midspan

The bending moment due to beam self-weight at a distance 59.5 feet from the end of the beam is:

\[ M_g = (0.5)(0.799)(59.5)(119 - 59.5) = 1,414.3 \text{ ft-kips} \]

Concrete stress at top fiber of the beam:

\[ f_t = \frac{1,240.8}{767} - \frac{(1,240.8)(30.78)}{15,421} + \frac{(1,141.3)(12)}{15,421} = 1.618 - 2.477 + 1.101 = +0.242 \text{ ksi} \]

Compressive stress limit for concrete: +3.300 ksi  O.K.

Concrete stress at bottom fiber of the beam,

\[ f_b = \frac{1,240.8}{767} + \frac{(1,240.8)(30.78)}{14,915} - \frac{(1,141.3)(12)}{14,915} = 1.618 + 2.561 - 1.138 = +3.041 \text{ ksi} \]

Compressive stress limit for concrete: +3.300 ksi  O.K.
Assume that the stress in the strand at the time of prestressing, before any losses, is:

\[ 0.80f_{pu} = 0.80(270) = 216 \text{ ksi} \]

Then, the prestress force per strand before any losses is:

\[ 0.153(216) = 33.0 \text{ kips} \]

From Figure 9.6.7.2-2, harp angle,

\[ \psi = \tan^{-1} \left( \frac{72 - 7 - 11}{35.5(12)} \right) = 7.2^\circ \]

Therefore, hold-down force/strand = 1.05(force per strand)(sin \( \psi \))

\[ = 1.05(33.0) \sin 7.2^\circ = 4.3 \text{ kips/strand} \]

Note that the factor, 1.05, is applied to account for friction.

Total hold-down force = 12 strands(4.3) = 51.6 kips

The hold-down force and the harp angle should be checked against maximum limits for local practices. Refer to Chapter 3, Fabrication and Construction, Section 3.3.2.2, and Chapter 8, Design Theory and Procedures, for additional details.

### Summary Of Stresses at Transfer

<table>
<thead>
<tr>
<th></th>
<th>Top stresses ( f_t ) (ksi)</th>
<th>Bottom stresses ( f_{bt} ) (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>At transfer length section</td>
<td>+0.334</td>
<td>+2.946</td>
</tr>
<tr>
<td>At harp points</td>
<td>+0.062</td>
<td>+3.226</td>
</tr>
<tr>
<td>At midspan</td>
<td>+0.242</td>
<td>+3.041</td>
</tr>
</tbody>
</table>

Note that the bottom stresses at the harp points are more critical than the ones at midspan.

The total prestressing force after all losses, \( P_{pe} = 1,029.6 \text{ kips} \)

**Compression:**

Due to permanent loads, (i.e., beam self-weight, weight of slab and haunch, weight of future wearing surface, and weight of barriers), for load combination Service I:

- for the precast beam: \( 0.45f_c' = 0.45(7.0) = +3.150 \text{ ksi} \)
- for the deck: \( 0.45f_c' = 0.45(4.0) = +1.800 \text{ ksi} \)

Due to permanent and transient loads (i.e., all dead loads and live loads), for load combination Service I:

- for precast beam: \( 0.60f_c' = 0.60(7.0) = +4.200 \text{ ksi} \)
- for deck: \( 0.60f_c' = 0.60(4.0) = +2.400 \text{ ksi} \)

**Tension:**

For components with bonded prestressing tendons:

- for load combination Service III: \( -0.19 \sqrt{f_c'} \)
  - for precast beam: \( -0.19 \sqrt{7.0} = -0.503 \text{ ksi} \)
9.6.8.2 Stresses at Midspan

- Concrete stresses at the top fiber of the beam:
  
  To check top compressive stress, two cases are checked:

  1. Under permanent loads, Service I:
     
     Using bending moment values given in Table 9.6.4-1, concrete stress at top fiber of the beam is:
     
     \[
     f_{tg} = \frac{P_{pe}}{A} - \frac{P_{pe}e_c}{S_t} + \frac{(M_L + M_s)}{S_t} + \frac{(M_{wa} + M_b)}{S_{tg}}
     \]
     
     \[
     = \frac{1,029.6}{767} - \frac{(1,029.6)(30.78)}{15,421} + \frac{(1,390.7 + 2,126.9)(12)}{15,421} + \frac{(128 + 73)(12)}{63,151}
     \]
     
     \[
     = 1.342 - 2.055 + 2.737 + 0.038 = +2.062 \text{ ksi}
     \]
     
     Compressive stress limit for concrete: +3.150 ksi  O.K.

  2. Under permanent and transient loads, Service I:
     
     \[
     f_{tg} = +2.062 + \frac{(M_{LL,g})}{S_{tg}} = +2.062 + \frac{(2,115)(12)}{63,315} = +2.062 + 0.401 = +2.463 \text{ ksi}
     \]
     
     Compressive stress limit for concrete: +4.200 ksi  O.K.

- Concrete stress at the top fiber of the deck, Service I:
  
  Note: Compression stress in the deck slab at service loads never controls the design for typical applications. The calculations shown below are for illustration and may not be necessary in most practical applications.

  1. Under permanent loads:
     
     \[
     f_{tc} = \frac{(M_{wa} + M_b)}{S_{tc}} = \frac{(128 + 73)(12)}{57,307} = +0.042 \text{ ksi}
     \]
     
     Compressive stress limit for concrete: +1.800 ksi  O.K.

  2. Under permanent and transient loads:
     
     \[
     f_{tc} = \frac{(M_{wa} + M_b + M_{LL,t})}{S_{tc}} = \frac{(128 + 73 + 2,115)(12)}{57,307} = +0.485 \text{ ksi}
     \]
     
     Compressive stress limit for concrete: +2.400 ksi  O.K.

