i. Basics of Bridge Dynamic Analysis

• Single Degree-of-Freedom System

\[ m\ddot{y}(t) + c\dot{y}(t) + ky(t) = f(t) \]

• Multiple Degree-of-Freedom System

\[ M\ddot{u}(t) + C\dot{u}(t) + K\ddot{u}(t) = f(t) \]
ii. Vehicle-Bridge Interaction

- Aim: To analyze the effects of highway vehicle- or train-induced vibrations for impact analysis or fatigue or cracking analysis.

- In the modeling process, only the superstructure is of concern to be included in a beam, grid, or more sophisticated shell model.

- The contact force interacting with two substructures, the bridge and the vehicle/train, is time-dependent and nonlinear since the contact force might move from time to time.

- All vehicles possess the suspension system, either in air suspensions or steel-leaf suspensions. Air suspensions use hydraulic shock absorbers for damping while steel-leaf suspensions use steel strips to provide damping through Coulomb friction between steel strips.

iii. Pedestrian Bridge Vibrations

- Bridge can be in beam, grid, or more sophisticated shell model
- Truck can be modeled in details
- Study
  1. dynamic analysis of bridge due to moving vehicles
  2. fatigue life assessment
  3. quantification bridge durability
  4. heavy vehicle load investigation

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Figure 17.6 - Recommended peak acceleration for human comfort for vibrations due to human activities (Allen and Murray, 1993; AISC 1997)
Case Study: Millennium Bridge

- Crosses River Thames, London, England
- 474’ main span, 266’ north span, 350’ south span
- Superstructure supported by lateral supporting cables (7’ sag)
- Bridge opened June 2000, closed 2 days later

\[ \sum f(t) = P \left[ 1 + \sum a_i \cos(2\pi f_{step} t + \phi_i) \right] \]

Millennium Bridge

- Severe lateral resonance was noted (0.25g)
- Predominantly noted during 1st mode of south span (0.8 Hz) and 1st and 2nd modes of main span (0.5 Hz and 0.9 Hz)
- Occurred only when heavily congested
- Phenomenon called “Synchronous Lateral Excitation”

Possible solutions
- Stiffen the bridge
  - Too costly
  - Affected aesthetic vision of the bridge
- Limit pedestrian traffic
  - Not feasible
- Active damping
  - Complicated
  - Costly
  - Unproven
- Passive damping

Passive Dampers
- 37 viscous dampers installed
- 19 TMDs installed
Millennium Bridge

- Results
  - Provided 20% critical damping.
  - Bridge was reopened February, 2002.
  - Extensive research leads to eventual updating of design code.

**iv. Bridge Earthquake Analysis**

- **Figure 17.7** - Four distinct analytical procedures for seismic analysis.

- **Methods of Analysis**
  - Uniform Load Method (single mode, elastic)
  - Single Mode Spectral Analysis Method (single mode, elastic)
  - Multi Mode Spectral Analysis Method (multiple mode, elastic)
  - Elastic Time History (multiple mode, elastic)
  - Nonlinear Static Procedure (single DOF, nonlinear)
  - Nonlinear Dynamic Procedure (multi DOF, nonlinear)
iv. Bridge Earthquake Analysis

\[ m \ddot{u} + c \dot{u} + ku = -m \tau u_g(t) \]

Table 17.1 – Bridge seismic analysis types recommended by Caltrans

<table>
<thead>
<tr>
<th>Bridge Classification</th>
<th>Nonlinear Static</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equivalent Static Analysis (ESA)</td>
<td>Incremental Static Analysis (Pushover)</td>
</tr>
<tr>
<td>Ordinary Standard</td>
<td>A</td>
<td>R</td>
</tr>
<tr>
<td>Ordinary Nonstandard</td>
<td>N</td>
<td>R</td>
</tr>
<tr>
<td>Important</td>
<td>N</td>
<td>R</td>
</tr>
</tbody>
</table>

N: Not acceptable analysis type
A: Acceptable analysis type
R: Acceptable and strongly recommended analysis type, not necessarily comprehensive

