

# A Distributed Opportunistic Scheduling Protocol for Multi-Channel Wireless Ad-Hoc Networks

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**Abstract**—The topic of opportunistic scheduling for wireless ad-hoc networks has been studied for single-channel networks in several recent works. Since now many wireless systems provide multiple channels for data transmission, this problem is of practical interest for networks with multiple channels. In this paper, we study the problem of opportunistic scheduling for one type of ad-hoc networks where the wireless spectrum can be divided into multiple independent sub-channels for better efficiency. We start with a naive multi-channel protocol where the scheduling scheme is working independently from sub-channel to sub-channel. We show that the naive protocol can only marginally improve the system throughput. We then develop a protocol to jointly consider the opportunistic scheduling behavior across multiple sub-channels. We characterize the optimal stopping rule and present several bounds for the system throughput of the multi-channel protocol. We show that by joint optimization of the scheduling scheme across multiple sub-channels, the proposed protocol improves the system throughput considerably in contrast to that of single-channel systems.

## I. INTRODUCTION

In the past decade many papers have been published on the topic of opportunistic scheduling [1]. Instead of treating channel fading as a source of unreliability and trying to mitigate the channel fluctuations, fading can be exploited by transmitting information *opportunistically* when and where the channel is strong [2]. This problem is well studied for cellular-like networks where a central scheduler tries to optimize the overall system performance by selecting the *on-peak* user for data transmission [1]–[3]. In contrast, in ad-hoc networks it is required to access the medium and schedule data transmission in a decentralized fashion. So far few works have studied this problem in distributed scenarios. A few examples include rate adaptation with MAC design based on the RTS/CTS handshaking for IEEE 802.11 networks [4]–[6] and channel-aware ALOHA for uplink communications [7]–[9]. However, rate adaptation schemes focus on exploiting temporal opportunities, leaving the distributed medium access problem to the RTS/CTS mechanism [4]–[6]. On the other hand, channel-aware ALOHA associates the probability to access the uplink with channel quality assuming that each user knows its own channel state information (CSI) from the uplink [7]–[9]. These works ignore the overhead due to distributed medium access in wireless ad-hoc networks. One type of distributed opportunistic scheduling (DOS) problems is studied in [10], in which multiple links contend the shared

wireless medium and schedule data transmission using only local information in an ad-hoc network. In [10], the transmitter has no knowledge of other links' channel conditions, and even its own channel condition is not available before one successful probing. The link condition corresponding to one successful probing can either be good or poor due to channel fluctuations. In each round of channel probing, the winner decides whether or not to send data over the channel. If the winner gives up the current opportunity, all links re-contend the wireless medium again, in the hope that another link with better channel condition can utilize the channel after re-contention. The purpose of this procedure is to optimize the overall system throughput. It is shown in [10] that the decision on further channel probing or data transmission is based on local channel condition only, and the optimal strategy is a threshold policy. This problem is further studied in [11] where the winners' channel rates are not explicitly assumed to be independent. The system throughput is characterized in [11] under possible dependence of the winners' channel rates.

On the other hand, many wireless systems now provide multiple channels for data transmission [12]–[14]. Hence the opportunistic scheduling problem for such kind of wireless networks is of practical interest. In a multi-channel network, different channels experience independent fluctuations if the channel separation is greater than the coherent bandwidth. This fact substantially enhances the possibility that there exists at least one channel with good quality. This type of opportunistic scheduling problem has been discussed in [12]–[14] for centralized scenario. In this paper, we study the distributed opportunistic scheduling problem for ad-hoc networks where the wireless spectrum is divided into multiple independent channels. We develop a scheduling scheme that jointly exploits opportunism across multiple sub-channels. We characterize the optimal decision rule and present several bounds for the system throughput of the multi-channel protocol. Through numerical results we show that our proposed multi-channel opportunistic scheduling protocol can improve system performance considerably in contrast to that of single-channel systems.