- Tension stress at the bottom fiber of the beam, Service III:
  
  \[
  f_b = \frac{P_{pe}}{A} + \frac{P_{pe}e_c}{S_b} - \frac{(M_L + M_s)}{S_b} - \frac{(M_{wa} + M_b) + 0.8(M_{LL,t})}{S_{bc}}
  \]
  
  \[
  = \frac{1,029.6}{767} + \frac{(1,029.6)(30.78)}{14,915} - \frac{(1,390.7 + 2,126.9)(12)}{14,915}
  \]
  
  \[
  - \frac{(128 + 73)(12) + (0.8)(2,115)(12)}{20,070}
  \]
  
  \[
  = 1.342 + 2.125 - 2.830 - 1.132 = -0.495 \text{ ksi}
  \]
  
  Tensile stress limit for concrete: −0.503 ksi  O.K.
9.6.8.3 Fatigue Stress Limit

9.6.8.3.1 Positive Moment Section

Fatigue limit state is not checked in this example. For an example of this calculation, refer to Examples 9.2 and 9.4, Sections 9.2.9.3 and 9.4.8.3 respectively.

9.6.8.3.2 Negative Moment Section

In order to perform the fatigue check, the reinforcement of the section should be determined. Therefore, the fatigue check for the negative moment section is addressed in Section 9.6.9.2.1.

9.6.8.4 Summary of Stresses at Service Loads

<table>
<thead>
<tr>
<th></th>
<th>Top of Deck (ksi) Service I</th>
<th>Top of Beam (ksi) Service I</th>
<th>Bottom of Beam (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Permanent Loads</td>
<td>Total Loads</td>
<td>Permanent Loads</td>
</tr>
<tr>
<td>At midspan</td>
<td>+0.042</td>
<td>+0.485</td>
<td>+2.062</td>
</tr>
</tbody>
</table>

9.6.9 Strength Limit State

9.6.9.1 Positive Moment Section

Total ultimate bending moment for Strength I is:

\[ M_u = 1.25(DF) + 1.5(DW) + 1.75(LL + IM) \]  

[LRFD Tables 3.4.1-1&2]

At midspan of center span, (center span point 0.5):

\[ M_u = 1.25(1,390.7 + 2,126.9 + 73) + 1.5(128) + 1.75(2,115) \]

\[ = 4,488.3 + 192.0 + 3,701.3 = 8,381.6 \text{ ft-kips} \]

Average stress in prestressing steel when \( f_{pe} \geq 0.5 f_{pu} \):

\[ f_{ps} = f_{pu} \left( 1 - \frac{k}{d_p} \right) \]

[LRFD Eq. 5.7.3.1.1-1]

where

\[ k = 2 \left( 1.04 - \frac{f_{py}}{f_{pu}} \right) \]

[LRFD Eq. 5.7.3.1.1-2]

\[ = 0.28 \text{ for low-relaxation strands} \]

[LRFD Table C5.7.3.1.1-1]

\[ d_p = \text{distance from extreme compression fiber to the centroid of the prestressing tendons} = h - y_{bs} = 80.00 - 5.82 = 74.18 \text{ in.} \]

\[ c = \text{distance between the neutral axis and the compressive face, in.} \]

To compute \( c \), assume rectangular section behavior, and check if the depth of the neutral axis, \( c \) is equal to or less than \( t_c \):

[LRFD 5.7.3.2.2]

\[ c = \frac{A_p f_{pu} + A_f f_y - A'_f f'_y}{0.85 f_{c} b_1 + k A_{ps} f_{pu}} \]

[LRFD Eq. 5.7.3.1.1-4]
where

\[ A_{ps} = \text{area of prestressing steel} = 44 \times 0.153 = 6.732 \text{ in.}^2 \]

\[ f_{ps} = \text{specified tensile strength of prestressing steel} = 270 \text{ ksi} \]

\[ A_s = \text{area of mild steel tension reinforcement} = 0.0 \text{ in.}^2 \]

\[ f_y = \text{yield strength of tension reinforcement} = 60.0 \text{ ksi} \]

\[ A'_{s} = \text{area of compression reinforcement} = 0.0 \text{ in.}^2 \]

\[ f'_{y} = \text{yield strength of compression reinforcement} = 60.0 \text{ ksi} \]

\[ f'_c = \text{compressive strength of deck concrete} = 4.0 \text{ ksi} \]

\[ \beta_1 = \text{stress factor of compression block} \quad \text{[LRFD Art. 5.7.2.2]} \]

\[ = 0.85 \text{ for } f'_c \leq 4.0 \text{ ksi} \]

\[ = 0.85 - 0.05(f'_c - 4.0) \geq 0.65 \text{ for } f'_c > 4.0 \text{ ksi} \]

\[ b = \text{effective width of compression flange} = 111 \text{ in.} \]

\[ c = \frac{(6.732)(270) + 0.0 - 0.0}{(0.85)(4.0)(0.85)(111) + 0.28(6.732)} = 5.55 \text{ in.} < t_b = 7.5 \text{ in.} \quad \text{O.K.} \]

\[ a = \text{depth of the equivalent stress block} = \beta_1 c \quad \text{[LRFD Eq. 9.6.9.1-1]} \]

\[ = 0.85(5.55) = 4.72 \text{ in.} \]

Therefore, the assumption of rectangular section behavior is valid and the average stress in prestressing steel is:

\[ f_{ps} = 270 \left( 1 - 0.28 \frac{5.55}{74.18} \right) = 264.3 \text{ ksi} \]

Nominal flexural resistance:

\[ M_n = A_{ps} f_{ps} \left( d_p - \frac{a}{2} \right) \quad \text{[LRFD Eq. 5.7.3.2.2-1]} \]

