

"Distributed Control of a Timoshenko Beam"

(with Yan Zhuang)

Proceedings of the
Third International Conference
on Adaptive Structures

San Diego, CA
November 9-11, 1992

THIRD INTERNATIONAL CONFERENCE ON ADAPTIVE STRUCTURES

November 9-11, 1992
San Diego, California, U.S.A.

Edited by
**BEN K. WADA
MICHIHIRO NATORI
ELMAR BREITBACH**



DISTRIBUTED CONTROL OF A TIMOSHENKO BEAM

Yan Zhuang,¹ John S. Baras²

ABSTRACT

We consider a flexible beam clamped to a rigid base on one end with the other end free. In order to stabilize the beam vibration, we introduce active damping into the beam with feedback control using distributed actuators and sensors. We apply Timoshenko beam theory to model the substructure. Unlike the familiar Euler-Bernoulli beam model, the shear effect and the rotational inertia are included in the modeling of elastic deformation to obtain a more precise beam model. The piezoelectric theory is briefly reviewed. The piezoelectric ceramic material (PZT) is used to build the distributed actuator. The distributed sensor is made of piezoelectric polymer polyvinylidene fluoride (PVDF). The sensor and actuator are layers which are attached directly to both sides of the beam. Based on the constitutive properties and layer geometry, the models for sensor and actuator are developed. We then embed the static actuator and sensor models into the beam substructure to form the model of the composite beam. We design the feedback controller by Lyapunov direct methods based on the energy functional of the system. It is proved that the derived controller can extract energy from the system and increase system damping. The resulting closed loop system is asymptotically stable. Since this method does not depend on model truncation, we further analyze the combined effect of the distributed controller on the relevant vibration modes. The integrated sensor and actuator can monitor the oscillation and suppress the unwanted modes. It is possible to suppress the selected modes by choosing the appropriate actuator layout. It is also shown that by properly installing the sensor and determining the sensor shape function, we can further extract and manipulate the sensor signal for our control need.

INTRODUCTION

Flexible structure control and stabilization have long been an interesting field. Its applications can be found in many areas such as vibration suppression in mechanical

¹Electrical Engineering Department and Institute for Systems Research, The University of Maryland, College Park, MD 20742, U.S.A.

²Martin Marietta Chair in Systems Engineering, Electrical Engineering Department and Institute for Systems Research, The University of Maryland, College Park, MD 20742, U.S.A.

engineering and structure control in civil engineering. One of the challenges comes from the applications in control of flexible aerospace systems and robotics. Due to the limited launching load, the space structures are usually large in size, light in mass and hence weakly damped. In order to implement attitude control of increasing pointing precision, active damping is required to enhance the system's stability. Problems in controller design for large flexible space structures include quickly damping out the pointing errors resulting from step disturbances, or nonzero initial conditions (e.g., resulting from slewing) and maintaining the desired attitude as close as possible in the presence of disturbances. We mainly address the damping problem here.

Proper modeling is essential to control system design and to avoid spillover due to the infinite dimensional nature of these systems. Modeling the structure's damping, appropriate interface conditions, different geometric configurations and various composite materials is a challenging research topic. Even a reasonable mathematical model may involve large number of coupled variables, nonlinearities and complicated boundary conditions. It is a formidable task to fully explore the solution and the stability of the solution, if not impossible. It is natural that current research in dynamic control of flexible structures mainly focuses on the basic components of the whole structure such as string, beam and plate. Boundary control of flexible beams is addressed in many articles such as [6], [9]. In [7] a detailed analysis and references on this problem are provided.

We are interested in distributed active damping using smart materials. The actuator considered here is a spatially distributed one, made of piezoelectric ceramic material (PZT) which is glued to the beam. Its constitutive property, i.e. its strain and stress relation, is influenced by the external voltage applied to it. Bonding or embedding segmented elements of this material in a structure would allow the application of localized strain developed in the actuator to be transferred to the structure whose deformation can be controlled. Under proper bonding conditions, the coupling between the actuator and the substructure can be determined to implement the control mechanism. The dynamical behavior of the composite beam can be changed by implementing appropriate control algorithms. In [4], [1], active vibration control is described using spatially distributed actuators. The PVDF sensor is bonded to the beam in a similar way. The output voltage is a functional of beam curvature. Unlike the conventional point sensor, this is a distributed one. Lee and Moon [8], Cudney [5] and Tzou [10] provide some detailed explanation of the nature of piezoelectric actuators and sensors.