This paper is organized as follows. We first introduce our system model in Section II. We explain our motivation for the multi-channel scheme and develop our protocol in Section III. We characterize the optimal rules and system throughput for the multi-channel protocol in Section IV, and show our

numerical results in Section V. Finally we conclude this paper in Section VI.

## II. SYSTEM MODEL

Similar to [10], [11], we assume there are  $M$  links sharing the wireless medium in an ad-hoc network without any centralized coordinator. To access the wireless medium, all links have to probe first. Suppose these links adopt a fixed probing duration  $\tau$ . A collision channel model is considered, where a link wins the channel if and only if no other links are probing simultaneously. If link  $m$  probes the medium with a fixed probability  $p(m)$ , the duration of the  $n$ -th round of channel probing is

$$T_n = \tau \cdot K_n, \quad (1)$$

where  $K_n$  is the number of probes before the channel is won by some link  $s_n$ . Hence  $K_n$  has a geometric distribution  $\text{Geom}(p_s)$ , where its parameter

$$p_s = \sum_{m=1}^M p(m) \prod_{i \neq m} [1 - p(i)] \quad (2)$$

is the successful probing probability. At the end of the  $n$ -th round, the winner  $s_n$  has an option to send data through the wireless channel at a rate of  $R_n$ , or to give up. Here we denote the channel rate for link  $m$  as  $R(m)$ , and the channel rate for the winner in the  $n$ -th round as  $R_n$ .

To opportunistically schedule transmissions in a distributed fashion, all  $M$  links cooperate to optimize the average network throughput. At the end of the  $n$ -th round,  $s_n$  makes a decision on whether or not to utilize the channel for data transmission, where  $s_n$  sends data over the channel only when  $R_n$  is satisfactory. If  $s_n$  gives up the opportunity, all links re-contend the wireless medium. This procedure repeats until some link finally utilizes the channel for data transmission. The goal is that all links cooperate to make the channel accessible by someone with satisfactory transmission rate. This problem can be modeled as an optimal stopping problem [15], [16]. In the  $n$ -th round, the winner  $s_n$  observes the probing duration  $T_n$  and the transmission rate  $R_n$ . Hence the sequence of  $\sigma$ -fields [15], [16] can be written as

$$\mathcal{F}_n = \{R_1, T_1; R_2, T_2; \dots; R_n, T_n\}. \quad (3)$$

At time  $n$ , based on the observation  $\mathcal{F}_n$  the winner  $s_n$  makes a decision on whether to stop or not to maximize the system throughput.

We study this problem under the constant access time (CAT) model [11], [13], where the total duration of the channel probing and data transmission is a constant, i.e.  $T_p + T_d = T$ . We adopt this model so that the beginning of each block  $T$  on different sub-channels can easily be synchronized in a multi-channel network. Note that the duration of channel probing is a random variable depending on the stopping time  $N$ , i.e.  $T_{p,N} = \sum_{i=1}^N T_i$ .

## III. THE MULTI-CHANNEL OPPORTUNISTIC SCHEDULING PROBLEM

In this section, we describe the opportunistic scheduling problem for multi-channel ad-hoc networks and develop a protocol that enables joint optimization across multiple sub-channels.

### A. Motivation for the Multi-Channel Problem

We consider a wireless ad-hoc network with available bandwidth  $W$ . There are a total of  $M$  links competing the medium in a cooperative and opportunistic manner. The whole spectrum can be directly used as one single channel, using the distributed opportunistic scheduling protocols described in [11]. We assume a homogeneous network where the channel statistics are identical for different links. Now we are interested in better efficiency by dividing the whole bandwidth into  $J$  sub-channels, where  $J < M$ . We assume each sub-channel has a bandwidth of  $\frac{W}{J}$ , and the sub-channels are orthogonal and hence their channel fadings are independent from each other. This is a common assumption in literature. For example, wireless networks with independent sub-channels have been discussed in [12]–[14]. We denote the time as  $t$  and the number of active probing links on the  $j$ -th sub-channel at time  $t$  as  $M_t^{(j)}$ . For simplicity we use the scenario  $M_t^{(j)} = \frac{M}{J}$  to illustrate our idea.

