

Flexible Moment Connections for Unbraced Frames Subject to Lateral Forces—A Return to Simplicity

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It seems that there has been confusion among structural engineers about the type of construction referred to in the AISC *Load and Resistance Factor Design Specification for Structural Steel Buildings*, since 1986, as Partially Restrained or PR. The general concept has been of interest to the authors for many years and has been the topic of several of their papers. The purpose of this paper is to acquaint the profession with a longstanding and successfully applied approach to structural steel frame design, herein called “Flexible Moment Connections (FMC),” and to compare this approach to the Specification-defined PR approach. In addition, the goal is to show that although much has changed in the profession, including specifications and the tools for their application, FMC design remains an acceptable and economical approach for steel structures.

The “Flexible Moment Connections” approach has been permitted in this country and around the world since at least the 1910s (Fleming, 1915). The basic principles of the FMC approach are to treat the beams as simply connected under gravity loads but as moment connected under lateral loads. The approach used for these historic designs has been referred to as “Type 2 with wind,” “Semi-rigid,” “Smart Connections,” “Flexible Wind Connections,” or with the current British term “Wind-Moment Connections” (Salter, Couchman, and Anderson, 1999).

These historical approaches appear to have first been recognized in United States (U.S.) specifications through the AISC Specification in 1946, as Type 2 with wind. Perhaps the first U.S. paper to address the actual connection moment-

rotation capacity and suggest that an approach other than the Type 2 with wind approach be used, was that of Rathbun (1935). What may be the first paper to discuss the actual response of frames designed with the Type 2 with wind approach was that of Surochnikoff (1950), although the method had been in common use for more than 40 years at the time. Another early paper that addressed the seeming paradox of connections knowing when to resist moment and when not to resist moment was presented by Disque (1964). An uncounted number of buildings have been successfully designed with this approach, including such well-known structures as the Empire State Building and the UN Secretariat as well as a large number of unnamed buildings of a common nature. However, current buildings do not exhibit the same level of extra unaccounted for stiffness, such as stiff masonry infill walls, that those earlier buildings exhibited. Thus, since the Type 2 with wind approach continues to be used by the profession, it seems appropriate that the approach should be reassessed and, if proven viable in today’s world of structural engineering, updated as a tool for today’s designers.

It is important to recognize that the proposed approach rests on a significant number of approximations regarding connection stiffness, frame behavior, column effective length, and bending moment amplification. As with any design approach, it must be carried out using good judgment and a thorough understanding of the assumptions made.

BASIC UNDERSTANDING

Connections

In order to understand FMC, it is first necessary to understand the general behavior of a beam-to-column connection. Figure 1 illustrates the moment rotation behavior of three generic connections. One that exhibits a small amount of rotation with a large amount of moment is noted as a rigid connection. A second connection that exhibits a large amount of rotation with a small amount of moment is noted as simple. The third connection is noted as a semi-rigid connection and provides some moment restraint while permitting some rotation. Semi-rigid connections can fall anywhere between simple and rigid as shown. In general, connections that are capable of resisting at least 90 percent of the beam fixed-end

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moment are referred to as rigid. Those that offer enough ductility to accommodate beam end rotation while resisting no more than 20 percent of the fixed-end moment are referred to as simple. Any connection that is capable of resisting a moment between these limits while permitting some rotation must be treated as semi-rigid. It should be clear that in order to make this distinction, something about the beam to which the connection is attached as well as the details of the connection must be known. Numerous researchers have presented the details of connection behavior and at least two collections of this data have been presented (Goverdhan, 1983; Kishi and Chen, 1986). A mathematical model for the semi-rigid connection will be discussed later.

With the introduction of the *Load and Resistance Factor Design Specification for Structural Steel Buildings* in 1986, new terms were introduced to define connection behavior. The rigid connection became known as fully restrained or FR and all other connections, both semi-rigid and simple, became known as partially restrained or PR. That is, simple connections were redefined as a special case of PR moment connections. In keeping with this, for the remainder of this paper, the term PR will be used to refer to all connections that are not FR. Attention will first be given to the influence of connection behavior on members and then a more detailed discussion of connection behavior will be presented.

Beams Carrying Gravity Load Only

For a symmetrical, uniformly-loaded beam, connected to rigid supports and assumed to behave elastically, the end rotation of the member is directly related to the load magnitude. This is most easily described through the use of the

classic Slope-Deflection Equation such that

$$M = \frac{wL^2}{12} - \frac{2EI}{L}\theta \tag{1}$$

where

- M = end moment
- θ = end rotation

This equation is shown as a straight line in Figure 2 and is referred to as the “beam line.”

Superposition of a PR moment connection curve from Figure 1 with the beam line from Figure 2 is shown in Figure 3. Equilibrium is attained at the intersection of these two curves, shown as point *a*. For the nonlinear connection curve shown, this point is not easily obtained. However, if the connection were to be modeled as a straight line, then a simple mathematical solution could be easily found. The straight-line connection model shown in Figure 3 has a slope of $K = M/\theta$ and intersects the beam line at the same point as the actual connection curve. Solving for the connection rotation from this relationship and substituting into Equation 1, the moment in the connection and on the end of the beam at equilibrium is

$$M = \frac{\frac{wL^2}{12}}{1 + \frac{2EI}{KL}} = \frac{\frac{wL^2}{12}}{(1 + 2u)} \tag{2}$$

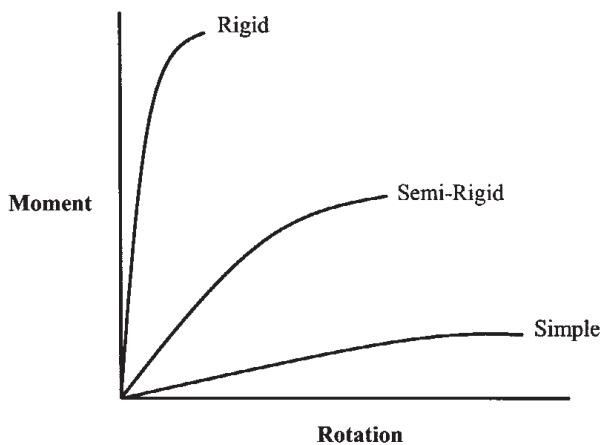


Fig. 1. Typical moment-rotation curves for the three connection types.

