Introduction

Following subjects are covered:
- Pure torsion
- Shear center
- Torsional differential equation
- Torsional stresses
- Analogy between torsion and plane bending
- Open vs closed thin-wall sections

Reading:
- Chapters 8 of Salmon & Johnson
- AISC Design Guide 9 – Torsional Analysis of Structural Steel Members

Torsion of a Prismatic shaft

\[ T = \int r^2 \frac{d\Phi}{dz} GdA = GJ \frac{d\Phi}{dz} = GJ \Phi \]

(S & J 8.2.5)

\[ \tau_t = \frac{\Phi}{t} = G\tau t/J \]

(S & J 8.2.6)

Torsion of Homogeneous Sections

- For Circular Section w/diameter t
  \[ J = \text{polar moment of inertia} = \frac{\pi t^4}{32} \]
  \[ \tau_t = \frac{16 T}{\pi t^3} \]
  (S & J 8.2.8)

- For rectangular section w/thickness t
  \[ \tau_t = \frac{T}{t/J} \]
  \[ J = K_2 b t^3 \]
  (S & J 8.2.13)

- For I-shaped, Channel, and Tee Section
  \[ J = \sum 1/3 b t^3 \]
  (S & J 8.2.14)
### Torsion of a Rectangular Section

Shear strain = change in this angle =
\[ \gamma \approx 2 \left( \frac{db}{dz} \right) \left( \frac{t}{2} \right) = \frac{db}{dz} \frac{t}{2} \]

### Stresses on Thin-wall Open Sections in Bending

Shear Center – forces acting through the shear center will cause no torsional stresses

### Shear Center

- **a. Y-axis**
  - let \( V_y = 0 \)
  - \( y_0 = -\frac{1}{V_x} \int_0^y (\pi) r ds \) (S & J 8.4.3)

- **b. X-axis**
  - let \( V_x = 0 \)
  - \( x_0 = \frac{1}{V_y} \int_0^x (\pi) r ds \) (S & J 8.4.4)

### Channel of Example 8.4.1

See handout for both (a) Force method and (b) Numerical method
Common Torsional Loadings

- Pure Torsion (Resisting moment of an unrestrained cross section)
- Warping Torsion (Resisting moment of a restrained cross section)
- Total Torsional Resisting Moment
  \[ M = G J \Phi' - E C_w \Phi'' \]

Solution to the Torsional Differential Equation

- Particular solution
  \[ \Phi_p = A \sinh \beta z + B \cosh \beta z + C \]
  \[ \Phi_p = C_1 + C_2 z + C_3 z^2 + \ldots \ldots \]

- Loading Condition
  - Constant
  - Uniformly distributed
  - Linearly varying

- Boundary Conditions
  - \( \Phi = 0 \) No rotation Pinned or fixed end
  - \( \Phi' = 0 \) Section cannot warp Fixed end
  - \( \Phi'' = 0 \) Section can warp freely Pinned or free end

Torsional Stresses

- Pure Torsional Shear Stresses \( \tau_t = G t \Phi' \)
- Warping Shear Stresses \( \tau_{ws} = -E S_{ws} \Phi'' / t \)
- Warping Normal Stresses \( \sigma_{ws} = E W_{ns} \Phi'' \)

- Torsional Properties
  - \( J = \) Torsional Constant (in\(^4\))
  - \( C_w = \) Warping Constant (in\(^6\))
  - \( W_{ns} = \) normalized warping function at pt. S (in\(^2\))
  - \( S_{ws} = \) Warping statical moment (in\(^4\))
  - \( Q = \) Statical moment (in\(^3\))
Torsional Case of Example 8.5.1

Case of Example 8.5.1. Concentrated torsional moment at midspan; torsionally simply supported

Direction and Distribution of Shear Stress in I-shaped Sections

Normal and Shear Stresses of an Open Section

Normal stress distribution when warping is restrained

Warping of Cross-section

- Total normal stress: a combination of axial stress, major axis bending stress, lateral bending stress, and warping normal stress (left).
- Total shear stress is the sum of vertical shear stress, horizontal shear stress, St. Venant torsional shear stress (generally relatively small), and warping shear stress (right).
Data for Example 8.5.2

Analogy between Flexure and Torsion

Comparison of lateral shear on flange due to warping torsion with that from simple lateral flexure analogy

Example 8.6.1
By Flexural Analogy

Table 8.5.1 Summary of Stresses for Example 8.5.2
Shear Flow in a Closed Thin Wall Section

\( \tau_t \) (shear flow) is constant with walls assumed thin.

Forces on a Cut Thin-wall Section

\[ T = GJ\theta \]
\[ J = \frac{4A^2}{\int ds/t} \]

Normal and Shear stresses of a Close Section

Figure 8.9 illustrates the general box girder normal stresses which can occur in a curved or skewed box-shaped girder.

Closed box sections are extremely efficient at carrying torsion by means of St. Venant torsional shear flow (Figure 8.10). When combined with vertical shear in the webs, this shear flow is always subtractive in one web and additive in the other.

Sections for Example 8.10.1

- 10" diam. pipe
  \( A = 15.7 \text{ sq in.} \)
  \( J = 393 \text{ in}^4 \)

- 12 \times 6 \text{ structural tubing}
  \( A = 15.9 \text{ sq in.} \)
  \( J = 288 \text{ in}^4 \)

- Channel
  \( A = 16.0 \text{ sq in.} \)
  \( J = 4.1 \text{ in}^4 \)