

## Cooperative Networked Systems: Multiple Graphs, Coalitional Games, New Probabilistic Models

John S. Baras

Institute for Systems Research and  
Department of Electrical and Computer Engineering and  
Fischell Department of Bioengineering  
Applied Mathematics, Statistics and Scientific Computation Program  
University of Maryland College Park  
{baras@umd.edu}

### Summary

Networked systems are ubiquitous. A taxonomy of networked systems includes infrastructure and communication networks, social and economic networks, biological networks and biological swarms, robotic swarms, and several other categories. Most networked systems include physical components and cyber components implemented in various ways. Thus many networked systems come from the domain of cyberphysical systems (CPS). Cooperative networked systems represent from a certain abstract point of view a distributed and (at least partially) asynchronous implementation of the well known cycle “sense-decide-actuate” of control systems. Yet the interaction of dynamics, sensing, control and decision making, with network structures has created the need for a new scientific basis for the modeling, analysis, design and performance evaluation of such networked systems. For example, efforts to develop a new “Network Science” have emerged. Similarly, efforts to develop new theories of computation that emphasize multiple agents interacting over networks, beyond the conventional concepts and principles of computation and computability, have been also undertaken very recently. In another direction, several efforts have been undertaken very recently to develop models for such systems that integrate continuous and event-triggered dynamics within a framework that permits validation and testing of correctness. Thus the consideration of cooperative networked systems have raised fundamental new questions and challenges.

In this lecture we consider networked systems from various domains including control, communications, sensing, sociology, economics and biology. We first describe a general model for modeling such systems that involves several interacting dynamic multigraphs. These multigraphs evolve in at least three planes. At the higher plane (layer) we have the network of cognitive agents, where decisions are made and executed. At the intermediate plane (layer) we have the information network, where data, models, observations and signaling are represented. At the lower plane (layer) we have the communication network that supports the information and agent networks. There are several ways to capture these ideas and principles, and the one presented in the lecture is one of the simpler possible representations. The lower layer is more connected to the physical layer, while the middle and higher layer are more logical (i.e. cyber). The networks involve have links and nodes that are annotated by weights that can be scalar, vector or even policies and rules. Furthermore the networks are dynamic. The resulting dynamic models are very complex and require a combination of methods from analysis, algebra, logic and optimization. The simplest possible model involves two interacting multigraphs: (a) the *collaboration multigraph*, which describes the time varying relation of collaboration between the agents (that is it answers the question: who has to collaborate with whom and when?); and (b) the *communication multigraph*, which describes the time varying communications that occur between the agents (that is it answers the question: who has to communicate with whom and when?) This representation is an important departure from typical models of cooperative networked systems used in the literature where the two multigraphs are considered as identical. We link these concepts to ideas from distributed computing, distributed programs and distributed computer hardware. We also link this representation to the behavior (what the system does?) and structure (what the system consists of?) models used in modern model-based-systems-engineering. We describe a novel path-oriented characterization of these activities in networked dynamic systems. The new framework allows us to pose several problems as generalized information flows and their control.

Next we introduce three fundamental problems and challenges emerging from this framework, which we consider essential for progress in cooperative networked systems. The first addresses the joint analysis of the collaboration and communication multigraphs and their impact on the performance of the networked system. We show initial results in this direction that indicate that progress on this problem is essential for understanding *autonomy*. We

develop the framework of constrained coalitional games and we show that it provides a promising methodology for modeling and analyzing the interaction between the collaboration and communication multigraphs. We show that it captures in a fundamental way the basic tradeoff of benefits vs. cost of collaboration, in networked systems. We demonstrate that various simple models of constrained coalitional games can explain network formation and the emergence or not of collaboration. Towards developing fundamental principles for autonomy we investigate the following key question: is it better to have a self-organized communication multigraph capable of supporting (in a scalable and fast manner) a rich set of distributed tasks (collaboration multigraphs), or a communication topology optimized to a particular distributed task? We demonstrate by several examples (some from biology) that the preferred solution from an autonomy perspective appears to be the former. We also link this challenge to generalized networks, by interpreting the generalized weights of nodes as generalized potentials and the generalized weights of links as generalized resistances. We discuss connections to optimization and dynamic games (including various forms of potential games), statistical physics, and several properties of the resulting systems such as stability and robustness.

