Dynamic Response of Pedestrian Bridges/Floor Vibration and Various Methods of Vibration Remediation

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Presentation

- Brief overview of structural vibration
- Understanding how people perceive and react to unwanted vibration
- General response of pedestrian bridges to vibration
- Various design guidelines
- Damping
- Bridge case study

Structural Vibration

- Stiffness Force: \( F_S = -kx \)
- Damping Force: \( F_D = -cx' \)
- External Force: \( F_E(t) \)
- Inertial Force

\[
mx''(t) + cx'(t) + kx(t) = F_e(t)
\]
### Structural Vibration

**Free Vibration**

\[ mx''(t) + cx'(t) + kx(t) = 0 \quad x(0) = 0 \quad x'(0) = 0 \]

**Solution**

\[ x(t) = e^{-\zeta \omega_n t} \left\{ x_o \cos(\omega_d t) + \frac{\zeta \omega_n x_o + x'(0)}{\omega_n \sqrt{1 - \zeta^2}} \sin(\omega_d t) \right\} \]

\[ x'(t) = e^{-\zeta \omega_n t} \left\{ x'_o \cos(\omega_d t) - \frac{\omega_n x_o + \zeta x'_o}{\sqrt{1 - \zeta^2}} \sin(\omega_d t) \right\} \]

\[ \omega_n^2 = \frac{k}{m} \quad 2\zeta \omega_n = \frac{c}{m} \quad \omega_d = \omega_n \sqrt{1 - \zeta^2} \]

### Structural Vibration

**Forced Vibration**

\[ mx''(t) + cx'(t) + kx(t) = F_e(t) \]

**Solution**

\[ x(t) = x_o e^{-\zeta \omega_n t} \left\{ x'_o \cos(\omega_d t) - \frac{\omega_n x_o + \zeta x'_o}{\omega_n \sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_d t) \right\} + \]

\[ x'(t) = x'_o e^{-\zeta \omega_n t} \left\{ x'o \cos(\omega_d t) - \frac{\omega_n x_o + \zeta x'_o}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_d t) \right\} + \]

\[ x''(t) = x''_o e^{-\zeta \omega_n t} \left\{ x''_o \cos(\omega_d t) + \frac{\omega_n x_o + \zeta x'_o}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_d t) \right\} \]

### Human Perception

**Steady State Forcing Function**

\[ F_e(t) = F_o \sin(\omega_o t) \]

**Solution**

\[ x_{ss}(t) = \frac{F_o}{\sqrt{k(1 - r^2)^2 + (2\zeta r)^2}} \left\{ -2\zeta r \cos(\omega_o t) + (1 - r^2) \sin(\omega_o t) \right\} \]

\[ x'_{ss}(t) = \frac{F_o \omega_o}{\sqrt{k(1 - r^2)^2 + (2\zeta r)^2}} \left\{ (1 - r^2) \cos(\omega_o t) + 2\zeta r \sin(\omega_o t) \right\} \]

**Human Response**

- Present: Not perceived
- Perceived: Does not annoy
- Perceived: Annoys and disturbs
- Perceived: Severe enough to cause illness

**Peak acceleration limits**

<table>
<thead>
<tr>
<th>Situation</th>
<th>Building in Strong Wind</th>
<th>Public Transportation</th>
<th>Building in Earthquake</th>
<th>Amusement Park Ride</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Acceleration (% g)</td>
<td>0.5 – 10</td>
<td>51 – 102</td>
<td>204 – 458</td>
<td>≤458</td>
</tr>
</tbody>
</table>
Peak Acceleration for Human Comfort for Vibrations

Design Guide 11 Fig. 2.1 Recommended peak acceleration for human comfort for vibrations due to human activities

Pedestrian Bridge Response

- Vertical Vibration
- Lateral Vibration

Pedestrian Bridge Response

- Vertical Vibration (also apply to floor vibration)

\[ \sum F(t) = P[1 + \sum \alpha_i \cos(2\pi f_{\text{step}} t + \phi_i)] \]