Table 17.2 - Performance Approach

<table>
<thead>
<tr>
<th>Probability of Exceedance For Design Earthquake Ground Motions</th>
<th>Performance Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rare Earthquake (MCE) 3% in 75 years</td>
<td>Life Safety Operational</td>
</tr>
<tr>
<td>Service</td>
<td>Significant disruption Immediate</td>
</tr>
<tr>
<td>Damage</td>
<td>Significant Minimal</td>
</tr>
<tr>
<td>Frequent of Expected Earthquake 50% in 75 years</td>
<td>Service Immediate Immediate</td>
</tr>
<tr>
<td>Damage</td>
<td>Minimal Minimal to none</td>
</tr>
</tbody>
</table>

iv. Bridge Earthquake Analysis

Static push-over analysis is an attractive tool for performance assessment because it involves less calculation than nonlinear dynamic analysis, and uses a response spectrum rather than a suite of ground accelerograms. Its main weakness is that it uses static analysis to capture dynamic effects, and hence may be inaccurate.

Figure 17.9 - Linear vs. Nonlinear time history analysis for a 9-Span bridge model (THA – Time-History Analysis).

Figure 17.11 – Types of Analytical Models

Figure 17.12 – Illustration of a spine model
iv. Bridge Earthquake Analysis

The superstructure is idealized using equivalent linear elastic beam-column elements.

- Effective bending stiffness - the moment of inertia \( I_{\text{eff}} \)
  
  \[ E_c I_{\text{eff}} = \frac{M_y}{\phi_y} \quad (17.18) \]

- Shear stiffness parameter \((GA)_{\text{eff}}\) for pier walls in the strong direction
  
  \[ (GA)_{\text{eff}} = G_c A_{cw} \frac{l_{\text{eff}}}{l_g} \quad (17.19) \]

- Effective torsional moment of inertia \( J_{\text{eff}} \)
  
  \[ J_{\text{eff}} = 0.2J_g \quad (17.20) \]

iv. Bridge Earthquake Analysis

**Soil Stiffness:** Abutment longitudinal stiffness \( K_{\text{eff}} \) due to passive soil pressure uniformly distributed over the height \( (H_w) \) and width \( (W_w) \) of the backwall or diaphragm.

\[ P_p = p_p H_w W_w \quad (17-21) \]

- For integral- or diaphragm-type abutments, equivalent linear secant stiffness, \( K_{\text{eff}} \) is
  
  \[ K_{\text{eff}} = \frac{P_p}{(F_w H_w)} \quad (17-22) \]
### iv. Bridge Earthquake Analysis

#### Table 17.4 – Stiffness of Circular Surface Footing ($K_0$)

<table>
<thead>
<tr>
<th>Degree of Freedom</th>
<th>Equivalent Radius $R$</th>
<th>Stiffness $K_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Translation</td>
<td>$R_0 = \sqrt{\frac{4BL}{\pi}}$</td>
<td>$4GR/(1 - v)$</td>
</tr>
<tr>
<td>Lateral Translation (Both)</td>
<td>$R_1 = \frac{4BL(4L^2 + 4L^2)}{6\pi}$</td>
<td>$16GR^2/3$</td>
</tr>
<tr>
<td>Torsion Rotation</td>
<td>$R_2 = \frac{(2B)^2(2L)}{3\pi}$</td>
<td>$8GR^2/(1 - v)$</td>
</tr>
<tr>
<td>Rocking about 2</td>
<td>$R_3 = \frac{(2B)(2L)^{3/2}}{3\pi}$</td>
<td>$8GR^2/(1 - v)$</td>
</tr>
</tbody>
</table>

#### Figure 17.13 – Half-spaced method for spread footings (NHI 1996)

#### Figure 17.14 – Shape factor ($\alpha$) for rectangular footing (NHI 1996)

#### Figure 17.15 – Embedment factor ($\beta$)

#### Figure 17.16 – Modeling soil flexibility

<table>
<thead>
<tr>
<th>Foundation Type</th>
<th>Modeling Method I</th>
<th>Modeling Method II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spread Footing</td>
<td>Rigid</td>
<td>Foundation spring required if footing flexibility</td>
</tr>
<tr>
<td></td>
<td></td>
<td>contributes more than 20% to pier displacement</td>
</tr>
<tr>
<td>Pile Footing with Pile Cap</td>
<td>Rigid</td>
<td>Foundation spring required if footing flexibility</td>
</tr>
<tr>
<td></td>
<td></td>
<td>contributes more than 20% to pier displacement</td>
</tr>
<tr>
<td>Pile Bent/Drilled Shaft</td>
<td>Estimate depth to fixity</td>
<td>Estimate depth to fixity or soil springs based on P-y</td>
</tr>
<tr>
<td></td>
<td></td>
<td>curves</td>
</tr>
</tbody>
</table>