The second fundamental problem addresses the development of a taxonomy of collaboration and communication multigraphs, from the perspective of system performance. The emphasis is on finding classes of graph topologies with favorable tradeoff between performance improvement (i.e. benefit) of collaborative behaviors vs. the cost of collaboration (e.g. implied by communications). We investigate the interrelationship between the collaboration and communication graphs in networked control systems and the role of the communication graph topology, among the collaborating agents, in improving the performance of distributed algorithms on graphs, such as convergence speed. We show that Small World graphs emerge as a good tradeoff between performance and efficiency in consensus problems, where the latter serves as a prototypical coordination problem. We show that these ideas lead to a self-organization of the communication graph in a two level hierarchy that leads to substantial speed of execution of distributed algorithms by the agents. We discuss extensions to expander graphs and present several results on designing communication topologies for collaborative control, some inspired from biology. We discuss several properties of expander graphs and link them to solutions of graph optimization problems. An intuitive explanation, for several of these useful properties, is that expander graphs are “globally sparse but locally rich”. We discuss the problem of controlling the topology of the communication graph so as to remain within the class of expander graphs and show that it is a relatively easy task to accomplish such topology control via a distributed manner. We link again several of these results to generalized networks, generalized potentials and generalized resistances.

The third fundamental problem addresses the need for different probability models for such cooperative networked systems. We take a fresh look at the probability and information models that can support the analysis and synthesis of distributed, networked multi-agent systems operating with at least partial asynchrony and with partial information. We relate the new framework, emerging from the results of the first and second problems, and its basic constructs, to information and control patterns, generalized information theory and entropy, and to distributed computing with local states. This new framework indicates the need for a new kind of probability over dynamical logical structures that is reminiscent of the axiomatic framework of quantum physics. We demonstrate that such systems cannot support probability models based on Kolmogorov-type models including cartesian products of the same. The probabilities needed have to be built on a logic model that is based on observables and on the geometry of subspaces of finite dimensional Hilbert spaces rather than the logic of subsets of sets. The reason is primarily that these asynchronous, distributedly operating and partially observed systems, do not provide the data needed to support the rigorous construction of a Kolmogorov-style model. We show that with these new probability models and the associated information models several difficult problems in distributed inference and control of networked systems become convex optimization problems. A consequence (result) of key significance is a suggested and more accurate way of modeling the information patterns involved in such systems. We link these results to similar very recent work on distributed information retrieval systems over the Web, certain logics and certain types of dynamical games with partial information. We also link these models to a new view of what “computation” means in cooperative networked systems.

We close with a brief list of interesting future research directions.

## References

1. J.S. Baras, "Noncommutative Probability Models in Quantum Communication and Multi-Agent Stochastic Control", *Recherche di Automatica*, Vol. X, No. 2, pp. 21-265, December 1979.