- P = Person’s weight
- \( \alpha_i \) = Dynamic coefficient for the harmonic force
- i = Harmonic multiple (1, 2, 3…)
- \( f_{\text{step}} \) = Step frequency of activity
- t = time
- \( \phi_i \) = Phase angle for the harmonic

Pedestrian Bridge Response

- Lateral Vibration

Synchronous Lateral Excitation
Design Guidelines

- Serviceability (i.e. functional, usable)
  - Stiffness
  - Resonance

- Resonance
  - Frequency matching
  - Uncomfortable/damaging vibration
  - Unfavorable perception

AVOID RESONANCE!

Design Guidelines

- Natural Frequency
  \[ f = \frac{\pi}{2} \sqrt{\frac{\text{stiffness}}{\text{mass}}} = \frac{\pi}{2} \sqrt{\frac{g}{\Delta}} \]
  Ex.) Uniformly loaded simple beam:
  \[ f_n = 0.18 \sqrt{\frac{g}{\Delta}} \]
  \[ \Delta = \frac{5wL^4}{384EI} \]

Bridge Design Guidelines

- Natural Frequency (Vertical Vibration)
  - Limiting values (Bridge)
    - AASHTO
      - \( f \geq 3.0 \text{ Hz} \)
      - \( f \geq 2.85\ln(180/W) \)
      - \( W \geq 180e^{0.35f} \)
      - Special cases: \( f \geq 5.0 \text{ Hz} \)
      - \( f_o \geq 5.0 \text{ Hz} \)
      - \( a_{\text{max}} \leq 0.5(f_o)^{1/2} \text{ m/s}^2 \)
      - \( a_{\text{max}} = 4\pi^2 f_o^2 y_s K\psi \)
      - \( F = 180\sin(2\pi f_o T) \text{ N} \)
      - \( v_t = 0.9 f_o \text{ m/s (} \geq 2.5 \text{ m/s per Ontario Code)} \)

<table>
<thead>
<tr>
<th>Ratio ( h/l )</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.6</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>0.6 or less</td>
<td>0.9</td>
</tr>
</tbody>
</table>
British Design Guidelines

\[ a_{\text{max}} = 4\pi^2 f_0^2 y_s K\Psi \]

Design Guidelines

- Natural Frequency (Vertical Vibration)
  - Limiting values
  - AASHTO
  - British Code (1978 BS 5400)
  - AISC/CISC Steel Design Guide Series 11

\[
\frac{a_p}{g} = \frac{P_o e^{-0.35 f_n}}{\beta W} \leq 1.5\% \text{ (Indoor walkways)}
\]

\[
\leq 5.0\% \text{ (Outdoor bridges)}
\]

Table 19. Logarithmic decrement of decay of vibration \( \delta \)

<table>
<thead>
<tr>
<th>Bridge superstructure</th>
<th>( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel with asphalt or epoxy surfacing</td>
<td>0.03</td>
</tr>
<tr>
<td>Composite steel/concrete</td>
<td>0.04</td>
</tr>
<tr>
<td>Prestressed and reinforced concrete</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Response to Sinusoidal Force

Resonance response function

\[
a/g = \frac{R_\text{f} P}{\beta W} \cdot \cos(2\pi f_{\text{on}} t)
\]

Simplified design criterion

\[
\frac{a_p}{g} = \frac{P_0}{\beta W} e^{-0.35 f_n} \leq \frac{a_0}{g}
\]

\( a/g\), \( a_0/g \) = ratio of the floor acceleration to the acceleration of gravity; acceleration limit \( f_n \) = natural frequency of floor structure

\( P_0 \) = constant force equal to 0.29 kN (65 lb.) for floors and 0.41 kN (92 lb.) for footbridges

Steel Framed Floor System

- The combined Beam or joist and girder panel system
- Spring in parallel (a & b) or in series (c & d)

System frequency

\[
\frac{1}{f_n^2} = \frac{1}{f_j^3} + \frac{1}{f_g^3}
\]