We first take a look at the average waiting time for any given link to access the medium. For a single-channel network, a given link  $m$  is able to access the current block with a probability  $\frac{1}{M}$ . Since the procedure is independent from block to block, the average waiting time before link  $m$  can send data through the wireless medium is  $MT$ , i.e.  $M$  blocks. For a multi-channel system, link  $m$  is able to access the current block with a probability  $\frac{1}{M_t^{(j)}} = \frac{J}{M}$ . Hence the average delay for link  $m$  to access the medium is  $\frac{MT}{J}$ . This is only  $\frac{1}{J}$  of that of a single-channel network. Hence multi-channel protocols can considerably reduce the average waiting time for any given link to access the medium.

Now we consider the system throughput. Intuitively speaking, the system throughput is determined by how likely a “good” link can be found in the network. For a single-channel network, we assume the probability that the current captured channel rate for a given link being “good” is  $P_g$ . Hence the probability that the wireless medium will be utilized by a “good” link is  $MP_g$ . For the multi-channel network, on the other hand, we need to find  $J$  “good” links. This is because the bandwidth of each sub-channel is only  $\frac{W}{J}$ . Suppose the probability that the current captured transmission rate being “good” on a given sub-channel is  $\tilde{P}_g$ . If we consider the scheduling is independent between sub-channels, this probability is  $J \cdot \frac{M}{J} \tilde{P}_g = M\tilde{P}_g$ . There is no big difference compared to the single-channel scenario, since we can treat  $P_g \approx \tilde{P}_g$  if the bandwidth  $W$  is evenly allocated to each sub-channel. On the other hand, if the distributed opportunistic scheduling is jointly designed across all sub-channels, this probability becomes  $\binom{M}{J} \tilde{P}_g$ . Hence this probability will be improved

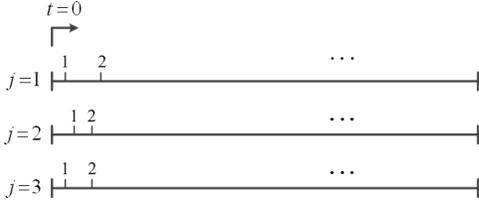


Fig. 1. The channel probing within one block duration  $T$  for a network with 3 sub-channels.

considerably when there are enough number of sub-channels  $J$ . It gives us some hint on the benefit from joint optimization across multiple sub-channels. However, it is tricky to design protocols that can work in a distributed scenario to achieve the opportunism introduced by these sub-channels.

### B. The Multi-Channel Opportunistic Scheduling Protocol

Similar to the single-channel scenario, we consider a collision model for each sub-channel where channel probing is required before accessing any sub-channel. The channel probing is still independent from sub-channel to sub-channel due to lack of centralized coordinator. Suppose at time  $t$  there are  $M_t^{(j)}$  links actively probing the  $j$ -th sub-channel with a fixed probability  $p$ . The  $j$ -th sub-channel is won by some link after a duration of  $\tau K_{n_j}^{(j)}$  and is captured at a transmission rate of  $R_{n_j}^{(j)}$ . Here  $n_j = n_j(t)$  is the index for the round of successful channel probing on the  $j$ -th sub-channel. We can see  $K_{n_j}^{(j)}$  has a geometric distribution with parameter  $p_{s,t}^{(j)}$ , where  $p_{s,t}^{(j)}$  is the successful probing probability on the  $j$ -th sub-channel at time  $t$  and it depends on  $M_t^{(j)}$ .