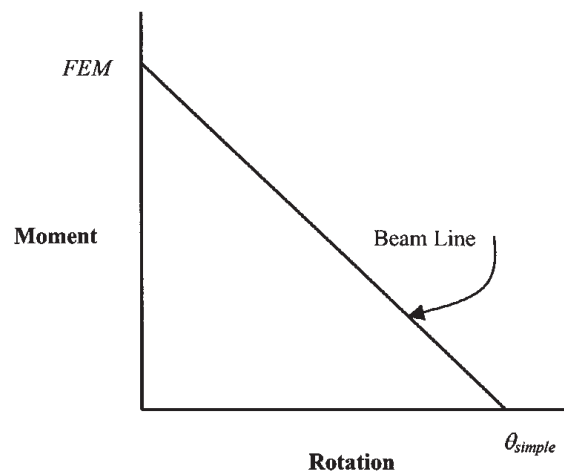


Fig. 2. Beam line for symmetrically loaded beam.

where

$$u = \frac{EI}{KL} \quad (3)$$

Due to symmetry, the moment at the center of the beam span can be found by subtracting the end moment from the simple beam moment. A plot of both the end and mid-span moments as a function of the beam/connection stiffness ratio, u , is given in Figure 4. It can be seen that as the end moment decreases, the mid-span moment increases. The maximum limits are the fixed-end moment for the beam end and the simple beam moment at mid-span. At the intersection of these curves, the end and centerline moments are the same. If a connection could be built with the required moment ro-

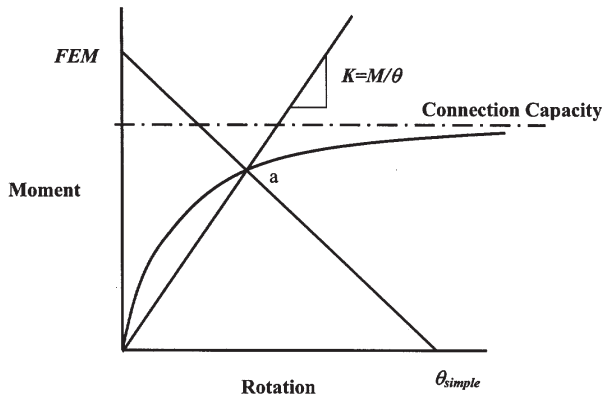


Fig. 3. Beam line and connection curve—equilibrium.

tation behavior, this would be the most economical case for beam design. This is the same result that would be obtained if the plastic mechanism moments were found for beams with connections capable of attaining the plastic flexural strength of the member. It is also noted that 90 percent of the fixed-end moment results when $u = 0.055$ and 20 percent of the fixed-end moment occurs when $u = 2.0$. A more complete discussion of the beam/connection relationship can be found in a paper by Geschwindner (1991).

Beams Carrying Lateral Load Only

Since it is the intention here to also address the influence of PR moment connections on lateral systems, it is important to first study these members in a simplified format. A simple rigid portal frame with lateral load H is shown in Figure 5a.

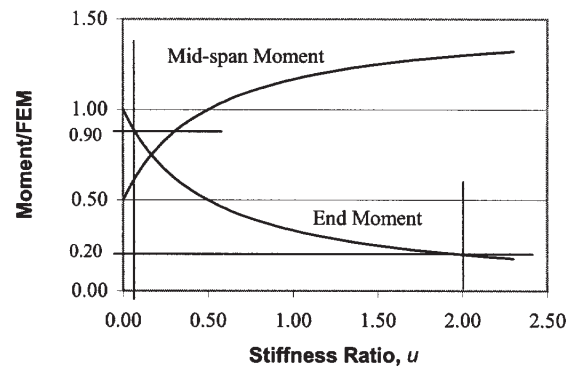


Fig. 4. Beam response as a function of connection stiffness.

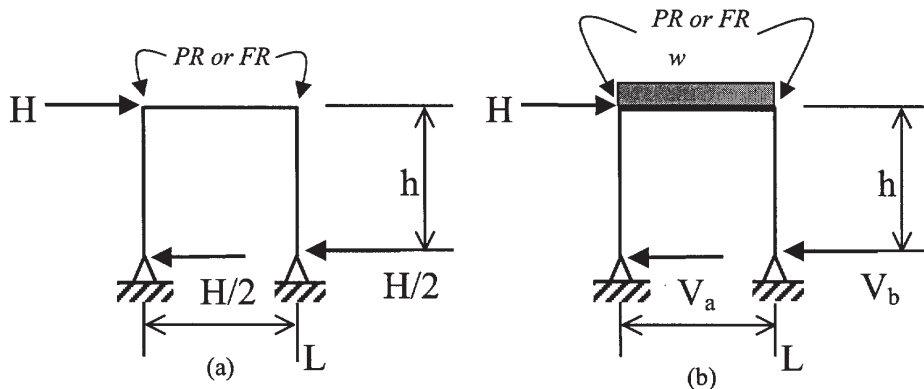


Fig. 5. Portal frame with lateral and gravity load.

A statically indeterminate analysis will show that the frame response is indeed asymmetric and the horizontal shears at the column supports are each $H/2$ as shown in the figure. If the rigid connections are now replaced with linear PR moment connections that exhibit the same behavior for both positive and negative rotation, asymmetry will still prevail and the support shears will again be $H/2$. By plotting the beam end moment as a function of beam and connection stiffness, similar to what was shown in Figure 4, it can be shown that the end moment remains unchanged, as long as the connections continue to behave the same under positive and negative rotation. However, the rotation required for moment equilibrium changes as the connection stiffness changes. This rotation will, of course, yield a corresponding lateral displacement for the frame, which will increase as the connection stiffness decreases.