\( f_s \) = 0.18 \sqrt{\frac{8}{(\Delta + \Delta_\delta)}}

Equivalent panel weight

\[
W = \frac{\Delta_j}{\Delta_j + \Delta_\delta} W_j + \frac{\Delta g}{\Delta_j + \Delta_\delta} W_g
\]
Design Guidelines

• Natural Frequency (Lateral Vibration)
  – Step frequency ½ vertical
  – 1996 British Standard BS 6399
    • 10% vertical load
  – Per ARUP research
    • \( f \geq 1.3 \) Hz
  – Rule of thumb
    • Lateral limits ½ vertical limits

Design Guidelines

• Stiffening
  – Uneconomical
  – Unsightly

• Damping
  – Inherent damping \( \leq 1\% \)
  – Mechanical damping devices

Damping

• Coulomb Damping

\[
F_d = mx'' + kx
\]

\[
x = \left(x_o - \frac{F_d}{k}\right) \cos \omega t + \frac{F_d}{k}
\]

\[
x_{t=\pi/\omega} = -x_o + \frac{2F_d}{k}
\]

Damping

• Viscous Damping

\[
x(t) = x_{max} e^{-\zeta\omega t} \sin(\omega_d t + \phi)
\]

\[
\zeta = \frac{1}{2n\pi} \ln \left(\frac{1}{\delta}\right)
\]

Welded steel, prestressed concrete, well detailed reinforced concrete. \( 0.02 \leq \zeta \leq 0.03 \)

Reinforced concrete with considerable cracking. \( 0.03 \leq \zeta \leq 0.05 \)
Damping

- Mechanical dampers
  - Active dampers (not discussed here)
    - Expensive
    - Complicated
    - No proven examples for bridges (prototypes currently being tested for seismic damping)

Damping

- Mechanical dampers
  - Passive dampers
    - Viscous Dampers
    - Tuned Mass Dampers (TMDs)
    - Viscoelastic Dampers
    - Tuned Liquid Dampers (TLDs)

Viscous Dampers

\[ F_D = c(x')^\eta \]
Dampers

Tuned mass damper

\[ \beta_s = \frac{1}{2} \sqrt{\frac{m}{M}} \]

Ex) Consider mass ratio = 0.01
\[ \beta_s = 0.05 \] (5% damping)

Dampers

Viscoelastic Dampers

Comparison of Damper Devices

(1) Tuned mass dampers
(2) Liquid column vibration absorbers
(3) Tuned liquid column dampers

TMD/TLD: additional mass generating counteracting inertia forces
Tuned Mass Damper of Taipei 101

Application of Tuned Mass Damper

The TMD under testing of walking & jumping in the airport

The TMD used in the passenger foot-bridge

Based Isolation System

Case Study: Millennium Bridge

- Crosses River Thames, London, England
- 474’ main span, 266’ north span, 350’ south span

- Superstructure supported by lateral supporting cables (7’ sag)
- Bridge opened June 2000, closed 2 days later, nicknamed “Wobbly Bridge”
Millennium Bridge
• Severe lateral resonance was noted (0.25g)
• Predominantly noted during 1\textsuperscript{st} mode of south span (0.8 Hz) and 1\textsuperscript{st} and 2\textsuperscript{nd} modes of main span (0.5 Hz and 0.9 Hz)
• Occurred only when heavily congested
• Phenomenon called “Synchronous Lateral Excitation”

Millennium Bridge
• Possible solutions
  – Stiffen the bridge
    • Too costly
    • Affected aesthetic vision of the bridge
  – Limit pedestrian traffic
    • Not feasible
  – Active damping
    • Complicated
    • Costly
    • Unproven
  – Passive damping

Millennium Bridge
• Passive Dampers
  – 37 viscous dampers installed
  – 19 TMDs installed

Millennium Bridge
• Results
  – Provided 20% critical damping.
  – Bridge was reopened February, 2002.
  – Extensive research leads to eventual updating of design code.