Since the channel probing is independent for different sub-channels,  $n_j(t)$  is generally asynchronous for different  $j$ . This is illustrated in Fig. 1, where the numbers above each sub-channel indicate  $n_j$  at different time  $t$ . We can see that sub-channel 2 has its first winner link later than sub-channel 1 and 3, while sub-channel 1 has its second winner link later than sub-channel 2 and 3 respectively. Whenever any sub-channel is won by some link that is actively probing that sub-channel, we say one *event* happens in the system. Now we take a look at the whole procedure from time  $t = 0$ . We already know that it takes a duration of  $\tau K_{n_j}^{(j)}$  for the  $n_j$ -th event to happen on the  $j$ -th sub-channel. Hence it takes a duration of  $\tau \min K_{n_j}^{(j)}$  for the first event *ever* to happen in this network. Similarly, starting from the first event, it takes a duration of  $\tau \min K_{n_j}^{(j)}$  for the second event to appear in the system, and so on. We denote the minimum duration across different sub-channels as

$$\tilde{K}_n = \min_{j \in \mathcal{J}_t} K_{n_j}^{(j)}, \quad (4)$$

where  $n = n(t)$  is the index for the round of successful probing in the system, and  $\mathcal{J}_t$  is the set of sub-channels that have not been utilized for data transmission until time  $t$ . We can see that  $\tilde{K}_n$  is the shortest time interval between any two events (not necessarily originated from the same sub-channel) in the system.

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1: Each link  $m$  picks one sub-channel;
2:  $\mathcal{J}_t \leftarrow \{1, 2, \dots, J\}$ ;
3: while  $\mathcal{J}_t \neq \emptyset$  do
4:   for each  $j \in \mathcal{J}_t$  do
5:     links probe the  $j$ -th sub-channel;
6:   end for
7:   if link  $m$  wins some sub-channel  $j$  then
8:      $m$  makes a decision on whether to send data on
9:      $j$  or not;
10:    if  $m$  decides to utilize sub-channel  $j$  then
11:       $m$  sends data over sub-channel  $j$  until the end
12:      of this block;
13:      sub-channel  $j$  is deleted from  $\mathcal{J}_t$ ;
14:    end if
15:   end if
16: end while

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Fig. 2. The Distributed Opportunistic Scheduling Protocol for Multi-Channel Networks

Hence for a multi-channel network, it is not necessary to trace all events on a specific sub-channel and design optimal stopping rule for that sub-channel. Instead the protocol could make a decision as soon as there is a new event available in the system, no matter from which sub-channel this event is originated. As a result, one major difference of the multi-channel protocol is that the decision making at different time instances could be based on observations from different sub-channels. The full protocol can be described in Fig. 2.

Note that in Fig. 2, there are generally multiple winners on different sub-channels at the same time and hence multiple decision makings based on different instant transmission rates.

## IV. PERFORMANCE ANALYSIS FOR THE MULTI-CHANNEL PROTOCOL

In this section, we analyze the proposed multi-channel protocol and characterize its system throughput. We present lower and upper bounds on the system throughput under various constraints.

In this paper, we only characterize system performances for *homogeneous* networks, where the distributions of the transmission rates are identical with respect to different links or sub-channels. To facilitate our performance analysis, we make some additional assumptions as we did for single-channel systems in [11]:

- [A1] The channel rates can only take values in  $(0, +\infty)$ ;
- [A2] The probing duration  $\tau$  is much smaller compared to the block length, i.e.  $\tau \ll T$ .