Beams Carrying Combined Gravity and Lateral Loads

The next step in developing a basic understanding is to put the gravity system and the lateral system together. The uniformly-loaded beam is now made part of a simple portal frame shown in Figure 5b. The beam line for a given load magnitude and the linear connection line are shown again in Figure 6. Note that for gravity load only, equilibrium is given at the intersection of these two lines and is noted as point a for the left end of the beam and point a' for the right end. If the lateral load is then applied, the windward connection unloads and the moment in the connection reduces as the rotation reduces, shown as b . For the leeward connection, the moment increases as the rotation increases, shown as b' . If the gravity load is increased or decreased while the lateral load is maintained, points b and b' move up or down the

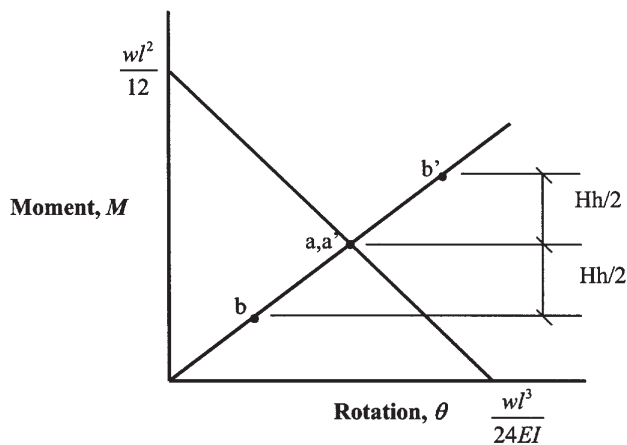


Fig. 6. Beam line with linear connection stiffness.

connection line while maintaining their separation along the connection line. If the lateral load is reduced, points b and b' move toward each other, each moving the same amount; and if the lateral load is increased, they move apart. A similar discussion for the nonlinear connection will be presented later.

PR MOMENT CONNECTION MODEL

As was mentioned earlier, a number of connection moment-rotation models have been proposed in the literature (Richard, 1961; Kennedy, 1969; Frye and Morris, 1975; Krishnamurthy, Huang, Jeffrey, and Avery, 1979; Mayangarum, 1996). For the discussion here, the Three Parameter Power Model, which has been used for a wide range of connection types and is calibrated for a variety of connection element properties (Kim and Chen, 1998), will be used. Figure 7 shows power model curves for a top- and seat-angle, double web-angle connection with top- and seat-angle thickness indicated. It is important to note that the accuracy with which the model predicts the true connection behavior is quite important. An increase or decrease in thickness of the top and bottom angles yields the curves above and below the original curve. A beam line is superimposed on the connection curves and it is seen that there can be a significant difference in the moment magnitude at the point of equilibrium. Referring back to the curves of Figure 4, it can be seen that there is a potential for significant error in the calculated versus actual beam moments. In addition, a review of the development of any connection behavior prediction equation will reveal that even for those that are accurate, they are only accurate within some specified range for specific parameters.

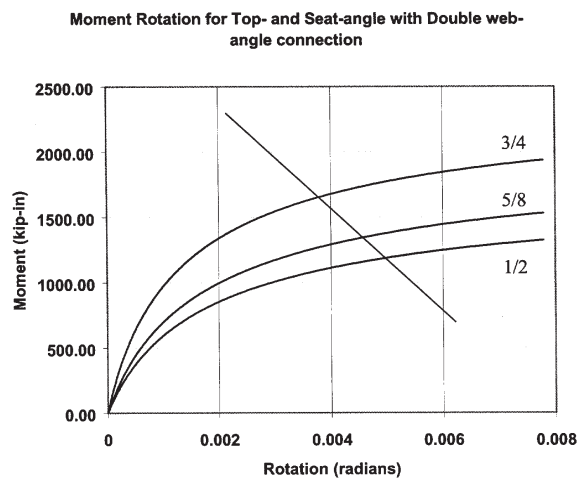


Fig. 7. Influence of top- and seat-angle thickness on connection response.

In addition to requiring an accurate model of the connection as load is applied, it is necessary to have a model for unloading and reloading if a detailed analysis of a PR connected frame is to be carried out. The normal assumption is that a connection loads along the moment-rotation curve and unloads linearly with a slope equal to the initial slope of the curve. This type of model has been verified in numerous connection tests and has been used in dynamic frame analysis by Khudada and Geschwindner (1997) and static frame analysis by Rex and Goverdhan (2002). Since the work of Rex and Goverdhan was applied to a real building structure and followed the analysis through several cycles of load, it will be instructive to review their results.

Figure 8 shows the power model connection curve for the top and bottom angle connection similar to that used by Rex and Goverdhan (2002). It should be noted that Rex and Goverdhan used the Mayangarum model (Mayangarum, 1996) for their analysis. Using the general results of their analysis and the presentation by Sourchnikoff (1950), the response of the PR moment connection can be described. Point *a, a'* represents the equilibrium position under gravity load for this connection on the end of a uniformly loaded beam in a lateral load resisting system. When lateral load is applied, as presented earlier for the linear connection, the windward connection unloads and the leeward connection loads. Since unloading is expected to follow the initial connection stiffness, point *a* moves down the straight line to point *b*. The loading connection continues up the connection curve to point *b'*. The first thing to notice here is that the response is not the symmetric response used earlier for the linear connection model. This is quite important since it significantly complicates the analysis. No longer are the lateral

load moments the same on each end of the member and of course no longer will the response take on the simple form of points *b* and *b'* moving up and down the connection curve as discussed earlier.

Many possible sequences of loading could be considered from this stage on but for the current discussion, a few specific cases will be presented. Consider that one half of the lateral load is removed. The response moves to points *c* and *c'*, both represented by a linear change. Now reapplying the half lateral load, the response is again represented by points *b* and *b'*. Removal of all lateral load is indicated by points *d* and *d'*.

Since the lateral load must be considered to act in both directions, application of the negative lateral load at stage *d* would show the connection to move from *d* and *d'* to *e* and *e'*. When all lateral load is removed, the connection moves to *f* and *f'*. If the gravity and lateral loads just discussed were the maximum loads that the structure would experience, it is seen that the response from this stage on, through the remaining life of the structure, would be linear and follow a response indicated by the initial stiffness of the connection. This sequence of responses will be valid provided that the lateral load does not cause the connection to reverse moment or load it beyond its strength.

It is clear that the engineer has no firm knowledge of the actual sequence or magnitude of the load at any stage. Rather, the engineer has knowledge of the design maximums. For their analysis, Rex and Goverdhan (2002) presented a sequence of load applications that included seven distinct cases with a maximum of 12 different load steps. From their detailed analysis it is seen that at ultimate loading, the connection is behaving as though it were a linear connection for both loading and unloading. It is also clear that the connections have undergone some permanent rotation.

In addition, it is noted that the actual structure response cannot be exactly determined, regardless of the sophistication of the analysis approach, since the true load sequencing cannot be determined. This suggests that perhaps a simplified approach that accounts for the worst case, without requiring the specialized computer software used by Rex and Goverdhan, might prove useful. It is the intention of this paper to present such an approach.

THE FLEXIBLE MOMENT CONNECTION APPROACH

There are two simplifying design assumptions made for implementation of the FMC approach:

1. The beams will be designed as simple beams for gravity load only.
2. The connections will be designed for lateral load moments only.