This equation is a simplified form of LRFD Equation 5.7.3.2.2-1 because no compression reinforcement or mild tension reinforcement is considered and the section behaves as a rectangular section.

\[ M_n = (6.732)(264.3) \left( 74.18 - \frac{4.72}{2} \right) / 12 = 10,648.9 \text{ ft-kips} \]

Factored flexural resistance:

\[ M_r = \phi M_n \quad \text{[LRFD Eq. 5.7.3.2.1-1]} \]

where \( \phi = \text{resistance factor} \quad \text{[LRFD Art. 5.5.4.2.1]} \)

\[ = 1.00 \], for flexure and tension of prestressed concrete

\[ M_r = 10,648.9 \text{ ft-kips} > M_u = 8,381.6 \text{ ft-kips} \quad \text{O.K.} \]

Total ultimate bending moment for Strength I is:

\[ M_u = 1.25(DC) + 1.5(DW) + 1.75(LL+IM) \quad \text{[LRFD Tables 3.4.1-1&2]} \]

At the pier section:

\[ M_u = 1.25(-197) + 1.5(-345) + 1.75(-2,327.7) = -4,837.2 \text{ ft-kips} \]
Notes:
1. At the negative moment section, the compression face is the bottom flange of the beam and is 26 in. wide.
2. This section is a nonprestressed reinforced concrete section, thus $\phi = 0.9$ for flexure.

Assume the deck reinforcement is at mid-height of the deck. The effective depth:

$$d = 72 + 0.5 + 0.5(7.5) = 76.25 \text{ in.}$$

$$R_u = \frac{M_u}{\phi bd^2} = \frac{4,837.2 (12)}{(0.9)(26)(76.25)^2} = 0.427 \text{ ksi}$$

$$m = \frac{f_y}{0.85f'_c} = \frac{60}{(0.85)(7.0)} = 10.084$$

$$\rho = \frac{1}{m} \left[ 1 - \frac{1}{1 - \frac{2R_u m}{f_y}} \right] = \frac{1}{10.084} \left[ 1 - \frac{1}{1 - \frac{2(0.427)(10.084)}{60}} \right] = 0.00739$$

$$A_s = (pbd) = (0.00739)(26)(76.25) = 14.65 \text{ in.}^2$$

This is the amount of mild steel reinforcement required in the slab to resist the negative moment. Assume that the typical deck reinforcement consists of a bottom mat of #5 bars @ 12 in. and a top mat of #4 @ 12 in. for a total $A_s = 0.20 + 0.31 = 0.51 \text{ in.}^2/\text{ft}$. Since the LRFD Specifications do not provide guidance on the width over which this reinforcement is to be distributed, it is assumed here to be the same as the effective compression flange width which was determined earlier to be 111 in.

The typical reinforcement provided over this width is equal to $(111 \times 0.51/12) = 4.72 \text{ in.}^2$. Therefore, the required additional reinforcement at the negative moment section $= 14.65 - 4.72 = 9.93 \text{ in.}^2$.

Provide 18 #7 bars additional reinforcement at 4 in. spacing (2 #7 bars in each space between #4 bars).

$A_s = 18(0.60) = 10.80 \text{ in.}^2$

Therefore, the total $A_s$ provided $= 10.80 + 4.72 = 15.52 \text{ in.}^2 > 14.65 \text{ in.}^2$ O.K.

Compute the capacity of the section in flexure at the pier:

Compute the depth of the compression block:

$$a = \frac{A_s f_y}{0.85 b f'_c} = \frac{15.52(60)}{(0.85)(26)(7.0)} = 6.02 \text{ in.}$$

Note that this value is slightly larger than the flange thickness of 6.0 in. However, the adjustment in the moment capacity, $\phi M_n$, when using a more accurate non-rectangular section analysis, is extremely small.

$$\phi M_n = \phi A_s f_y \left( d - \frac{a}{2} \right) = 0.9(15.52)(60.0) \left( 76.25 - \frac{6.02}{2} \right) / 12$$

$$= 5,115.1 \text{ ft-kips} > 4,837.2 \text{ ft-kips} \quad \text{O.K.}$$

With time, creep of concrete members heavily pretensioned, may cause camber growth. Because this bridge is designed to have rigid connections between beams at the piers, camber growth is restrained. As a result, time-dependent positive moments will develop. Therefore, it is recommended that a nominal amount of positive moment continuity reinforcement be used over the piers to control potential cracking in this region. A common way to provide this reinforcement is to extend approx-
approximately 25 percent of the strands from the bottom flange and bend them up into the
diaphragm. Another common detail is the addition of a quantity of mild steel rein-
forcement required to resist a moment equal to 1.2 \( M_{cr} \). This reinforcement is also
extended from the ends of the beam and bent up into the diaphragm.

The fatigue limit state and crack control for the negative moment zone over the piers
are important design criteria that must be checked. This zone is expected to be
cracked due to service loads and the steel stress range is expected to be significantly
high.

For moment calculations, the fatigue truck loading must be introduced to the three-
span continuous structure. The resulting moments are then used to determine
whether or not the stress range in the longitudinal reinforcement steel is within the
acceptable limits.

In order to control flexural cracking, the tensile stress in the mild steel reinforcement
at service limit state, should not exceed the value given by LRFD Eq. 5.7.3.4.1.

This section is a prestressed section.