We take a look at the channel probing and decision making procedure described by Fig. 2. Suppose the  $j$ -th sub-channel is won by some link after a duration of  $\tilde{K}_n$ , which makes it the  $n$ -th round of successful channel probing for the multi-channel system. Suppose sub-channel  $j$  is then captured by

the winner  $s_n$  at rate  $\tilde{R}_n$ .<sup>1</sup> Hence the reward is

$$Y_n = \frac{\tilde{R}_n \cdot (T - \tau \sum_{i=1}^n \tilde{K}_i)}{T} \quad (5)$$

if the winner  $s_n$  decides to utilize the channel, and is 0 otherwise. We can rewrite it as

$$Y_n = \frac{T - \tau \sum_{i=1}^n \tilde{K}_i}{T/\tilde{R}_n},$$

and the optimization is now reduced to maximize the rate of return [15], [16]. To do this, we need to characterize the probabilistic distribution of  $\tilde{K}_n$ .

*Lemma 1:*  $\tilde{K}_n$  has a geometric distribution  $\text{Geom}(\tilde{p}_{s,t})$  with parameter

$$\tilde{p}_{s,t} = 1 - \prod_{j \in \mathcal{J}_t} [1 - p_{s,t}^{(j)}], \quad (6)$$

where  $p_{s,t}^{(j)}$  is the successful probing probability on the  $j$ -th sub-channel at time  $t$ .

*Proof:* It is easy to compute the CDF of  $\tilde{K}_n$  based on (4) if we notice that  $K_{n_j}^{(j)}$  are independent geometric distributions with parameter  $p_{s,t}^{(j)}$ . ■

Now that  $\tilde{K}_n$  has a geometric distribution  $\text{Geom}(\tilde{p}_{s,t})$ , we can apply a similar procedure as in [11] to characterize the optimal stopping rule.

*Lemma 2:* Suppose at time  $t$ , the set of sub-channels that have not been utilized for data transmission is  $\mathcal{J}_t$ . Then the optimal stopping rule is

$$N^* = \min \left\{ n \geq 1 : \tilde{R}_n \geq \lambda_n^* \cdot \frac{T}{T - \tau \sum_{i=1}^n \tilde{K}_i} \right\}, \quad (7)$$

where  $\lambda_n^*$  is the solution to

$$E \left[ 1 - \frac{\tau}{T} \left( \sum_{i=1}^n \tilde{K}_i - \tilde{K}_{n+1} \right) - \frac{\lambda}{\tilde{R}_n} \right]^+ = \frac{\tau}{T \cdot \tilde{p}_{s,t}}. \quad (8)$$

The optimal system throughput  $\lambda^*$  is the solution to

$$E \left[ 1 - \frac{\lambda}{\tilde{R}_n} \right]^+ = \frac{\tau}{T \cdot \tilde{p}_{s,t}}. \quad (9)$$

Here the successful probing probability  $\tilde{p}_{s,t}$  is defined in (6).

*Proof:* The proof can be obtained in a similar way as the proof of Theorem 1 in [11]. ■

From Lemma 2, we can see the optimal reward and stopping rule only depend on the cumulative durations  $\tau \sum_{i=1}^n \tilde{K}_i$  for channel probing and the captured instant channel rate  $\tilde{R}_n$ . Both of them are readily available for the winners' decision makings in a *distributed* setting, even though physically these events might be originated from different sub-channels.

We now take a look at the whole decision making procedure. Once a sub-channel is utilized for data transmission, it will not be involved in the channel probing until the beginning

<sup>1</sup>Strictly speaking, they should be denoted as  $s_n^{(j)}$  and  $\tilde{R}_n^{(j)}$  respectively, since there might be multiple winners on different sub-channels at time  $t$ . Here we ignore the superscript when discussing one of these winners.

of the next block. Hence the cardinality of  $\mathcal{J}_t$  (denoted as  $J_t = \|\mathcal{J}_t\|$ ) decreases whenever there is a decision to stop. All sub-channels will be eventually utilized for data transmission. Hence there should be  $J$  decisions that are to stop in the end. The decreasing of  $J_t$  will affect the successful probing probability (6) for the multi-channel system and hence the system throughput.

We first characterize the optimal reward when the successful probing probability  $\tilde{p}_{s,t}$  is varying as the procedure moves on.