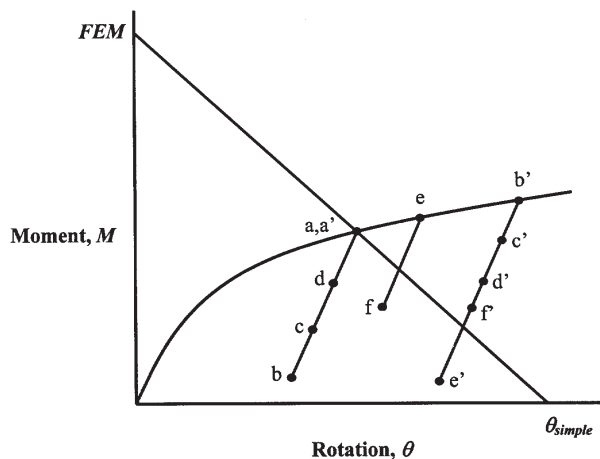


Fig. 8. Behavior of a partially restrained connection under load.

Using the PR moment connection response developed through the discussion of Figure 8, shakedown is seen as an important consideration. The foundation of the FMC approach as originally presented by Disque (1964, 1975) was an understanding of shakedown and its influence on the structure. Surochnikoff (1950) also presented the shakedown response, although the detailed computer analysis of a nonlinear PR moment connection, such as presented by Rex and Goverdhan (2002), had not been developed at that time. The term shakedown simply means that the connection, after proceeding through a series of load applications, responds to all additional loads as if it were a linear connection. The FMC approach relies on the connection exhibiting this shakedown behavior. Although it does not depend on the detailed moment rotation curves for the connections, it does depend on the connections reaching a predictable plastic moment capacity and being sufficiently ductile. With FMC, an alternative approach is available to take advantage of some of the advantages of the PR connection while using a simplified design approach.

Since the connection in the FMC approach will be designed for the lateral load moment only, it is unlikely that the connections will exhibit the complete range of behavior of the PR moment connection shown in Figure 8 without reaching the plastic capacity of the connection. Thus, the connection will be modeled as shown in Figure 9, where the connection reaches its plastic moment capacity and continues to hold that level of moment while undergoing plastic deformations. A beam line has been superimposed on the figure to show that the point of equilibrium under gravity load, a and a' , is likely to occur in the plastic response region of the connection. If lateral load is then applied to the structure, the

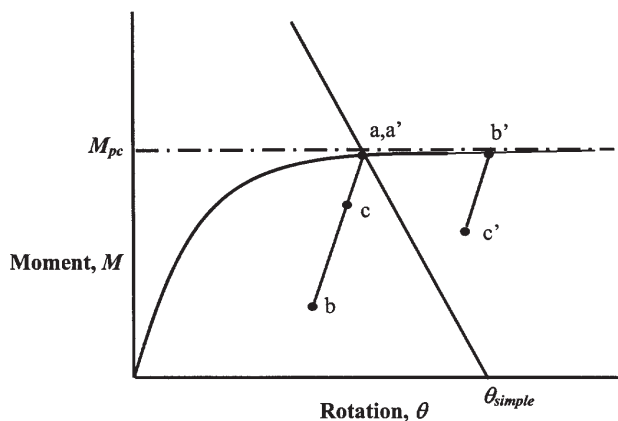


Fig. 9. FMC connection assumed response.

windward connection will unload and move to point b , while the leeward connection will attempt to load and will move to point b' . When the lateral load is removed, the connections move to points c and c' . With continued cycles of load, the connection will shake down in a way similar to the PR moment connection, except that there will be significantly more permanent deformation taking place than was exhibited in the PR moment connection. As a result of this permanent deformation, the beam will experience positive moments in the connections when all loads are removed. When gravity load is reapplied, these positive end moments will offset the negative gravity moments in the beam so that the beam response approaches that of a simple beam. Thus, it will be appropriate to design the beam as a simple beam for gravity loads.

DESIGN APPROACH—STRENGTH

The original design approach (Fleming, 1915; Surochnikoff, 1950; Disque, 1964, 1975) started with the assumption that the beams would be designed for the simple beam moment. It is seen from Figure 4 that this is the most conservative approach since, regardless of the connection stiffness, the gravity load beam moment can never be greater than this moment. In the British approach (Salter and others, 1999; Hughes, Brown, and Anderson, 1999) the beams are designed for 90 percent of the simple beam moments to recognize that there will always be some restraint applied to the beam-ends and the mid-span moment will always benefit from that restraint. The general assumption presented earlier was that any connection capable of resisting no more than 20 percent of the fixed-end moment could be considered a simple connection. In that case, none of the beam gravity moment would be transmitted to the column or the connection. The 10 percent of the simple beam moment used in the British approach is 15 percent of the fixed end moment, so, by the previous assumptions, the connection would still be considered simple with no gravity moment applied to the column. For consistency with the historical approach, the beam will be designed for the simple beam moment in the example that follows.

With the design assumption that all windward connections behave linearly after shakedown and all leeward connections act as plastic hinges, a simplified distribution of lateral load can be made using a method similar to the portal method. A portion of a three-bay frame is shown in Figure 10. The assumptions are made that:

1. Hinges form at the mid-height of the columns.
2. The lateral load is distributed as shear in the columns with magnitudes shown.

Since the leeward column is attached to the beam with a connection that cannot resist any additional moment due to the lateral load, it cannot participate in the lateral load re-

sistance. Thus, each beam-column combination is the same, will resist the lateral load equally, and the connection moments are

$$M_{conn.} = (V_1 + V_2) \frac{h}{2} \quad (4)$$

where

- V_1 = the lateral shear above the story divided by the number of connections acting to resist that shear
- V_2 = a similar force for the column below the story

The beam, too, must be capable of resisting this moment at its ends but it need not resist any moment resulting from the gravity load at its ends.

The column is the final element in the load path for the connection moment. If the connection is designed for the lateral load moment only, then the columns, according to their stiffnesses, must resist that moment. If the connection has been designed to resist the lateral load moment plus 10 percent of the simple beam gravity moment, then the columns must be sized for that moment.