We consider here the beam as a flexible structure and study its modeling and active damping. The Timoshenko beam model is used in the analysis. Since the model accounts for the shear effect and the rotational inertia, it represents more precisely the physical nature of the beam than the Euler-Bernoulli beam model does. We first discuss the modeling of the beam and the distributed actuator. A static model of the actuator coupled into the structure is developed. Next the sensor model is addressed. We then discuss the controller design using Lyapunov methods. We finally investigate the actuator and sensor shapes and their impact on the system's elastic modes.

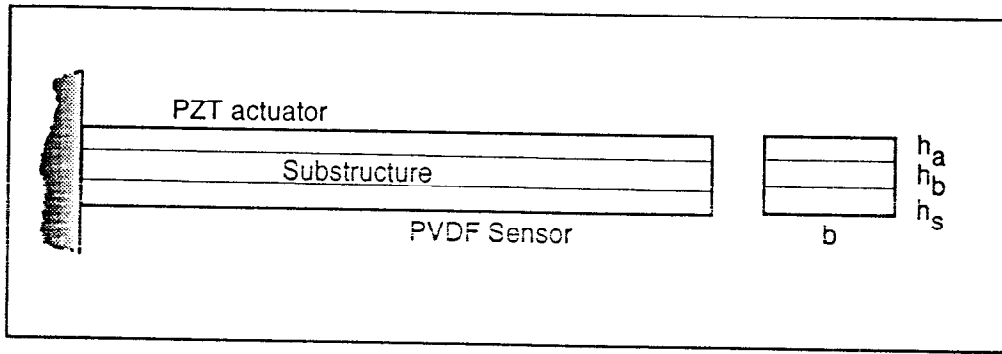


Figure 1: The composite beam

SYSTEM MODEL

The system model consists of the beam substructure with actuator and sensor layers glued on both sides of the beam. We consider the beam and the actuator model here.

One approach to build the desired actuator is to take advantage of the special constitutive properties of certain materials. The actuation produced is due to the property change of the material subject to certain stimulation other than the external actuation force. Such materials are the so called smart materials. Once properly embedded to the structure the induced actuation will produce bending or stretching or both to control the structure deformation. Since the actuator can be built into the structure the overall structure design can be optimized. One of the advantages of using smart materials as sensors and actuators is that that the after-fact structure change and additional weight can be avoided.

Piezoelectric actuators were used as elements of intelligent structures by Crawley and de Luis [4]. Bailey and Hubbard [1] have used PVDF actuators to control the vibration of a cantilever beam. The control voltage applied across the actuator is the sign of the tip rotation velocity multiplied by a constant so as to introduce active damping. We studied the active damping problem with distributed actuator and sensor with the rotational inertial included in the beam model [2].

Figure 1 shows the structure of the beam with both the sensor and the actuator layers glued together.

In this figure, h stands for the thickness of the different layers of the composite beam. The subscripts s , b and a denote sensor, beam and actuator respectively. The stress-strain relation for the piezoelectric material is similar to that of the thermoelastic materials, with the thermal strain term replaced by the piezoelectric strain Λ . The constitutive equation of the actuator is given by

$$\sigma = E_a(\epsilon - \Lambda) \quad (1)$$

where Λ is the actuation strain due to the external electric field, and ϵ is the strain without external electric field. E_a is the Youngs' modulus of the actuator, σ is the stress of the actuator. The actuation strain is given by

$$\Lambda(x, t) = \frac{d_{31}}{h_a} V(x, t) \quad (2)$$

where d_{31} is the piezoelectric field and strain field constant. $V(x, t)$ is the distributed voltage. Suppose the bonding between the actuator layer and the beam is perfect, i.e., there is no shear lag layer in between them, the induced strain has two effects on the beam. One effect is that it introduces a longitudinal strain ϵ_l to insure a force equilibrium along the axial direction. The steady state value of ϵ_l can be derived by solving a force equilibrium equation. The second effect is that the net force in each layer acts through the moment arm with the length from the midplane of the layer to the neutral plane of the beam. The result of the actuation produces the bending moment which is introduced as the control mechanism. Taking a similar approach as in [1], the actuation moment can be expressed as

$$M_a = K_a \Lambda(x, t) \quad (3)$$

where K_a is a constant depending on the geometry and the materials of the beam.

