BLOCK DIAGONAL DOMINANCE AND THE DESIGN OF DECENTRALIZED COMPENSATION

by

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ABSTRACT

Title of Thesis: Block Diagonal Dominance and the Design of Decentralized Compensation

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A frequency domain design method for control systems via decentralized feedback compensation is presented using transfer function models for the system input-output dynamics. The proposed design method is an extension of Rosenbrock's Inverse Nyquist Array method for the design of linear multivariable systems. The technique, based on the concept of block diagonal dominance for rational transfer function matrices, allows characterization of a control system as an interconnection of weakly interacting subsystems. The flexibility of the method with respect to the partitioning and measures of gain employed leads to improved estimates for overall system stability under decentralized compensation. Examples are included which illustrate the theory and its application by computer-aided design. Various extensions are suggested for further research.

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INTRODUCTION

(1.0) PRELIMINARIES

During the present decade several methods for the design of multivariable system compensation via transfer function (or frequency domain) techniques have been developed [1]-[9]. These techniques draw on the breadth of experience available from the classical frequency domain design techniques of Nyquist, Bode, and Nichols, and extend the results to the current spectrum of multi-input/multi-output problems facing the modern control systems engineer. By a multivariable control system we mean one in which the introduction of a signal at one of several inputs to the plant will have an effect seen at perhaps all of the several outputs of the plant which are to be controlled. Thus in the general case of multivariable feedback compensation the correction or error signal applied to each of the inputs to the plant is a function of all of the outputs which are to be controlled. If the plant to be controlled can be modeled by a linear time-invariant model a useful representation employs Laplace transforms to describe the input-output dynamic behavior in a matrix of transfer functions.

Alternatively, a plant dynamic structure can be modeled by a first order matrix differential equation, the state space model. The state space model describes in detail the internal structure of a dynamical system. As such there are infinitely many state space models which can be written to describe the input-output behavior of a given plant. Design techniques which utilize the state space description require certain internal variables, called state variables, to be available in the realization of a feedback compensator. When these variables are not available, a dynamic feedback compensator is required which will asymtotically estimate the state variables.

This brings us to the first major advantage of the frequency domain design techniques for compensation over the state space techniques. The methods employing transfer function descriptions of the input-output dynamics allow greater control over the required dynamic degree of a feedback compensator. This is true not only because the input-output description alone is used but also because the classical frequency domain techniques allow order reductions or model simplifications such as the dominant pole approximation to be recognized during the design procedure. Additionally, it is well known that design specifications in the frequency domain can often be more meaningful than the performance measures employed in state space (or time domain methods). As an example of the latter, consider the specification of bandwidth and damping ratio for a servomechanism as opposed to the specification of a measure of waveform energy that will be available from a performance index in an LQG design.

The multivariable frequency domain design techniques [1]-[9] have several aspects in common. They are typically supported by interactive computer codes which often make use of graphical descriptions of the parameter space. The designer can inject at this point his intuitive insight in order to achieve design goals. These design methods typically employ one or both of the following techniques during the design procedure:

(1) Algebraic manipulations of the transfer function matrix viewed as a matrix rational expression are performed to place the matrix in some standard form (e.g. upper triangular),
(2) Sequential design methods are employed where the classical single-input/single-output techniques are applied sequentially to each input-output pair in such a way that overall system stability is guaranteed (or other specifications are met).

The former procedure suffers again from the inability to limit the dynamic degree of the compensator as a result of algebraic manipulations on a rational matrix. The latter technique, when used separately, is not recursive in the sense that once a loop design is fixed for the i^{th} input-output pair it need not be changed during the design of the next loop; but the procedure does not guarantee system robustness in the face of a failure in one loop.

(1.1) INTERCONNECTED SYSTEMS AND DECENTRALIZED COMPENSATION

The above situation describes the most general case of compensation of a multivariable plant and is often called centralized compensation. We will now look at the motivation for what is called decentralized compensation of control systems.

As the demand for high performance automatic control systems increases the size and complexity of the design problem increases in two important ways:

(1) more information must be processed regarding a larger number of controlled parameter,
(2) the flow of information develops increasingly complex networks.

The result is that modern control systems have increased dimensionality not so much due to an increase in dynamic complexity as to an increase in the number of influence and observation parameters (i.e. inputs and outputs). Often, previously isolated systems become joined forming large networks as in power distribution networks.

Both the system analyst and the engineer are faced with the problem of reducing the dimensionality of large scale problems before solutions can be found. This is often motivated by the computational and arithmetic complexity of these problems. Perhaps more important is the fact that the design engineer or systems analyst looses intuitive insight into the cause and effect relationships which become blurred by the size of the problem.

Several order reduction and model simplification techniques are currently in use [10]-[14]. They include:

(1) techniques for aggregation of high order models into lower dimensioned problems,
(2) dominant mode or time scaling techniques,
(3) techniques for decentralization or decoupling into subsystems.

In the decentralized approach a large scale system is viewed as an interconnection of subsystems that are in some sense dominant over the interconnections. The partitioning of the large scale problem into weakly interacting subsystems may be implicit in the design. Design specifications or physical constraints can naturally partition the vectors of inputs and outputs conformally.