*Lemma 3:* Suppose the successful probing probability  $\tilde{p}_{s,t}$  in Lemma 2 is varying as the procedure moves on. Suppose before one winner decides to stop, the minimum and maximum of  $\tilde{p}_{s,t}$  are  $\tilde{p}_{s,min}$  and  $\tilde{p}_{s,max}$  respectively. Then the optimal system throughput  $\lambda^*$  for this decision can be bounded as

$$\lambda_{min}^* \leq \lambda^* \leq \lambda_{max}^*, \quad (10)$$

where  $\lambda_{min}^*$  is the system throughput if the successful probing probability is always  $\tilde{p}_{s,min}$ , and  $\lambda_{max}^*$  is the system throughput if the successful probing probability is always  $\tilde{p}_{s,max}$ .

*Proof:* The proof is straight-forward if we notice that the optimal reward  $\lambda^*$  in (9) monotonically increases as  $\tilde{p}_{s,t}$  increases. ■

To calculate the system throughput, note that the successful probing probability  $\tilde{p}_{s,t}$  increases as  $J_t$  increases. Hence  $\tilde{p}_{s,t}$  reaches its maximal value in the beginning when  $J_t = J$ . Based on this we can get an upper bound on the system throughput. To simplify our notation, we further make the following assumptions:

[A3] Each sub-channel has the same number of links, i.e.  $M_t^{(j)} = M_t^{(1)}$  for  $j = 1, \dots, J$ ;

[A4] All links are probing with the same probability, i.e.  $p_{(m)} = p$  for  $m = 1, \dots, M$ .

Thus all sub-channels have the same successful probing probabilities, i.e.  $p_{s,t}^{(j)} = p_{s,t}^{(1)}$  for  $j = 1, \dots, J$ .

*Theorem 1:* The system throughput of Fig. 2 is at most  $J\lambda_0^*$ , where  $\lambda_0^*$  is the solution to

$$E \left[ 1 - \frac{\lambda}{\tilde{R}_n} \right]^+ = \frac{\tau/T}{1 - [1 - p_{s,t}^{(1)}]^J}. \quad (11)$$

We can also have a lower bound on the system throughput if  $J_t$  is available for optimal decision making through some means.

*Theorem 2:* If  $J_t = \|\mathcal{J}_t\|$  is available for decision making in Fig. 2, the system throughput is at least  $\sum_{j=1}^J \gamma_j^*$ , where  $\gamma_j^*$  is the solution to

$$E \left[ 1 - \frac{\gamma}{\tilde{R}_n} \right]^+ = \frac{\tau/T}{1 - [1 - p_{s,t}^{(1)}]^j}. \quad (12)$$

*Proof:* We can see when  $J_t = j$ , the optimal system throughput is  $\gamma_j^*$ . Hence if there is at most one sub-channel that is decided to be utilized for data transmission at any time  $t$ , the total network throughput will be exactly  $\sum_{j=1}^J \gamma_j^*$ .

Now suppose  $J_t = j$ . At this time there are still  $j$  sub-channels involved in active channel probing and decision

making. Suppose  $\Delta$  sub-channels are decided to be utilized for data transmission at some point. Then the reward from these sub-channels are  $\Delta \cdot \gamma_j^*$ . We can easily see that

$$\Delta \cdot \gamma_j^* > \sum_{i=j-\Delta+1}^j \gamma_i^*. \quad (13)$$

To bound the system throughput, iterate  $j$  from the very beginning  $j = J$  and apply (13) when multiple sub-channels are decided to be utilized for data transmission at the same time. ■

Unfortunately, in ad-hoc networks  $J_t$  is not readily available for decision making. Starting from  $J_t = J$ , more and more sub-channels will be eventually utilized for data transmission as the procedure moves on. At any time more than one decisions over multiple sub-channels might be made to stop. Hence  $J_t$  is a random process which depends on the channel probing and decision making behavior. One solution to this problem is to conservatively use a *fixed* small  $J_0$  as the true  $J_t$  for decision making. We can get a lower bound on the system throughput if the protocol works in this way.