We use the Timoshenko beam model to describe the dynamical behavior of the beam. Unlike the Euler-Bernoulli beam model, the Timoshenko model contains the rotational inertial and shear effect of the actual beam. The analysis of the latter is more complicated. The beam model is given as

$$\rho A \frac{\partial^2 w}{\partial t^2} = kAG \left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial \Phi}{\partial x} \right), \quad (4)$$

$$\rho I \frac{\partial^2 \Phi}{\partial t^2} = EI \frac{\partial^2 \Phi}{\partial x^2} + kAG \left(\frac{\partial w}{\partial x} - \Phi \right). \quad (5)$$

Here t is the time variable and x is the space coordinate along the beam in its equilibrium configuration. $w(x, t)$ is the displacement of the centroid from its equilibrium line which is described by $w = 0$. Φ denotes the deflection curve when the shearing force is neglected. The total slope of deflection is

$$\frac{dw}{dx} = \Phi + \beta,$$

where β is the angle of shear. ρ , I and E are mass density, moment of inertial of cross section and Young's modulus respectively. k is a numerical factor depending on the shape of the beam while A and G are area of cross section and modulus of elasticity in shear.

The bending moment of the composite beam without actuation is

$$M_b = EI \frac{\partial \Phi}{\partial x}, \quad (6)$$

where

$$EI = E_a I_a + E_b I_b + E_s I_s. \quad (7)$$

The bending moment of the beam with actuation moment is

$$M = M_a + M_b \quad (8)$$

Using this augmented moment to replace the moment term in the original formulation of equations of motion, we obtain

$$\rho A \frac{\partial^2 w}{\partial t^2} = kAG \left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial \Phi}{\partial x} \right), \quad (9)$$

$$\rho I \frac{\partial^2 \Phi}{\partial t^2} = EI \frac{\partial^2 \Phi}{\partial x^2} + kAG \left(\frac{\partial w}{\partial x} - \Phi \right) + c \frac{\partial V}{\partial x} \quad (10)$$

with boundary conditions

$$\begin{aligned} w(0, t) &= 0, \\ \Phi(0, t) &= 0, \\ \frac{\partial w(L, t)}{\partial x} - \Phi(L, t) &= 0, \\ EI \frac{\partial \Phi(L, t)}{\partial x} &= 0, \end{aligned} \quad (11)$$

where

$$c = \frac{d_{31}}{h_a} K_a.$$

The distributed voltage $V(x, t)$ is the control applied to the system. Equations (9) (10) and boundary condition (11) form the control system model. The actuation appears in the system in the form of a distributed bending moment in the rotational equation (10).

SENSOR MODEL

A spatially distributed sensor is modeled here to provide the sensing signal for the control system in consideration. The distributed sensor is the one whose output is a function of structural responses at different locations. It can be a group of point sensors or a spatially continuous one or the combination of the both. These structural responses are obtained either discretely or continuously in space. Using the latter has the advantage of simplifying the complicated computations based on the point measurements since the sensor geometry can be tailored to provide the necessary computation. The spatial aliasing from an array of sensors can be avoided. Typical noncausal sensor dynamics such as gain rolloff without phase shift is possible by using distributed sensors [3].

Piezoelectric polymer polyvinylidene fluoride (PVDF) is strain sensitive and relies on the applied strain to produce electrical charge. The amount of electrical charge is proportional to the amount of strain induced by the structure. This process is the reverse to piezoelectric actuation. It is also based on the constitutive property of the sensor material. The induced charge per unit length from the strain is

$$q(x, t) = -E_s d_{31} \epsilon_s. \quad (12)$$

where E_s is Young's modulus of the sensor material.