2. J.S. Baras, "Distributed Asynchronous Detection: General Models", invited paper, *Proceedings of the 26th IEEE Conference on Decision and Control*, pp. 1832-1835, Los Angeles, California, December 9-11, 1987.
3. J. S. Baras, "Multi-Agent Stochastic Control: Models Inspired from Quantum Physics", *Proceedings of 2003 International Conference, Physics and Control (PhysCon)*, pp. 747-758, Saint Petersburg, Russia, August 20-22, 2003.
4. W. Xi, X. Tan, J.S. Baras, "Gibbs Sampler-based Self-organization of Autonomous Swarms", *Automatica*, pp. 1107-1119, July 2006.
5. T. Jiang and J. Baras, "Trust Evaluation in Anarchy: A Case Study on Autonomous Networks", *Proceedings of the 25th IEEE Conference on Computer Communications (Infocom06)*, Barcelona, Spain, April 25-27, 2006.
6. T. Jiang and J. S. Baras, "Fundamental Tradeoffs and Constrained Coalitional Games in Autonomic Wireless Networks", *Proceedings of 5th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks*, pp. 1-8, Limassol, Cyprus, April 16-20, 2007.
7. J. S. Baras and P. Hovareshti, "Effects of Topology in Networked Systems: Stochastic Methods and Small Worlds", *Proceedings of the 47th IEEE Conference on Decision and Control*, pp. 2973-2978, Cancun, Mexico, December 9-11, 2008.
8. P. Hovareshti, J. S. Baras and V. Gupta, "Average Consensus over Small World Networks: A Probabilistic Framework", *Proceedings of the 47th IEEE Conference on Decision and Control*, pp. 375-380, Cancun, Mexico, December 9-11, 2008.
9. P. Purkayastha and J.S. Baras, "An Optimal Distributed Routing Algorithm using Dual Decomposition Techniques", *Communications in Information and Systems, Brockett Legacy Issue, International Press*, Vol. 8, Issue 3, pp 277-302, 2008.
10. T. Jiang and J. S. Baras, "Coalition Formation Through Learning in Autonomic Networks", *Proceedings of the International Conference on Game Theory for Networks (GameNets09)*, pp. 10-16, Istanbul, May 13-15, 2009.
11. J. S. Baras, P. Hovareshti and S. Perumal, "Event Triggered Distributed Collaborative Control", Invited paper, *Proceedings of the European Control Conference 2009 (ECC'09)*, Budapest, Hungary, August 23-26, 2009.
12. J. S. Baras, T. Jiang and P. Hovareshti, "Coalition Formation and Trust in Collaborative Control", Invited paper, *Proceedings of the European Control Conference 2009 (ECC'09)*, Budapest, Hungary, August 23-26, 2009.
13. J. S. Baras and P. Hovareshti, "Efficient and Robust Communication Topologies for Distributed Decision Making in Networked Systems", *Proceedings of the 48th IEEE Conference on Decision and Control (CDC 2009)*, pp. 3751-3756, Shanghai, China, December 16-18, 2009.
14. J.S. Baras and G. Theodorakopoulos, *Path Problems in Networks*, Lecture Notes, Synthesis Lectures Series, Morgan & Claypool Publishers, January 2010.
15. X. Tan, W. Xi and J. S. Baras, "Decentralized Coordination of Autonomous Swarms Using Parallel Gibbs Sampling", *Automatica*, Vol. 46, No. 12, pp. 2068-2076, 2010.
16. P. Hovareshti and J. S. Baras, Efficient communication Infrastructures for Distributed Control and Decision Making in Networked Stochastic Systems, *Proceedings of the 19th International Symposium on Mathematical Theory of Networks and Systems (MTNS 2010)*, Budapest, Hungary, July 5-9, 2010.
17. K. Somasundaram and J. S. Baras, "Path Optimization and Trusted Routing in MANET: An Interplay Between Ordered Semirings", *Proceedings of The Second International Conference on Networks & Communications (NetCoM - 2010)*, pp. 88-98, Chennai, India, December 27-29, 2010.
18. K. Somasundaram and J. S. Baras, "Solving Multi-metric Network Problems: An Interplay Between Idempotent Semiring Rules", *Linear Algebra and Applications, Special Issue on the occasion of 1st Montreal Workshop on Idempotent and Tropical Mathematics, April 1, 2011*, doi:10.1016/j.laa.2011.02.055
19. I. Matei and J. S. Baras, "The Geometry of a Probabilistic Consensus of Opinion Algorithm", *Proceedings of The 2011 American Control Conference*, San Francisco, CA, June 29-July 01, 2011.
20. J.S. Baras, P. Hovareshti and H. Chen, "Motif-based Communication Network Formation for Task Specific Collaboration in Complex Environments", *Proceedings of The 2011 American Control Conference*, San Francisco, CA, June 29-July 01, 2011.