If a frame designed according to the principles just stated were to be subjected to a plastic mechanism analysis, it would have more than sufficient strength. For the gravity load mechanism, the beam would have more than sufficient strength due to the influence of the connection moment that has, to this point, been ignored. For the lateral load mechanism, the connections have been designed for the required strength using one less connection than would be involved in the mechanism failure and the column strength will be selected to be greater than the connection strength so that the mechanism will occur under a lateral load greater than

the factored load. However, this load case could never occur since there will always be some gravity load. A combined gravity and lateral mechanism would occur at a load greater than the factored lateral and gravity loads because of the influence of the beam strength and reduced factored load in the combined load case. Thus, the structure will have sufficient strength at the ultimate load.

DESIGN APPROACH—STIFFNESS

The historic approach to FMC normally ignored issues of stiffness. Even the 1989 *ASD Specification* for Type 2 construction simply required that “connections and connected members have adequate capacity to resist wind moments.” This was generally interpreted to mean strength only. In the 1999 *AISC Load and Resistance Factor Design Specification for Structural Steel Buildings* (AISC, 2000), hereafter referred to as the *AISC LRFD Specification*, the definition of Type PR construction requires that “the connections and connected members must be adequate to resist the factored lateral loads.” Along with permitting the approach for low seismic forces (seismic design in which the seismic response modification factor R is taken as 3 or less) as well as wind load, this statement is generally understood to mean that stiffness too must be considered.

With the British Wind-Moment Connection approach (Salter and others, 1999), the procedure for checking lateral displacement is quite simple. For structures within the limits of their studies, the structure is analyzed as a rigid frame and the resulting lateral displacements are multiplied by a factor between 1.5 and 2.0. Although this may seem somewhat crude in the face of today’s advanced computer approaches to structural analysis, it is not an oversimplification when weighed against the lack of sophistication that goes into establishing the drift limits normally used for design purposes.

Driscoll (1976) presented an approach where beam stiffness was modified to account for connection flexibility. These reduced stiffnesses could then be used in a rigid frame analysis to determine frame displacements.

If desired, any one of a number of more sophisticated analysis approaches may be taken to assess building drift. However, to do so would take away some of the simplicity of the FMC approach. The key to any of these more accurate approaches is proper modeling of the connections to meet the needs of the analysis. Since it has already been established that FMC will eventually behave linearly under lateral load, an analysis including initial connection stiffness could be used. A more accurate approach would be to use a secant stiffness as the model or, if even more accurate drift calculations are desired, more detailed connection models could be used.

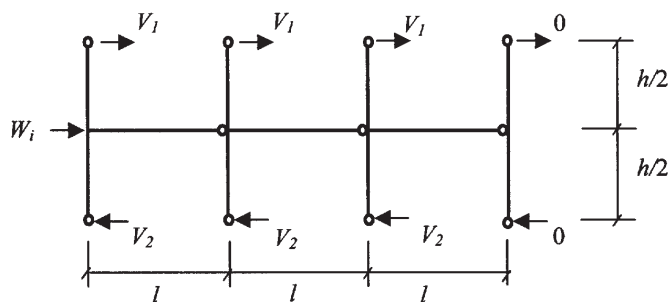


Fig. 10. Distribution of lateral load in frame according to FMC assumptions.

Incorporation of some connection stiffness in a drift calculation is an added aspect of FMC over the historic approaches. Since significant research into connection behavior has been carried out over recent years, there are published models giving the initial connection stiffness and the complete connection curve. For example, the work of Kennedy (1969) provides a method for initial stiffness determination for end plate connections while Mayangarum (1996) and Kim and Chen (1998) provide guidance for top- and bottom-angle, double web-angle connections. Guidance for other types of connections is available in the literature.

DESIGN APPROACH—STABILITY

As was the case with stiffness, the historic approaches to FMC generally ignored issues of stability, perhaps since the *ASD Specification* did not specifically require that it be addressed. Hughes and others (1999) suggested that, within the bounds of the structures they studied, stability was not an issue. But, they also noted that design for stability is closely linked to the code-specified strength equations. Disque (1975) discussed stability issues and included the influence of the connection loading and unloading. He also accounted for the influence of the leeward column, which does not contribute to the lateral stiffness and should be treated as a leaning column, but he did not address the influence of the initial connection stiffness for the unloading connection.

To address stability, consider first the portion of the structure shown in Figure 10 where the far ends of the beams are connected to the columns with pins. These pinned connections represent the connections that are loaded and not able to carry additional moment, since they have already reached their plastic strength. These beams are assumed to be rigidly connected to the columns at the near ends. Thus, as far as the columns are concerned, the beams appear to have a reduced stiffness when compared to beams with rigid connections at each end. This reduced stiffness may be included in the determination of the effective length factor by using a modified beam length in the determination of the G term used in the effective length nomograph. To account for the far end being pinned, the beam length should be taken as twice the actual beam length. Thus,

$$G = \frac{\sum \frac{EI_c}{L_c}}{\sum \frac{EI_g}{2L_g}} = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_g}{2L_g}} \quad (5)$$

Another approach was presented by Driscoll (1976) and restated by Christopher and Bjorhovde (1999) where the stiffness ratios used with the nomograph are modified for the

beam with flexible connections such that

$$G = \frac{\sum \frac{EI_c}{L_c}}{\sum C^*} \quad (6)$$

where the term C^* represents the effective stiffness of the beam and its connections.

For a beam in a sidesway permitted (moment) frame with a pin at one end and a connection with stiffness K_i at the other (*the FMC equation*),

$$C^* = \frac{1}{2} \left(\frac{EI_g / L_g}{1 + 3u} \right) \quad (7)$$

and for a beam with connections at each end with stiffness K_i (*the PR equation*),

$$C^* = \left(\frac{EI_g / L_g}{1 + 6u} \right) \quad (8)$$

Since, in the historical approach and the FMC approach, when considering column stability, the windward connection is assumed to be rigid, that is, one with a sufficiently high initial stiffness so that if it could maintain its stiffness throughout its loading, it would resist 90 percent of the beam Fixed End Moment, then $u = 0.055$ and Equation 7 yields

$$C^* = 0.429 \frac{EI_g}{L_g} \quad (9)$$

This compares to a conservative value of

$$C^* = 0.5 \frac{EI_g}{L_g} \quad (10)$$

which results from the use of a beam with length equal to twice the actual length as presented in Equation 5.

Using Equation 8, which assumes linear PR moment connections at each end of the beam with an initial stiffness ranging between $u = 0.055$ and 2.0 (the range of PR response) the multipliers become 0.75 and 0.077, respectively. For the typical case of two beams framing into the column, the summation would be 1.5 and 0.15, respectively.