The sensor strain is related to the beam curvature by

$$\epsilon_s = \frac{h_b + h_s}{2} \frac{\partial \Phi}{\partial x}. \quad (13)$$

This is based on the assumption that the neutral layer of bending is close to the geometric centroid of the composite beam. The electrical charge along the beam is

$$\begin{aligned} Q(x, t) &= \int_0^x q(x, t)F(x)dx \\ &= -E_s d_{31} \frac{h_b + h_s}{2} \int_0^x F(x) \frac{\partial \Phi}{\partial x} dx, \end{aligned} \quad (14)$$

where $F(x)$ is the weight function or shape function of the sensor; it is the local width of the electrodes covering both sides of the sensor. Since the PVDF is only a thin layer, its shape can be easily changed according to different needs for interpreting the sensor signals. The output of the sensor is

$$\begin{aligned} V_s(x, t) &= \frac{Q(x, t)}{C} \\ &= -K_s \int_0^x F(x) \frac{\partial \Phi}{\partial x} dx \end{aligned} \quad (15)$$

where

$$K_s = \frac{E_s d_{31} (h_b + h_s)}{2C} \quad (16)$$

is a constant with C being the capacitance between the electrodes of the sensor layer. Suppose the sensor covers the whole beam, then

$$V_s(t) = -K_s \int_0^L F(x) \frac{\partial \Phi}{\partial x} dx. \quad (17)$$

Equation (17) is the sensor output equation. The output voltage is the weighted integral of the beam curvature. Integrating by parts the right hand side of the above expression once in the spatial variable, we have another form of the sensor output.

$$V_s(t) = -K_s \Phi(L, t)F(L) + K_s \int_0^L \Phi(x, t) \frac{\partial F(x)}{\partial x} dx. \quad (18)$$

We shall observe later that the format of Equation (18) can be manipulated according to our control needs.

CONTROL ALGORITHM

We design the control algorithm by Lyapunov's direct method since no model truncation is required with this method. The energy functional is used to measure the amount of vibration of the system. We seek a control law in such a way that active damping is introduced into the system and the closed loop system is asymptotically stable.

Given the system (9), (10) with boundary conditions (11) and an energy functional $E(t)$, we need to find a control $V(x, t)$ such that

$$\lim_{t \rightarrow \infty} E(t) = 0. \quad (19)$$

It suffices to find a control $V(x, t)$, such that

$$\frac{dE(t)}{dt} < 0, \text{ for } t > 0. \quad (20)$$

We define the energy function as follows:

$$E(t) = \frac{1}{2} \int_0^L \left\{ \rho A \left[\frac{\partial w}{\partial t} \right]^2 + \rho I \left[\frac{\partial \Phi}{\partial t} \right]^2 + K \left[\frac{\partial w}{\partial x} - \Phi \right]^2 + EI \left[\frac{\partial \Phi}{\partial x} \right]^2 \right\} dx. \quad (21)$$

where

$$K = kAG.$$

The first two terms in the integral are the kinetic energy terms from vertical displacement and cross section rotation of the beam. The third term is the energy due to shear deformation while the last term represents the stored energy from bending.

Taking the derivative of $E(t)$ with respect to time, we have

$$\begin{aligned} \frac{dE(t)}{dt} &= \int_0^L \left\{ \rho A \frac{\partial w}{\partial t} \frac{\partial^2 w}{\partial t^2} + \rho I \frac{\partial \Phi}{\partial t} \frac{\partial^2 \Phi}{\partial t^2} \right. \\ &\quad \left. + K \left(\frac{\partial w}{\partial x} - \Phi \right) \left(\frac{\partial^2 w}{\partial x \partial t} - \frac{\partial \Phi}{\partial t} \right) + EI \frac{\partial \Phi}{\partial x} \frac{\partial^2 \Phi}{\partial x \partial t} \right\} dx. \end{aligned} \quad (22)$$

Integrating by parts in the spatial variable and incorporating the system equations (9) and (10) into it, we obtain the simplified form

$$\frac{dE(t)}{dt} = K \frac{\partial w}{\partial t} \left(\frac{\partial w}{\partial x} - \Phi \right) \Big|_0^L + \frac{\partial \Phi}{\partial t} EI \frac{\partial \Phi}{\partial x} \Big|_0^L + \int_0^L c \frac{\partial \Phi}{\partial t} \frac{\partial V}{\partial x} dx. \quad (23)$$

Further, using the boundary conditions (11), we arrive at

$$\frac{dE(t)}{dt} = \int_0^L c \frac{\partial \Phi}{\partial t} \frac{\partial V}{\partial x} dx. \quad (24)$$

The first two terms in (23) vanish.