#### Figure 17.17 – Pushover force-deformation (P-d) or moment rotation (M-\(\Theta\)) curve

- Plastic rotation capacity angle, $a$ from $B$ to $C$
- Ultimate rotation angle, $b$ from $B$ to $E$ ($1.5$ times the plastic angle)

**Modal Pushover Analysis (MPA)** - pushover analyses are carried out separately for each significant mode, and the contributions from individual modes to calculated response quantities (displacements, drifts, etc.) are combined using an appropriate combination rule (SRSS or CQC).
iv. Bridge Earthquake Analysis

Figure 17.18 – Plan and elevation views of illustration example 1 (FHWA 1996)

Figure 17.20 - Finite element model of illustration example 1 (FHWA 1996)

Figure 17.22 – Deformed shape of Mode 2 (T2 = 0.5621s)

iv. Bridge Earthquake Analysis

Figure 17.23 – Comparison of different methods by deck displacement

Figure 17.24 – Comparison of different methods by deck displacement (PGA = 0.30g)

v. Blast loading Analysis

Analysis for blast-resistant design:
1) **Equivalent static analysis** (neglecting the inertial effects of members in motion)
2) **Single-degree-of-freedom (SDOF) linear/nonlinear dynamic analysis** (considered the current state-of-practice method which ignores higher-order failure, allowing for the analysis of a large number of load cases, bridge types, and structural configurations)
3) **Multi-degree-of-freedom (MDOF), uncoupled/ coupled, nonlinear dynamic analysis**

Modified Friedlander exponential decay equation

\[ p(t) = p_m [1 - \frac{t}{t_p}] e^{-\alpha t/t_p} \]

Figure 17.10 – Pressure time-history for free field blast (TMS-1300 1990)
vi. Wind Analysis

Wind induces two typical aerodynamic phenomena in long span bridges:
- **Fluttering** is an aerodynamic instability that may cause failure of the bridge
- **Buffeting** is an aerodynamic random vibration that may lead to fatigue damage, excessive vibration, and large displacements.

“Torsional flutter”: Tacoma Narrows Bridge

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vi. Wind Analysis

- Aerodynamic loading is commonly separated into self-excited and buffeting forces.
- The self-excited forces acting on a unit deck length are expressed as a function of the so-called flutter derivatives (Scanlan 1978a), which can be expressed as:
  \[
  \{F_{se}\} = \begin{pmatrix} L_{se} \\ D_{se} \\ M_{se} \end{pmatrix} = U^2 [F_d](q) + U^2 [F_v](q) \quad (17.16)
  \]
- The buffeting forces (Scanlan 1978b) are expressed as
  \[
  \{F_b\} = \begin{pmatrix} L_b \\ D_b \\ M_b \end{pmatrix} = \bar{U}^2 [C_b](\eta)
  \]

---

vi. Wind Analysis

- Wind is a dynamic load. However, it is generally approximated as a uniformly distributed static load on the exposed area of a bridge.
- For typical girder and slab bridges (based on 100 mph)
  - SPAN ≥ 125\(\): 0.05 ksf, transverse, 0.02 ksf, longitudinal
  - SPAN < 125\(\): 0.10 ksf, transverse, 0.04 ksf, longitudinal
- For the strength limit state, wind on the structure is considered for the Strength III and Strength V load combinations. For Strength III, the load factor for wind on structure is 1.40 but live load is not considered. Therefore, for this design example, only the Strength V load combination will be investigated. The Strength III load combination is likely to be more critical when checking wind load effects during construction.

<table>
<thead>
<tr>
<th>STRENGTH-III</th>
<th>(\gamma_p)</th>
<th>1.00</th>
<th>1.40</th>
</tr>
</thead>
<tbody>
<tr>
<td>STRENGTH-V</td>
<td>(\gamma_p)</td>
<td>1.35</td>
<td>1.00</td>
</tr>
</tbody>
</table>