To continue to investigate the impact of the FMC assumption on column stability, it will be useful to consider two beams with PR moment connections framing into a column. Setting the PR model, two times Equation 8, equal to Equation 9, and solving for the connection stiffness ratio, it is seen that $u = 0.61$. This indicates that the FMC approach for determining column effective lengths is conservative for all but the most flexible PR moment connections, that is, those with $u > 0.61$.

A close look at ΣC^* reveals that it is essentially a way to determine the number of equivalent rigidly connected girders that resist column buckling. For the PR equation, which is a connection with K_i at each end, with $u = 0$ (an infinitely stiff connection, thus actually a FR moment frame), the multiplier is 2, since there are two beams framing into the column. Figure 11 shows the equivalent number of beams versus the beam to connection stiffness ratio, u , where u varies from 0.0 to 2.0. This is the full range for PR and FR behavior.

Driscoll provided a range of initial stiffness for the three types of connections shown in Figure 12. Figure 11 shows bands representing the number of equivalent girders that correspond to the range of those connection stiffnesses. In addition, the value of 0.5 proposed when using the FMC approach is indicated. It is seen that for connection types B and C, the FMC approach is conservative, while it falls at approximately the mid range for connection type A. Thus, again, the FMC approach is shown to be conservative for all but the most flexible connections.

An additional stability issue is the influence of the leeward column, which must be treated as a leaning column. There are numerous approaches to account for leaning columns but the one that seems most appropriate for the FMC approach

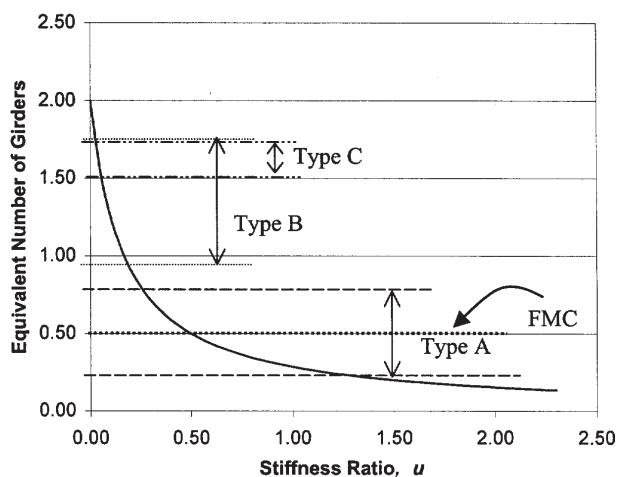


Fig. 11. Equivalent number of girders based on connection type.

is that proposed by Yura (1971). The Yura approach requires that all columns participating in the lateral load resisting system together resist the destabilizing effects of the full gravity load, including that which is assigned to the leaning columns. This is accomplished by designing the lateral load resisting columns to carry the total gravity load in the plane of the frame. As seen for the frame of Figure 10, the columns carrying the lateral load equally can also be designed to carry the gravity load equally and thus satisfy the Yura criteria. The leeward column will be the same as the others based on the case of lateral load in the opposite direction.

Any other factors that impact frame stability can be incorporated in the FMC approach as they would for any other approach, including any other leaning columns (Geschwindner, 2002) in the building and such factors as column inelasticity (the stiffness reduction factor).

LIMITATIONS

A number of authors have suggested that the FMC approach is appropriate, as long as it is used for structures that fit within the range of their studies. One of the first such studies was presented by Ackroyd (1987). The conclusions of that work state that the method is acceptable for structures up to 10 stories in height. It should be noted that the study assumed fixed bases for all columns.

Hughes and others (1999) presented another, more recent, study. Their study formed the basis of the British publication on wind-moment design (Salter and others, 1999) in which the method is limited to frames from two to four stories and two to four wind-resisting bays. This limitation is set for structures specifically designed according to their rules. In addition they propose a standard set of connections that will

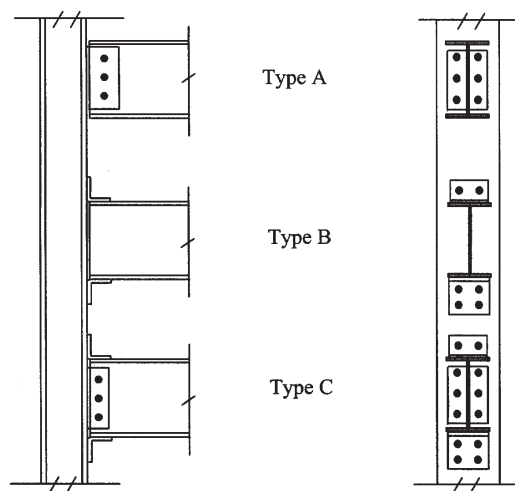


Fig. 12. Three types of semi-rigid connections.

perform as expected, as long as they are designed in accordance with the provided design charts.

Although there may be some limit to the applicability of the FMC approach, the design approach presented here should permit the design engineer to make a conscious decision as to the appropriateness of the design, based on the results of the analysis and the final member and connection sizes.

ECONOMICS

It is clear from the foregoing discussion that the assumptions made in the FMC approach will lead to larger beams than might be necessary for an FR moment frame structure. However, it is not as clear that the beams will be larger than those resulting from a PR design, since in the PR approach shakedown may reduce the restraining moments for gravity loading and the beams may be designed for something close to the simple beam moment anyway.

Since the connections that result from the FMC approach need only be designed to resist the lateral load moment, they will surely be more economical than a similar style FR connection that must resist a higher moment resulting from a rigid frame analysis.

The columns in the FMC frame are assumed to be oriented for strong axis bending in the frame and to be part of a braced frame in the weak axis, perpendicular direction. Thus, depending on connection stiffness and the relative strengths of the column strong and weak axes, there is the possibility that columns will turn out to be the same for the FR, PR, and FMC approaches.

Although much of the work of structural analysis and design today is carried out through computer software, the buildings that would most benefit from the FMC approach

are those of a magnitude where hand calculations would be appropriate. For cases where a PR analysis is to be carried out, the FMC approach will also provide a reasonable starting point.

DESIGN EXAMPLE

The 4-bay, 2-story frame shown in Figure 13 will be used to demonstrate the FMC approach. This frame was originally used by Deierlein (1992) and again by Christopher and Bjorhovde (1998, 1999). For this example, all beams and columns will be taken as ASTM A992, with $F_y = 50$ ksi, and all connection elements will be taken as ASTM A36, with $F_y = 36$ ksi. The frame is assumed braced normal to the plane. Girders are assumed sufficiently braced against lateral-torsional buckling so that the full plastic strength of the member may be reached.