Let the control $V(x, t)$ be decomposed as the product of a spatial function and a temporal function,

$$V(x, t) = v(x)q(t) \quad (25)$$

Here $v(x)$ is the actuator shape function and $q(t)$ is the coordinate function. Substituting $V(x, t)$ into (23), we obtain

$$\frac{dE(t)}{dt} = cq(t) \int_0^L \frac{\partial \Phi}{\partial t} \frac{dv(x)}{dx} dx. \quad (26)$$

Function $q(t)$ can be determined by using sensor output V_s from (18) with $F(x)$ compact on $[0, L]$,

$$\begin{aligned} q(t) &= -\frac{dV_s(t)}{dt} \\ &= -K_s \int_0^L \frac{\partial \Phi}{\partial t} \frac{dF(x)}{dx} dx. \end{aligned} \quad (27)$$

The corresponding control $V(x, t)$ is given by

$$V(x, t) = -K_s v(x) \int_0^L \frac{\partial \Phi}{\partial t} \frac{dF(x)}{dx} dx \quad (28)$$

Then the rate of energy change (26) becomes

$$\begin{aligned} \frac{dE(t)}{dt} &= -cq(t) \int_0^L \frac{\partial \Phi}{\partial t} \frac{dv(x)}{dx} dx \\ &= -cK_s \int_0^L \frac{\partial \Phi}{\partial t} \frac{\partial F(x)}{\partial x} dx \int_0^L \frac{\partial \Phi}{\partial t} \frac{dv(x)}{dx} dx. \end{aligned} \quad (29)$$

It is sufficient to select functions $F(x)$ and $v(x)$ such that

$$\frac{dF(x)}{dx} \frac{dv(x)}{dx} > 0 \quad (30)$$

which gives

$$\frac{dE(t)}{dt} \leq 0. \quad (31)$$

The closed loop system is energy dissipative. The feedback control (26) can be expressed in another form as

$$V(x, t) = -K_s v(x) \left. \frac{\partial \Phi}{\partial t} F(x) \right|_0^L + K_s v(x) \int_0^L \frac{\partial^2 \Phi}{\partial x \partial t} F(x) dx. \quad (32)$$

The feedback control (26) is velocity feedback and actually provides damping of Voigt type. The second term in (26) is the weighted integral of the rate of change of the beam curvature with respect to time. Since the curvature of the beam is proportional to the strain of the beam in this formulation, the controlled beam has an altered constitutive equation. The stress is no longer just proportional to the strain, but the linear combination of both strain and the rate of change of the strain with respect to time.

We may propose another control algorithm as

$$q(t) = C_1 V_s(t) + C_2 \frac{dV_s(t)}{dt}. \quad (33)$$

The corresponding control $V(x, t)$ appears in the form of

$$V(x, t) = -C_1 K_s v(x) \int_0^L \Phi \frac{dF(x)}{dx} dx - C_2 K_s v(x) \int_0^L \frac{\partial \Phi}{\partial t} \frac{dF(x)}{dx} dx. \quad (34)$$

We anticipate that this control algorithm would increase the natural frequency of the vibration of the beam. The integration of the weighted curvature along the beam is introduced in the feedback. It is similar to introducing position and velocity feedback in the traditional PID control. The constant C_1 is to be determined for the required performance.

MODAL ANALYSIS

We discussed sensor shape and actuator shape and their impact to the control of a cantilever beam in [2]. Different sensor outputs are available by choosing the appropriate sensor shape function $F(x)$. PVDF can be segmented along the beam to collect the signals from local regions. The sensor can provide deflection and strain information of the beam. A sensing network with simple computations shall produce different signals in one sample period for the need of the control system.