The maximum amount of prestressed and nonprestressed reinforcement should be
such that:

\[
\frac{c}{d_e} \leq 0.42
\]

[LRFD Eq. 5.7.3.1-1]

where

\[
d_e = \frac{A_{ps} f_p d_p + A_{f_s} d_s}{A_{ps} f_{ps} + A_{f_s} f_y}
\]

[LRFD Eq. 5.7.3.1-2]

Since \( A_s = 0 \), then \( d_e = d_p = 74.18 \text{ in.} \)

\[
\frac{c}{d_e} = \frac{5.55}{74.18} = 0.075 \leq 0.42 \quad \text{O.K.}
\]

At any section, the amount of prestressed and nonprestressed tensile reinforcement
should be adequate to developed a factored flexural resistance, \( M_r \), equal to the less-
er of:

- 1.2 times the cracking strength determined on the basis of elastic stress distribution
  and the modulus of rupture, and,
- 1.33 times the factored moment required by the applicable strength load combina-
tion.

Check at midspan:

The **LRFD Specifications** do not give a procedure for computing the cracking
moment. Therefore, the following equation adapted from the **Standard Specifications**, Art. 9.18.2.1 is used:

\[
M_{cr} = (f_t + f_{ps})S_{bc} - M_{d/(S_{bc}/S_b - 1)}
\]
where

\[ f_r = \text{modulus of rupture} = 0.24 \sqrt{f_c'} = 0.24 \sqrt{7.0} = 0.635 \text{ ksi} \] [LRFD Art. 5.4.2.6]

\[ f_{pb} = \text{compressive stress in concrete due to effective prestress force only (after allowance for all prestress losses) at extreme fiber of section where tensile stress is caused by externally applied loads} \]

\[ \frac{P_{pe} + P_{pc}}{S_b} = \frac{1,029.6}{767} + \frac{(1,029.6)(30.78)}{14,915} = 1.342 + 2.125 = 3.467 \text{ ksi} \]

\[ M_{d(nc)} = \text{moment due to non-composite dead loads, ft-kips} \]

\[ = M_g + M_s = 1,390.7 + 2,126.9 = 3,517.6 \text{ ft-kips} \]

\[ S_{bc} = \text{composite section modulus for the extreme fiber of section where the tensile stress is caused by externally applied loads} \]

\[ S_b = \text{non-composite section modulus for the extreme fiber of section where the tensile stress is caused by externally applied loads} \]

\[ M_{cr} = (0.635 + 3.467)(20,070/12) - (3,517.6) \left( \frac{20,070}{14,915} - 1 \right) = 5,644.8 \text{ ft-kips} \]

\[ 1.2M_{cr} = 1.2(5,644.8) = 6,773.8 \text{ ft-kips} \]

At midspan, the factored moment required by the Strength I load combination is:

\[ M_u = 8,381.6 \text{ ft-kips (as calculated in Section 9.6.)} \]

Therefore, \[ 1.33M_u = 1.33 \times 8,381.6 = 11,147.5 \text{ ft-kips} \]

Since \[ 1.2M_{cr} < 1.33M_u, \ 1.2M_{cr} \ (\text{Controls}) \]

\[ M_c = 10,648.9 \text{ ft-kips} > 1.2M_{cr} \ \text{O.K.} \]

Note: Contrary to the Standard Specifications, the LRFD Specifications state that this requirement be met at every section.

**9.6.10.2 Negative Moment Section**

**9.6.10.2.1 Maximum Reinforcement**

The maximum amount of prestressed and nonprestressed reinforcement shall be such that:

\[ \frac{c}{d_e} \leq 0.42 \] [LRFD Eq. 5.7.3.3.1-1]

where

\[ d_e = \frac{A_{ps}f_{pe}d_p + A_{fy}d_s}{A_{ps}f_{pe}} \] [LRFD Eq. 5.7.3.3.1-2]

Since \[ A_{ps} = 0 \], then \[ d_e = d_s = 76.25 \text{ in.} \]

\[ \beta_1 = \text{stress factor of compression block} \]

\[ = 0.85 \text{ for } f'_c \leq 4.0 \text{ ksi} \]

\[ = 0.85 - 0.05(f'_c - 4.0) \leq 0.65 \text{ for } f'_c \geq 4.0 \]

\[ = 0.85 - 0.05(7.0 - 4.0) = 0.70 \]
Note that the value of \( a \) used here is not exact because the geometry of the bottom flange must be accommodated. But since \( a \) is slightly larger than 6 in., the uniform width portion of the bottom flange, and since \( (c/d_s) \) is much lower than the maximum limit, further refinement is not warranted.

\[ c = \frac{a}{\beta_1} = \frac{6.02}{0.70} = 8.60 \text{ in.} \]

\[ c = \frac{8.60}{76.25} = 0.113 \leq 0.42 \quad \text{O.K.} \]

For nonprestressed sections, the minimum reinforcement provision may be considered satisfied if:

\[ \rho \geq 0.03 \frac{f'_c}{f_y} \]

where \( \rho \) = ratio of tension steel to gross area = \( A_s/(bd) = \frac{(15.52)}{(26)(76.25)} = 0.008 \)

\[ 0.008 \geq 0.03 \frac{7.0}{60} = 0.0035 \quad \text{O.K.} \]

At the negative moment section, the bottom flange of the precast beam acts as the compression block of the composite section. Therefore, the 28-day strength of the beam concrete, 7.0 ksi, is used.

The area and spacing of shear reinforcement must be determined at regular intervals along the entire length of the beam. In this design example, transverse shear design procedures are demonstrated below by determining these values at the critical section near the supports.