*Theorem 3:* If a fixed  $J_0$  is used in Fig. 2 to replace  $J_t$  when computing the optimal stopping rule, the system throughput is at least  $(J - J_0 + 1)\zeta^*$ , where  $\zeta^*$  is the solution to

$$E \left[ 1 - \frac{\zeta}{\hat{R}_n} \right]^+ = \frac{\tau/T}{1 - [1 - p_{s,t}^{(1)}]^{J_0}}. \quad (14)$$

*Proof:* To characterize the throughput from each decision, we divide the whole procedure into two phases:

- $J_t \geq J_0$ : The decision rule is more conservative as it is using a smaller  $\hat{p}_{s,t}$ . Hence the decision making will stop earlier and result in a reward  $\hat{\zeta}$ . Apparently we have  $\hat{\zeta} \geq \zeta^*$ . The first  $J - (J_0 - 1)$  sub-channels that are decided to be utilized for data transmission fall into this category.
- $J_t < J_0$ : The decision rule is more optimistic compared to the true situation. There is a chance that it will never stop properly since a larger threshold is used here. The worst case is that we get a total reward of 0 for these  $J_0 - 1$  sub-channels.

Now combine these two cases, we get a total throughput which is at least  $(J - J_0 + 1)\zeta^*$ . ■

## V. NUMERICAL RESULTS

In this section, we compare system throughput of Fig. 2 to that of the single-channel protocol under various constraints, as described by Theorem 1, 2 and 3 in Section IV.

We consider a wireless network with a total bandwidth  $W$ . Without loss of generality, we assume the bandwidth is 1 in certain units, e.g.  $W = 1$  MHz. We assume the wireless medium is Rayleigh fading within each block  $T = 1$ . Hence if the whole spectrum is used as a single wireless channel, its channel rate can be written as

$$R(h) = \log(1 + \rho h)$$

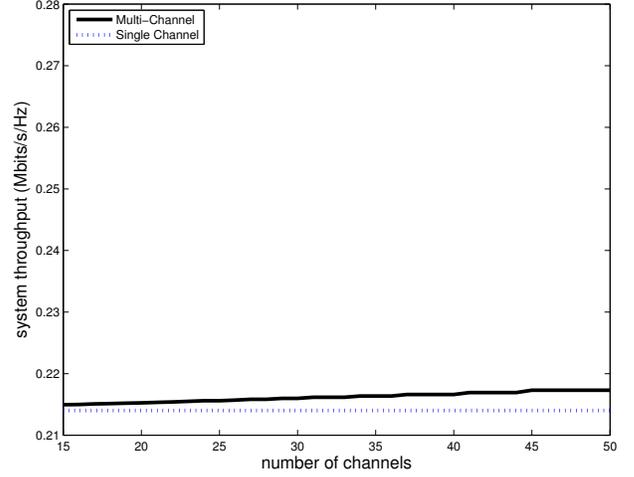


Fig. 3. System throughput with varying number of sub-channels in the network, where  $\tau = 0.02$ ,  $\rho = -10$  dB and  $\sigma = 1$ .

in Mbits/s/Hz, where  $\rho$  is the average signal-to-noise ratio (SNR), and  $h$  is the channel gain corresponding to Rayleigh fading. We write the probability density function (pdf) of  $h$  as

$$f(h; \sigma) = \frac{h}{\sigma^2} e^{-\frac{h^2}{2\sigma^2}}, \quad h > 0.$$

There are a total of  $M = 400$  links accessing the wireless medium with distributed opportunistic scheduling protocols.