We use a Galerkin procedure to implement modal expansion and to analyze the impact of the control system on the flexible beam. We write the beam lateral displacement $w(x, t)$ and cross section rotation $\Phi(x, t)$ as

$$w(x, t) = \sum_{k=1}^n P_k(x) d_k(t) \quad (35)$$

$$\Phi(x, t) = \sum_{k=1}^n \Psi_k(x) q_k(t) \quad (36)$$

where $P_k(x)$ and $\Psi_k(x)$ are the admissible functions belonging to $H_0^2[0, L]$. These functions can be chosen such that $\{P_k\}$ form a normalized orthogonal basis and $\{\Psi_k\}$ form another normalized orthogonal basis. $d_k(t)$ and $q_k(t)$ are their time coordinates respectively.

We rewrite here the control $V(x, t)$ from (25)

$$V(x, t) = v(x)q(t) \quad (37)$$

Substituting the modal forms (35) (36) and (25) into system's equations (9) and (10), multiplying P_l and Ψ_m to both sides of the first and second equation respectively, integrating both equation with respect to the spatial variable, we arrive at

$$\rho A \int P_k P_l dx \ddot{d}_k(t) = K \sum \int P_k^{(2)} P_l dx d_k(t) - K \sum \int \Psi_k^{(1)} P_l q_k(t) \quad (38)$$

$$\begin{aligned} \rho I \int \Psi_k \Psi_m dx \ddot{q}_k(t) &= K \sum \int P_k^{(1)} \Psi_m dx d_k(t) + \sum (EI \int \Psi_k^{(2)} \Psi_m dx \\ &\quad - K \int \Psi_k \Psi_m dx) q_k(t) + c \int v^{(1)} \Psi_m dx q(t). \end{aligned} \quad (39)$$

where \dot{d} and \ddot{d} stands for the first and second time derivative of the function $d(t)$, $P^{(i)}(x)$ stands for the i^{th} spatial derivative of $P(x)$.

The orthogonal property of the base functions $P_k(x)$ and $\Psi_k(x)$ yields

$$\rho A \ddot{d}_l(t) - K \sum \int P_k^{(2)} P_l dx d_k(t) + K \sum \int \Psi_k^{(1)} P_l q_k(t) = 0 \quad (40)$$

$$\begin{aligned} \rho I \ddot{q}_m(t) - K \sum \int P_k^{(1)} \Psi_m dx d_k(t) - \sum (EI \int \Psi_k^{(2)} \Psi_m dx \\ - K \int \Psi_k \Psi_m dx) q_k(t) - c \int v^{(1)} \Psi_m dx q(t) = 0 \end{aligned} \quad (41)$$

The last term in the second equation is the contribution of the distributed control.

Equations (40) and (41) can be rearranged into a $2n \times 2n$ matrix form

$$M\ddot{u}(t) + C\dot{u}(t) + Ku = 0, \quad (42)$$

where $u(t) = [d_1(t) \cdots d_n(t)q_1(t) \cdots q_n(t)]^t$ is the vector of coordinate functions. M is the mass matrix. It is diagonal in this case with the first n elements being ρA with the rest being ρI . C is the damping matrix whose nonzero elements are given by

$$c_{(n+l)(n+k)} = cK_s \int v^{(1)}(x)\Psi_l(x)dx \int F^{(1)}(x)\Psi_k(x)dx, \quad 1 \leq l, k \leq n \quad (43)$$

which are introduced by the feedback control $V(x, t)$. It is known from (27) that

$$\begin{aligned} q(t) &= -K_s \int_0^L \frac{\partial \Phi}{\partial t} \frac{dF(x)}{dx} dx \\ &= -K_s \int_0^L \Psi_k F^{(1)} dx \dot{q}_k(t) \end{aligned} \quad (44)$$

K is stiffness matrix with elements

$$\begin{aligned} k_{lk} &= -K \int P_k^{(2)} P_l dx, \\ k_{l(n+k)} &= K \int \Psi_k^{(1)} P_l dx, \quad 0 \leq l, k \leq n \\ k_{(n+l)k} &= K \int P_k^{(1)} \Psi_m dx, \\ k_{(n+l)(n+k)} &= K \int \Psi_k \Psi_m dx - EI \int \Psi_k^{(2)} \Psi_m dx. \end{aligned} \quad (45)$$

Observing (43), we notice that the damping introduced to the system depends on the sensor and actuator shape functions $F(x)$ and $v(x)$. The amount of damping involved is limited by the sensor and actuator coefficients K_s and c which are determined by the materials and their manufacturing. The coefficient c_{lk} describes the amount of damping introduced to the l th mode by the control system based on the measurement of the k th mode. The similarity holds for the actuator as well. Hence it is possible to enhance damping to some undesired elastic modes by modal analysis and by choosing the sensor and actuator shape functions $F(x)$ and $v(x)$ accordingly. The actual control systems can be implemented by segmented actuators and sensors with a switching network to provide different configurations in each sample period.