Transverse shear reinforcement must be provided when:

\[ V_u > 0.5\phi(V_c + V_p) \]

where

\( V_u \) = total factored shear force, kips
\( V_c \) = shear strength provided by concrete, kips
\( V_p \) = component of the effective prestressing force in the direction of the applied shear, kips
\( \phi \) = resistance factor

Critical section near the supports is the greater of:

\[ 0.5d_{vc} \cot \theta, \text{ or, } d_e \]

where

\( d_e \) = effective shear depth

= distance between resultants of tensile and compressive forces, \((d_e - a/2)\) but not less than \( 0.9d_e \) or \( 0.72h \)
where

- \(d_e\) = the corresponding effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement = 76.25 in.
- \(a\) = equivalent depth of the compression block = 6.02 in. (from negative moment flexural design)
- \(h\) = total height of the section = 80.0 in.
- \(\theta\) = angle of inclination of diagonal compressive stresses, assumed = 32° (slope of compression field)

\[d_v = 76.25 - 0.5(6.02) = 73.24 \text{ in.}\]
\[\geq 0.9d_e = 0.9(76.25) = 68.63 \text{ in.}\]
\[\geq 0.72h = 0.72(80.0) = 57.60 \text{ in.}\]

Therefore, \(d_v = 73.24 \text{ in.}\)

The critical section near the support is the greater of:

- \(d_v = 73.24 \text{ in.} \quad \text{(Controls)}\)
- \(0.5d_v\cot\theta = 0.5(73.24)\cot32° = 58.60 \text{ in.}\)

Because the width of the bearing is not yet determined, the width of bearing was conservatively assumed to be equal to zero for the computation of the critical section of shear, as shown in Figure 9.6.11-1. Therefore the critical section in shear is at a distance of \(0.5 + 0.5 + 73.19/12 = 7.10 \text{ ft}\) from the centerline of the first interior support (pier).

\[x/L = 7.10/120 = 0.059L \text{ from the centerline of the first interior support (pier)}\]
Using values from Table 9.6.4-1, compute the factored shear force and bending moment at the critical section for shear (center span point 0.059), according to Strength I load combinations.

\[
V_u = 1.25(42.3 + 64.6 + 7.8) + 1.50(14.2) + 1.75(137.3) = 405.0 \text{ kips}
\]

\[
M_u = 1.25(272.7 + 417.1 - 139.6) + 1.50(-244.4) + 1.75(-1717.8) = -2685.0 \text{ ft-kips}
\]

or,

\[
V_u = 0.9(42.3 + 64.6 + 7.8) + 1.50(14.2) + 1.75(137.3) = 364.8 \text{ kips}
\]

\[
M_u = 0.9(272.7 + 417.1 - 139.6) + 1.50(-244.4) + 1.75(-1717.8) = -2877.6 \text{ ft-kips}
\]

When determining \(M_u\) at a particular section, it is conservative to take \(M_u\) as the highest factored moment that will occur at that section, rather than the moment corresponding to maximum \(V_u\) (LRFD Art. C5.8.3.4.2). Therefore,

\[
V_u = 405.0 \text{ kips}
\]

\[
M_u = -2877.6 \text{ ft-kips}
\]

The contribution of the concrete to the nominal shear resistance is:

\[
V_c = 0.0316 \beta b v d_v \quad \text{[LRFD Eq. 5.8.3.3-3]}
\]

Several quantities must be determined before this expression can be evaluated.

Calculate strain in the reinforcement, \(\varepsilon_x\):

\[
\varepsilon_x = \frac{M_u + 0.5N_u + 0.5(V_u - V_p)\cot \theta - A_p f_{p0}}{2(E_A A_s + E_p A_p)} \leq 0.001 \quad \text{[LRFD Eq. 5.8.3.4.2-2]}
\]

where

- \(N_u\) = applied factored normal force at the specified section = 0
- \(V_p\) = component of the effective prestressing force in the direction of the applied shear = (force per strand)(number of draped strands)(sin \(\psi\))
- Force per strand = 23.4 kips
- From Section 9.6.7.5, \(\psi = 7.2^\circ\)
- \(V_p = (23.4)(12)\sin 7.2^\circ = 35.2 \text{ kips}\)
- \(f_{p0}\) = a parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete (ksi). For pretensioned members, LRFD Article C5.8.3.4.2 indicates that \(f_{p0}\) can be taken as the stress in the strands when the concrete is cast around them, which is the jacking stress, \(f_{pj}\) or 0.75\(f_{pu}\).

\[
f_{p0} = 0.75(270.0) = 202.5 \text{ ksi}
\]

- \(A_p = a\) area of prestressing steel on the flexural tension side of the member. The flexural tension side of the member should be taken as the half-depth containing the flexural tension zone as illustrated in LRFD Figure 5.8.3.4.2-3.
- \(A_p = 12(0.153) = 1.836 \text{ in.}^2\)
- \(A_s = \text{area of nonprestressing steel on the flexural tension side of the member} = 15.52 \text{ in.}^2\)
\[ \varepsilon_x = \frac{2.877.6(12)}{73.24} + 0 + 0.5(405.0 - 35.2)(\cot 32^\circ) - 1.836(202.5) }{2[29,000(15.52) + 28,500(1.836)]} = +0.394 \times 10^{-3} \]

9.6.11.2.1 Shear Stress

\[ \nu_u = \frac{V_u - \phi V_p}{\phi_b d_v} \]

where

\( \nu_u \) = shear stress in concrete

\( b_v \) = effective web width of the beam = 6 in.

\( V_p \) = component of the effective prestressing force in the direction of the applied shear (calculated in Sect. 9.6.11.2.1)

\[ \nu_u = \frac{405.0 - 0.9(35.2)}{(0.9)(6)(73.24)} = 0.944 \text{ ksi} \]

\( (\nu_u/f'_c) = (0.944/7.0) = 0.135 \]

Having computed \( \varepsilon_x \) and \( \nu_u/f'_c \), find a better estimate of \( \theta \) from LRFD Table 5.8.3.4.2-1. Since the computed value of \( \nu_u/f'_c \) is likely to fall between two rows in the table, a linear interpolation may be performed. However, for hand calculations, interpolation is not recommended (LRFD Art. C5.8.3.4.2). The values of \( \theta \) in the lower row that bounds the computed value may be used. Similarly, the values of \( \beta \) in the first column to the right of the computed value may be used. For this example, the applicable row and column are the ones labeled “\( \leq 0.150 \)” and “\( \leq 0.50 \)”, respectively. The values of \( \theta \) and \( \beta \) contained in the cell of intersection of that row and column are:

\( \theta = 32.1^\circ \) which is close to assumed \( \theta \) of 32.0°.