Step 1: Girders are designed for simple beam bending for the gravity only load case ($1.2D + 1.6L$).

Roof:

$$w_u = 1.2(0.625) + 1.6(1.125) = 2.55 \text{ kip/ft}$$

$$V_u = 2.55(12.5) = 31.9 \text{ kips}$$

$$M_u = 2.55(25)^2/8 = 199.2 \text{ kip-ft}$$

Select W16×31

$$\phi M_p = 203 \text{ kip-ft}$$

$$\phi V_n = 118 \text{ kips}$$

Floor (restricting depth of members to W18):

$$w_u = 1.2(1.875) + 1.6(1.25) = 4.25 \text{ kip/ft}$$

$$V_u = 4.25(12.5) = 53.1 \text{ kips}$$

$$M_u = 4.25(25)^2/8 = 332 \text{ kip-ft}$$

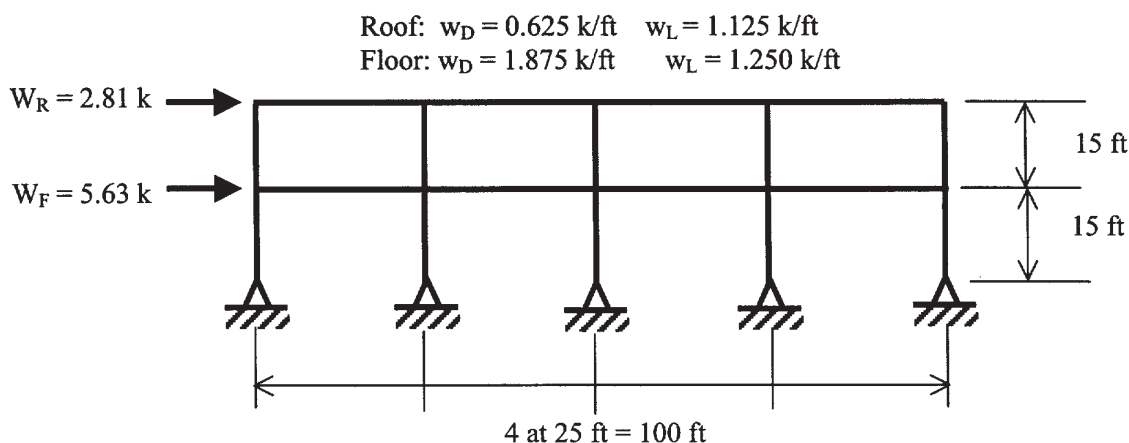


Fig. 13. FMC example problem (Deierlein, 1992).

Select W21×44

$$\phi M_p = 359 \text{ kip-ft}$$

$$\phi V_n = 196 \text{ kips}$$

Step 2: Lateral Load Distribution will be as shown in Figure 14. Note that the leeward column will not contribute to the resistance due to the hinge assumed at the beam connection. It is clear that each column-beam combination is the same, thus they will each resist an equal portion of the lateral load. Using the factored lateral load as discussed earlier,

$$W_R = 1.3(2.81) = 3.65 \text{ kips}$$

$$W_F = 1.3(5.63) = 7.32 \text{ kips}$$

Beam Shear

$$V_B = \frac{0.91(7.5) + 2.74(15)}{25} = 1.92 \text{ kips}$$

Connection Moment

$$M_{conn} = 1.92(25) = 48.0 \text{ kip-ft}$$

Step 3: Column design will assume that the column is a two-story column. Selecting the lower level column for strength under gravity load

$$P_u = [1.2(1.875 + 0.625) + 1.6(1.25) + (0.5(1.125))](25) = 139 \text{ kips}$$

The columns must also resist a moment equivalent to the connection design moment, which is the moment that was determined from the lateral analysis. Although the two columns at the joint should share this moment, it will conservatively be applied fully to the column being designed. Since this is a gravity only load case it is acceptable to assume that there is no lateral translation moment thus,

$$M_{nt} = 48.0 \text{ kip-ft}$$

$$M_{lt} = 0 \text{ kip-ft}$$

To start, a W10×39 is selected as a trial section.

For in-plane sway, the nomograph may be used to deter-

mine the effective length factors. For the pinned base, the recommended value of $G_B = 10$, will be used. For the upper end of the column, account must be taken for the beam with a pinned end; thus, its length will be doubled and only one beam can be used to restrain the column. Thus, conservatively neglecting the stiffness reduction factor, from Equation 5,

$$G_T = \frac{2\left(\frac{209}{15}\right)}{\left(\frac{843}{2(25)}\right)} = 1.65$$

which yields $k_x = 2.0$. Determining the critical length, with $k_y = 1.0$, the y-axis will still control and for $k_y L_y = 15$ ft,

$$\phi P_n = 267 \text{ kips}$$

The bending strength of the column is determined considering the lateral-torsional buckling length, $L_b = 15$ ft, resulting in

$$\phi M_n = 149.4 \text{ kip-ft}$$

Consideration of the second order amplification using the B_1 and B_2 approach of the AISC *LRFD Specification* (AISC, 2000) will show that the only factor needed is B_1 and it will be found to be 1.0, thus,

$$M_u = 1.0(48.0) = 48.0 \text{ kip-ft}$$

The interaction Equation H1-1a yields

$$\frac{139}{267} + \frac{8}{9} \left(\frac{48.0}{149.4} \right) = 0.81 < 1.0$$

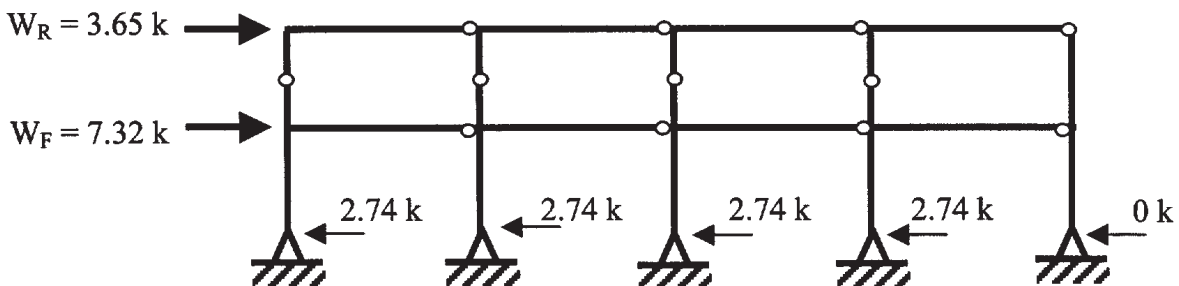


Fig. 14. FMC example lateral load distribution.