It is an interesting question to choose the shape functions for the distributed sensors and actuators. The purpose is to introduce damping to different vibration modes efficiently rather than excite some undesired modes. The concentrated points or regions of control moment should be placed away from the nodal points of the vibration modes to assert the maximum control effect. Our case here can be formulated as optimization of certain performance measurement functional by choosing the candidate functions $F(x)$ and $v(x)$ subject to some constraints on the control $V(x, t)$. We anticipate that the optimal layout of sensors and actuators provide good performances.

CONCLUSIONS

We have embedded a static distributed PZT actuator model and a distributed sensor model into the Timoshenko beam model to form a composite beam model. We then design a closed loop controller to introduce damping to the system using Lyapunov's direct method. The closed loop system is proved to extract energy from the system. This composite model is more precise in describing the movement of the beam compared to the Euler-Bernoulli beam model. We then use modal analysis to further explain the induced damping by the controller and the a method to choose the appropriate sensor and actuator shapes. It is possible to monitor and suppress the undesired modes by using suitably distributed sensors and actuators.

Further research is needed regarding the damping of beams with nonlinear deformation and comparing the results from the linear models with those from the nonlinear geometric exact model. An efficient numerical approach is also needed for the latter. The optimal distributed sensor and actuator layout is to be found through optimization with respect to the appropriate criteria. The real impacts of the distributed control system on the beam should be verified and further explored by experiments.

ACKNOWLEDGMENT

This research was partially supported by NSF grant NSFD CDR 8803012 through the Engineering Research Center's Program and AFOSR URI grant 90-01054.

REFERENCES

- 1 Bailey, T. and J.E. Hubbard Jr. 1985. "Distributed Piezoelectric-Polymer Active Vibration Control of a Cantilever Beam." Journal of Guidance, Control and Dynamics, 8(5):605-611. Sept.-Oct..
- 2 Baras, J.S. and Y. Zhuang. 1992. "Active Vibration Damping Using Smart Material." In Proceedings of the Workshop on Distributed Parameter Modeling and Control of Flexible Aerospace Systems. Williamsburg, Virginia, June 8-10. NASA Langley Flight Research Center.
- 3 Collina, S.A., D.W. Miller, and A.H. von Flotow. 1991. "Piezopolymer Spatial Filters for Active Structural Control." In C.A. Rogers and C.R. Fuller, editors, Recent Advances in Active Control of Sound and Vibration, pages 64-77. VPI&SU, Blacksburg, Virginia, April 15-17. NASA Langley Research Center and others, Technomic Publishing Co. Inc.
- 4 Crawley, E.F. and Javier de Luis. 1987. "Use of Piezoelectric Actuators as Elements of Intelligent Structures," AIAA Journal, 25(10):1373-1385.
- 5 Cudney, H.H. 1989. Distributed Structural Control Using Multilayered Piezoelectric Actuators. PhD thesis, SUNY Buffalo, NY.

- 6 Kim, J.U. and Y. Renardy. 1987. "Boundary Control of the Timoshenko Beam," SIAM Journal of Control and Optimization, 25(6):1417-1429, November.
- 7 Lagnese, J.E. 1989. Boundary Stabilization of Thin Plates. SIAM, Philadelphia.
- 8 Lee, C. K. and F. C. Moon. 1990. "Modal Sensors/Actuators," Journal of Applied Mechanics, 57:434-441, June. Trans. of the ASME.
- 9 Morgul, O. 1992. "Dynamic Boundary Control of a Euler-Bernoulli Beam." IEEE Trans. on Automatic Control, 37(5):639-642, May.
- 10 Tzou, H. S. 1989. "Integrated Distributed Sensing and Active Vibration Suppression of Flexible Manipulators Using Distributed Piezoelectrics," Journal of Robotic Systems, 6(6):745-767.