Thus, no further iteration is needed. However, if the designer desires to go through further iteration, it should be kept in mind that the position of the critical section of shear and consequently the values of \( V_u \) and \( M_u \) will need to be based on the new value of \( \theta \), 32.1°.

\( \beta = 2.36 \)

where \( \beta \) = a factor indicating the ability of diagonally cracked concrete to transmit tension; a value indicating concrete contribution.

The nominal shear resisted by the concrete is:

\[ V_c = 0.0316 \beta \sqrt{f'_c} b_v d_v \]

\[ = 0.0316(2.36) \sqrt{7.0(6)(73.24)} = 86.7 \text{ kips} \]

9.6.11.3 Contribution of Reinforcement to Nominal Shear Resistance

9.6.11.3.1 Requirement for Reinforcement

Check if \( V_u > 0.5\phi(V_c + V_p) \)

\[ V_u = 405.0 \text{ kips} > 0.5\phi(V_c + V_p) = 0.5(0.9)(86.7 + 35.2) = 54.9 \text{ kips} \]

Therefore, transverse shear reinforcement should be provided.
\[ V_u / \phi \leq V_n = V_c + V_s + V_p \]

where

\[ V_s = (V_u / \phi) - V_c - V_p = (405.0 / 0.9) - 86.7 - 35.2 = 328.1 \text{ kips} \]

\[ V_s = \frac{A_v f_y d_v (\cot \theta + \cot \alpha) \sin \alpha}{s} \]

where

- \( A_v \) = area of shear reinforcement within a distance \( s \), in.\(^2\)
- \( s \) = spacing of stirrups, in.
- \( f_y \) = yield strength of shear reinforcement, ksi
- \( \alpha \) = angle of inclination of transverse reinforcement to longitudinal axis = 90°

Therefore, area of shear reinforcement (in.\(^2\)) within a spacing, \( s \), is:

\[ \text{req'd } A_v = \frac{(sV_s)}{(f_yd_v\cot \theta)} \]

\[ = s(328.1)/(60)(73.24\cot 32°) = 0.047(s) \text{ in.}^2 \]

if \( s = 12 \text{ in.} \), the required \( A_v = 0.56 \text{ in.}^2 \)

Try #5 double legs @ 12 in. spacing.

\[ A_v \text{ provided} = (2)(0.31)\left(\frac{12}{12}\right) = 0.62 \text{ in.}^2/\text{ft} > A_v \text{ required} = 0.56 \text{ in.}^2/\text{ft} \quad \text{O.K.} \]

\[ V_s \text{ provided} = \frac{(0.62)(60)(73.24)\cot 32°}{12} = 363.2 \text{ kips} \]

**Spacing of Reinforcement**

Check maximum spacing of transverse reinforcement: \( \text{[LRFD Art. 5.8.2.7]} \)

Check if \( v_u < 0.125f'_c \)

\[ v_u = 0.944 \text{ ksi} \]

Since \( v_u > 0.125f'_c \)

Then \( s \leq 12 \text{ in.} \leq 0.4d_c = 0.4(73.24) = 29.3 \text{ in.} \)

Therefore, \( s \leq 12 \text{ in.} \)

Actual spacing, \( s = 12 \text{ in.} \quad \text{O.K.} \)

**Minimum Reinforcement Requirement**

The area of transverse reinforcement should not be less than:

\[ 0.0316 \sqrt{f_c} \frac{b_s}{f_y} \]

\[ 0.0316 \sqrt{7.0} \left( \frac{6(12)}{60} \right) = 0.100 \text{ in.}^2 < A_v \text{ provided} \quad \text{O.K.} \]
In order to ensure that the concrete in the web of the beam will not crush prior to yielding of the transverse reinforcement, the LRFD Specifications give an upper limit of $V_n$.

$$V_n = 0.25f'_c b_d d_v + V_p$$  \[LRFD \text{ Eq. 5.8.3.3-2}\]

Comparing this equation with LRFD Eq. 5.8.3.3-2, it can be concluded that,

$$V_c + V_s \leq 0.25f'_c b_d d_v$$

$$86.7 + 328.1 = 414.8 \text{ kips} \leq 0.25(7.0)(6)(73.24) = 769.0 \text{ kips} \quad \text{O.K.}$$

Using the foregoing procedures, the transverse reinforcement can be determined at increments along the entire length of the beam.

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### 9.6.12 INTERFACE SHEAR TRANSFER

#### 9.6.12.1 Factored Horizontal Shear

At the strength limit state, the horizontal shear at a section on a per unit basis can be taken as:

$$V_h = \frac{V_u}{d_v}$$  \[(LRFD \text{ Eq. C5.8.4.1-1)}\]

where

- $V_h = $ horizontal factored shear force per unit length of the beam, kips/in.
- $V_u = $ factored shear force due to superimposed loads, kips
- $d_v = $ distance between resultants of tensile and compressive forces, $(d_e - a/2)$

The LRFD Specifications does not identify the location of the critical section. For convenience, it will be assumed here to be the same location as the critical section for vertical shear, at point 0.059 of the center span.