Therefore, the column will be adequate for this loading case.

To check strength for the lateral load case, the gravity load is taken as

$$\begin{aligned} P_u &= [1.2(1.875 + 0.625) + 0.5(1.25 + 1.125)](25) \\ &= 105 \text{ kips} \end{aligned}$$

For this case, the moment in the connection is due to the lateral load, assuming that there is no moment due to the gravity load, thus,

$$\begin{aligned} M_u &= 0 \text{ kip-ft} \\ M_l &= 48.0 \text{ kip-ft} \end{aligned}$$

In order to account for the leaning column, that is the leeward column that does not contribute to the lateral resistance, all columns will be taken as the same shape. As mentioned earlier, this is consistent with the Yura (1971) approach for dealing with leaning columns. It also permits the assumption that

$$\frac{\sum P_u}{\sum P_{e2}} = \frac{P_u}{P_{e2}}$$

where

$$P_{e2} = \frac{\pi^2 (29000)(209)}{(2.0(15)(12))^2} = 461.6 \text{ kips}$$

and

$$B_2 = \frac{1}{1 - \frac{105}{461.6}} = 1.29$$

thus

$$M_u = 1.29(48.0) = 61.9 \text{ kip-ft}$$

Again using Equation H1-1a,

$$\frac{105}{267} + \frac{8}{9} \left(\frac{61.9}{149.4} \right) = 0.76 < 1.0$$

which again indicates that the column is adequate.

Step 4: Connection design follows the requirements of the AISC *LRFD Specification* (AISC, 2000) for a top- and seat-angle with double web angles connection. For the floor, the result is a L4×4×¾, 8-in. long with 13/16 × 1 in. vertical slots in the vertical leg for the top- and seat-angle, attached to the beam using two ¾-in. diameter A325-X bolts and to the column flange with two ¾-in. diameter A325-N bolts. The web angles are 2L3½×3×¼, 10-in. long with 13/16 × 1 in. horizontal slots in the 3 in. outstanding leg and three ¾-in. diameter A325-N bolts to the beam web and six to the column flange.

For the roof, the result is a L3½×3½×¾, 8-in. long for the top and seat, with 13/16 × 1 in. vertical slots in the vertical leg and two ¾-in. diameter A325-N bolts in each leg. The web angles are 2L3½×3×¼, 7-in. long with 13/16 × 1 in. horizontal slots in the 3-in. outstanding leg and two ¾-in. diameter A325-N bolts.

Step 5: Drift must be considered for the completed design. As mentioned in the text, there are a number of approaches that may be taken for these calculations. Using the frame modeled as a fully rigid structure with column bases pinned, a first-order stiffness analysis yields a total frame drift of 0.76 in. Accounting for the recommended stiffness of a pin base, as used in the design, the drift reduces to 0.57 in. Since this does not account for any of the flexibility of the connections, some modification should be considered. Using the British multiplier of 1.5, the deflection becomes 0.86 in. This is less than $H/400 = 0.9$ in., a common limitation used in design. However, the British approach is only calibrated for end plate connections so its appropriateness here is questionable.

A second approach would be to include the flexibility of the connections directly in the analysis program. Using the equations presented by Kim and Chen (1998), the connections designed for the example frame yield a stiffness of $K_{i(roof)} = 557,000$ in.-kip/rad and $K_{i(floor)} = 3,137,000$ in.-kip/rad. These are very stiff connections. Using these linear connection stiffnesses in a conventional stiffness analysis results in a building drift of 0.59 in. This is well below the suggested limit of $H/300$ and not significantly larger than that obtained for the fully rigid connection analysis. Of course if the drift limit were not satisfied, either a stiffer connection or a change in member sizes would reduce the drift, depending on the designer's wishes.

A 4-bay 2-story frame has been designed according to the FMC approach outlined in this paper. All requirements of the AISC *LRFD Specification* have been satisfied and the serviceability limit state of drift has been checked. A comparison of the results obtained here with those obtained by Deierlein (1992) shows that members sizes from the FMC approach are slightly larger than those obtained by Deierlein. Thus, the strength and stiffness of the structure obtained here are sure to be sufficient, although the detailed response may not be known. The drift calculated by Deierlein was also below reasonable design limits, although he did not present actual connection designs, so the drift expected in this design can be expected to meet those same limits.

CONCLUSIONS

The following conclusions can be stated based upon the foregoing discussions:

1. The basic principles of the FMC approach are founded in the design of steel frame buildings since the early 20th

century. Based on the presentation here, the latest understanding of stability and partially restrained moment connections have been merged with this historic approach. Through an example frame design, the FMC approach has been shown to be a viable approach within the current AISC *LRFD Specification*.

2. The FMC approach is simplified and straightforward, obviating the inherent complexities that may be encountered in a PR approach. Unless a more accurate drift analysis is desired, no PR moment connection data are required. When a more accurate drift analysis is desired, the initial or secant connection stiffness is needed.
3. The FMC approach does not depend, as the PR approach does, on the actual moment rotation curves of the connection. It is an approximation that was shown to be conservative.
4. With the PR approach, the actual structure response cannot be exactly determined, regardless of the sophistication of the analysis, unless the true load sequencing can be determined. This suggests that a simplified approach that takes advantage of the connection behavior and accounts for the worst case would be useful.
5. Although the member sizes determined through the FMC approach may not be the minimum-weight members that could have been found through another approach, they are reasonable and the connection design and fabrication are not as demanding as would be the case for FR frames. Thus, the system does appear to provide an economic alternative for the designer, particularly since connections (labor items) are simplified.
6. The FMC approach uses a well-established design philosophy and incorporates current thinking about frame behavior and design. Details of connection moment-rotation characteristics are not required to make the approach usable.
7. The FMC approach satisfies the existing AISC *LRFD Specification* and also permits checking of the drift serviceability limit state.
8. The example design that resulted from this FMC approach was shown to be adequate when compared to the design results presented by Deierlein (1992).

ACKNOWLEDGMENTS

The authors wish to thank Victor Shneur of Lejeune Steel Company for his connection designs, Charles Carter for his encouragement in the development of this paper, and all of the authors whose papers have been referenced, since it is

their work that has permitted us to put this presentation together.

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