For a multi-channel network, we split the total bandwidth evenly as  $\frac{W}{J} = \frac{1}{J}$ , where  $J$  is the number of sub-channels in the system. Accordingly the rate for each sub-channel can be written as

$$R^{(j)}(h) = \frac{1}{J} \log(1 + \rho h)$$

in Mbits/s/Hz, where  $j = 1, \dots, J$ .

We first show that it only marginally improves the system throughput if the opportunistic scheduling is working independently on each sub-channel. Fig. 3 shows the system throughput for this case, with parameters  $\tau = 0.02$ ,  $\rho = -10$  dB and  $\sigma = 1$ . The number of sub-channels  $J$  is varying from  $J = 15$  to  $J = 50$ . For comparison, the dotted line shows the system throughput for the single-channel network. For the single-channel system, the distributed opportunistic scheduling protocol is running where all links probe with probability  $p = \frac{1}{M}$ . For the multi-channel system, each sub-channel is running the single-channel protocol independently with  $p = 1/\lfloor \frac{M}{J} \rfloor$ . We can see it only yields a performance improvement of roughly 1.5% with  $J = 50$  sub-channels.

In Fig. 4, we show system throughput of the multi-channel opportunistic scheduling protocols based on various bounds discussed in Section IV. Similarly the dotted line shows the system throughput for the single-channel network. The dashdotted line shows the upper bound of the system throughput described in Theorem 1. We can see the system throughput quickly reaches a maximal value at a relatively medium  $J$ . It shows an increase of almost 22.4% in network throughput. The dashed line shows the lower bound of the network throughput

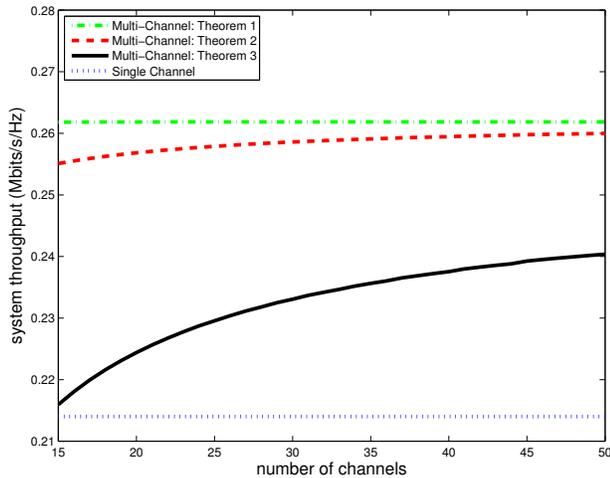


Fig. 4. System throughput with varying number of sub-channels in the network, where  $\tau = 0.02$ ,  $\rho = -10\text{dB}$  and  $\sigma = 1$ .

shown in Theorem 2, where  $J_t$  is available through other means for decision making. We can see that as  $J$  increases, it increases slower than the upper bound. For a large enough  $J$ , say  $J > 25$ , it shows an increase of 21.4% in network throughput. Finally, the solid line shows the network throughput of Fig. 2 if we simply use  $J_0 = 3$  in the decision making procedure. We can see that the network throughput increases much slower compared to the dashdot line. For  $J = 50$ , it shows an increase of 12.3% in the network throughput compared to the single-channel scenario. Hence even without any additional information, the distributed version of Fig. 2 can still improve the system throughput considerably.

## VI. CONCLUSIONS

In this paper, we studied one distributed opportunistic scheduling problem for ad-hoc networks with multiple independent sub-channels. The motivation is to divide the wireless spectrum into multiple sub-channels for better efficiency. We showed that a naive protocol where the opportunistic scheduling is designed independently within each sub-channel can only slightly improve the system throughput. We then came up with the idea of opportunistic scheduling across multiple sub-channels. We developed a multi-channel protocol for ad-hoc networks and analyzed its performance. We characterized the optimal decision rule and the system throughput. Through numerical results we showed that by joint optimization of the scheduling schemes across multiple sub-channels, the proposed protocol improves the network throughput considerably.

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