Using load combination Strength I:

$$V_u = 1.25(7.8) + 1.50(14.2) + 1.75(137.3) = 271.3 \text{ kips}$$

$$d_v = 73.24 \text{ in.}$$

Required $V_h = \frac{271.3}{73.24} = 3.70 \text{ kips/in.}$

Required $V_n = V_h/0.9 = 3.70/0.9 = 4.11 \text{ kips/in.}$

#### 9.6.12.3 Required Interface Shear Reinforcement

The nominal shear resistance of the interface plane is:

$$V_n = c A_{cv} + \mu[A_{sf} f_y + P_e]$$  \[LRFD \text{ Eq. 5.8.4.1-1)}\]

where

- $c = $ cohesion factor, ksi  \[LRFD \text{ Art. 5.8.4.2}\]
- $\mu = $ friction factor  \[LRFD \text{ Art. 5.8.4.2}\]
- $A_{cv} = $ area of concrete engaged in shear transfer, in.$^2$
- $A_{sf} = $ area of shear reinforcement crossing the shear plane, in.$^2$
Pc = permanent net compressive force normal to the shear plane, kips
f’y = shear reinforcement yield strength, ksi

For concrete placed against clean, hardened concrete with the surface intention-
ally roughened:  [LRFD Art. 5.8.4.2]
c = 0.1 ksi
μ = 1.0λ
where λ = 1.0 for normal weight concrete

The actual contact width, bν, between the deck and the beam is 42 in. Therefore,
Av = (42 in.)(1 in.) = 42 in.²/in.

LRFD Eq. 5.8.4.1-1 can be solved for Av as follows:
4.12 = 0.1(42) + 1.0[Av(60) + 0]
Av < 0

Since the resistance provided by cohesion is higher than the applied force, pro-
vide the minimum required interface reinforcement.

Minimum shear reinforcement, Av ≥ (0.05bν)/f’y  [LRFD Eq. 5.8.4.1-4]
where bν = width of the interface

From the design of vertical shear reinforcement, a #5 double-leg bar at 12-in. spacing
is provided from the beam extending into the deck. Therefore, Av = 0.62 in.²/ft.

Av = (0.62 in.²/ft) > (0.05bν)/f’y = 0.05(42)/60 = 0.035 in.²/in. = 0.42 in.²/ft O.K.

Consider further that LRFD Article 5.8.4.1 states that the minimum reinforcement
requirement may be waived if Vn/Av < 0.100 ksi.

4.11 kips/in./42.0 in. = 0.098 ksi < 0.100 ksi

Therefore, the minimum reinforcement requirement could be waived had it governed.

Provided Vn = (0.1)(42) + 1.0 \left[ \frac{0.62}{12} \frac{(60) + 0}{(60)} \right] = 7.3 kips/in.

0.2f’cAv = 0.2(4.0)(42) = 33.6 kips/in.
0.8Av = 0.8(42) = 33.6 kips/in.

Vn provided ≤ 0.2f’cAv O.K.  [LRFD Eq. 5.8.4.1-2]
≤ 0.8Av O.K.  [LRFD Eq. 5.8.4.1-3]

The LRFD Specifications state that if the reaction force or the load at the maximum
moment location introduces direct compression into the flexural compression face of
the member, the area of longitudinal reinforcement on the flexural tension side of the
member need not exceed the area required to resist the maximum moment acting
alone.
This reason that the longitudinal reinforcement requirement is relaxed for this condition, is based on the following explanation. At maximum moment locations, the shear force changes sign and, hence, the inclination of the diagonal compressive stresses also changes. At direct supports and point loads, this change of inclination is associated with a fan-shaped pattern of compressive stresses radiating from the point load or the direct support. This fanning of the diagonal stresses reduces the tension in the longitudinal reinforcement caused by the shear, i.e., angle $\theta$ becomes steeper.

The conditions mentioned above exist at the interior supports. Directly over the support, the angle $\theta$ becomes 90° and the contribution of shear to the longitudinal reinforcement requirement is zero. Therefore, at this location, the longitudinal reinforcement is sized only for the moment applied to the section and there is no need to check the minimum longitudinal reinforcement requirement.

However, for sections within a distance of $(d, \cot \theta)/2$ from the interior supports, the shear will again affect the required longitudinal reinforcement and the requirement must be checked. It should be noted that at locations near the interior supports of continuous members, the minimum longitudinal reinforcement requirement is used to check the quantity of reinforcement in the deck. The longitudinal reinforcement requirement must also be checked for the prestressing strands at the simply-supported ends of continuous span units. Refer to Design Example 9.4, Section 9.4.13.

\[ [\text{LRFD Art. 5.10.10}] \]

\[ [\text{LRFD Art. 5.10.10.1}] \]

Design of the anchorage zone reinforcement is computed using the force in the strands just before transfer.

Force in the strands before transfer $= F_{pi} = 44(0.153)(202.5) = 1,363.2$ kips

The bursting resistance, $P_r$, should not be less than 4.0% of $F_{pi}$.

\[ P_r = f_s A_s \geq 0.04 f_{pi} = 0.04(1,363.2) = 54.5 \text{ kips} \]

\[ [\text{LRFD Arts. 5.10.10.1 and C3.4.3}] \]

where

\[ A_s = \text{total area of transverse reinforcement located within the distance } h/4 \text{ from the end of the beam, in.}^2 \]

\[ f_s = \text{stress in steel, but not taken greater than 20 ksi} \]

Solving for the required area of steel, $A_s = 54.5/(20) = 2.73$ in.$^2$

At least 2.73 in.$^2$ of vertical transverse reinforcement should be provided at the end of the beam for a distance equal to one-fourth of the depth of the beam, $h/4 = 72/4 = 18.0$ in.

The shear reinforcement was determined in Section 9.6.11 to be #5 (double legs)@ 10 in. However, the minimum vertical reinforcement criteria controls. Therefore, for a distance of 18.0 in. from the end of the member, use 5 #5 @ 4 in. The reinforcement provided is $5(2)(0.31) = 3.10$ in.$^2 > 2.73$ in.$^2$, O.K.
**9.6.14.2 Confinement Reinforcement**

For a distance of 1.5d = 1.5(72) = 108 in., from the end of the beam, reinforcement is placed to confine the prestressing steel in the bottom flange. The reinforcement should not be less than #3 deformed bars, with spacing not exceeding 6.0 in., and shaped to enclose the strands.

**9.6.15 Deflection and Camber**

Deflections are calculated using the modulus of elasticity of concrete calculated in Section 9.6.2 and the moment of inertia of the non-composite precast beam.

### 9.6.15.1 Deflection Due to Prestressing Force at Transfer

Force per strand at transfer = 28.2 kips

\[ \Delta_p = \frac{P_i E I}{EI} \left( \frac{e_c^2}{8} - \frac{e^2}{6} \right) \]

where

- \( P_i = \) total prestressing force at transfer = 44(28.2) = 1,240.8 kips
- \( e_c = \) eccentricity of prestressing force at midspan = 30.78 in.
- \( e' = \) difference between eccentricity of prestressing force at midspan and end, as shown in Figure 9.6.15.1-1
  \[ e' = e_c - e_c = 30.78 - 19.42 = 11.36 \text{ in.} \]
- \( a = \) distance from end of beam to harp point = 35.5 ft
- \( L = \) beam length = 119.0 ft

\[ \Delta_p = \frac{1,240.8}{4,496(545,894)} \left( \frac{30.78(119x12)^2}{8} - \frac{11.36 (35.5x12)^2}{6} \right) = 3.79 \text{ in.} \]

### 9.6.15.2 Deflection Due to Beam Self-Weight

\[ \Delta_g = \frac{5wL^4}{384E_c I} \]

where \( w = \) beam self-weight, kip/ft

Deflection due to beam self-weight at transfer:

- \( L = \) overall beam length = 119 ft
Deflection due to beam self-weight at erection:
\[ \Delta_g = \frac{5 \left( \frac{0.799}{12} \right) (119 \times 12)^4}{(384)(4,496)(545,894)} = 1.47 \text{ in.} \]

Deflection due to beam self-weight at erection:
\[ \Delta_g = \frac{5 \left( \frac{0.799}{12} \right) (118 \times 12)^4}{(384)(4,496)(545,894)} = 1.42 \text{ in.} \]

\[ \Delta_s = \frac{5wL^4}{384E_cI} \]

where
- \( w \) = deck slab plus haunch weights, kip/ft
- \( L \) = span length between centerlines of bearings, ft
- \( E_c \) = modulus of elasticity of precast beam at 28 days

\[ \Delta_s = \frac{5 \left( \frac{1.222}{12} \right) (118 \times 12)^4}{(384)(5,072)(545,894)} = 1.93 \text{ in.} \]

\[ \Delta_{b\text{vws}} = 0.048 \text{ in.} \]

(This value was calculated using a continuous beam program.)

For midspan:
At transfer, \( (\Delta_p + \Delta_g) = 3.79 - 1.47 = 2.32 \text{ in.} \)
Total deflection at erection, using PCI multipliers (see the PCI Design Handbook)
\[ = 1.8(3.79) - 1.85(1.42) = 4.20 \text{ in.} \]

Long-Term Deflection
LRFD Article 5.7.3.6.2 states that the long-term deflection may be taken as the instantaneous deflection multiplied by a factor of 4.0, if the instantaneous deflection is based on the gross moment of inertia. However, a factor of 4.0 is not appropriate for this type of precast construction. Therefore, it is recommended that the designer follow the guidelines of the owner agency for whom the bridge is being designed, or undertake a more rigorous time-dependent analysis.

Live load deflection is not a required check, according to the provisions of the LRFD Specifications. Further, live load deflections are usually not a problem for prestressed concrete I- and bulb-tee shapes especially when they are constructed to act as a continuous structure under superimposed loads. If the designer chooses to check deflection, the following recommendations are from the LRFD Specifications.

Live load deflection limit: Span/800 = (120)(12)/800 = 1.80 in. [LRFD Art. 2.5.2.6.2]
If the owner invokes the optional live load deflection criteria specified in LRFD Article 2.5.2.6.2, the deflection is the greater of:

- that resulting from the design truck alone, or,
- that resulting from 25% of the design truck taken together with the design lane load

The *LRFD Specifications* state that all beams may be assumed to be deflecting equally under the applied live load and impact. Therefore, the distribution factor for deflection is calculated as follows:

\[
\text{(number of lanes/number of beams)} = \frac{3}{4} = 0.75
\]

However, it is more conservative to use the distribution factor for moment. The live load deflection may be conservatively estimated using the following formula:

\[
D = \frac{5L^2}{48EI} \left[ M_a - 0.1(M_a + M_b) \right]
\]

(Eq. 9.6.15.6-1)

where

- \( M_a \) = the maximum positive moment
- \( M_a \) and \( M_b \) = the corresponding negative moments at the ends of the span being considered.

The live load combination specified in LRFD Article 3.6.1.3.2 calls for the greater of design truck alone or 0.25 design truck plus lane load.

In this example, a conservative approximation may be made by using the positive moment for Service III load combination, 0.8 truck plus lane load, and by ignoring the effect of \( M_a \) and \( M_b \).

\[
\Delta_L = \frac{5(120 \times 12)^2}{48(5,072)(1,097,252)} \left[ 0.8 \times 2,115.0 \times 12 \right] = 0.79 \text{ in.} \downarrow < 1.80 \text{ in.} \quad \text{O.K.}
\]