In references [11]-[13] and [16]-[18] the authors develop stability theorems for interconnected systems with decentralized feedback compensation. The system may contain isolated non-linearities which obey certain sector properties. Stability results are then developed in terms of bounded input-bounded output stability utilizing norms defined on function spaces. The various theorems developed have in common a test matrix composed of gains (norms on certain function spaces) of the partitioned input-output structure of the interconnected system. If the test matrix is a member of a class of matrices known as Metzler matrices, then stability of the overall system is guaranteed. In general these results are achieved by the straight forward application of the small gain theorem [15] to various descriptions of the interconnected system structure. Several limitations are obvious. The results obtained can be extremely conservative since the function space norms are rather gross estimates of the system frequency response. The description of system performance or the design specification in terms of response time or bandwidth is not immediately apparent. That is the variation of system response with frequency is not fully utilized.

A typical result [17] for the system described by:

$$y_i = \sum_{j=1}^{n} H_{ij} e_j$$

$$e_i = x_i + z_i + w_i$$

$$z_i = \sum_{j=1}^{n} B_{ij} f_j$$

$$f_i = u_i + y_i + v_i$$

where v_i, w_i are reference signals, x_i, u_i are input signals, y_i, z_i are output signals, and H_{ij}, B_{ij} are transfer function matrices, is that stability is guaranteed if the test matrix

$$A = I - G(B)G(H)$$

has all its successive principle minors positive (i.e., A is an M-matrix). Here the following definitions are in use:

$$G(M) \triangleq \left[g(M_{ij}) \right],$$

the matrix of gains, where

$$g(H) = \sup_{\omega} |\hat{H}(j\omega)|,$$

with an appropriate matrix norm employed on the right hand side. Then the connection is made with the frequency domain description of a linear time-invariant system in such a way that accentuates the gross nature of the norm measures used.

It is worth mentioning at this point that the results of this thesis will utilize one of a group of theorems from linear algebra which includes the theorems on Metzler matrices due to Fiedler and Ptâk [25]. In particular, we will use frequency varying norms to describe system dynamics.

Some results are available for the decentralized stabilization problem via state space methods. For example, the results of Wang and Davison [11] and [12] for the decentralized pole placement problem are of theoretical significance. In [11] the authors extend the familiar concept of unobservable and uncontrollable modes of a system with centralized output feedback compensation to the more constrained problem of decentralized feedback compensation. The result provides decentralized (output feedback) dynamic compensation scheme which will stabilize the system, provided the "fixed modes" of the system defined by the given decentralized structure are in the open left half plane. The problem of constructing a decentralized feedback compensator of minimal order remains unresolved.

(1.2) DECENTRALIZED COMPENSATION IN THE FREQUENCY DOMAIN

In [1] and [2] Rosenbrock and his coworkers developed a practical technique for compensator design of multivariable systems which makes use of standard frequency domain techniques. The method, known as the Inverse Nyquist Array (INA), results in a feedback compensator which is diagonal. The method, which has been rather extensively used to design compensators for industrial processes [2], is supported by interactive computer codes and graphical displays.

The general multivariable control system for the INA method is shown in figure 1.1 below.

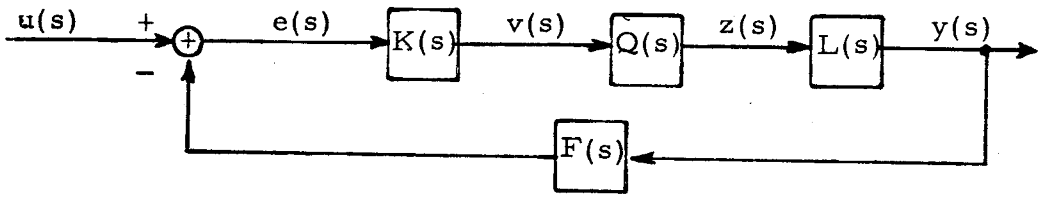


Figure 1.1 The general multivariable control system.

Here Q(s) is the $n \times \ell$ plant transfer function matrix,

K(s) is the $\ell \times m$ input compensator,

L(s) is the $m \times n$ output compensator,

and F(s) is the $n \times n$ diagonal feedback compensator.

Letting,

$$G(s) = L(s)Q(s)K(s)$$

be the $m \times m$ forward loop transfer function matrix, the closed loop transfer function matrix is,

$$H(s) = \left[I_m + G(s)F(s)\right]^{-1} G(s)$$

$$= G(s)\left[I_m + F(s)G(s)\right]^{-1}.$$

As its name implies, the INA method uses as a representation of the system dynamics the inverse of the transfer function matrix. Thus define the matrices

$$\hat{G}(s) \triangleq G^{-1}(s)$$

$$\hat{H}(s) \triangleq H^{-1}(s).$$

Provided the inverses exist we have the following relationship,

$$\hat{H}(s) = F(s) + \hat{G}(s).$$

Important motivations for using the inverse description can be found in [1] and [2].

The primary mathematical tool of the design method for our purposes is the use of the theorems of Gershgorin and Ostrowski (which bound the eigenvalues of a complex matrix) to support the diagonal or decentralized feedback matrix, F(s). The concept of diagonal dominance of a rational matrix on a contour D in the complex plane is of primary importance. A rational matrix Z(s) is diagonally dominant on the contour D in the complex plane, $\mathbb{C}$, if $z_{ii}(s)$ has no pole on D for i = 1, . . . , m and

$$\text{either } |z_{ii}(s)| - \sum_{\substack{j=1 \\ j \neq i}}^{m} |z_{ij}(s)| > 0, \quad i = 1, \ldots, m$$

for each s on D

$$\text{or} \quad |z_{ii}(s)| - \sum_{\substack{j=1 \\ j \neq i}}^{m} |z_{ji}(s)| > 0, \quad i = 1, \ldots, m.$$

The first step of the design method is to produce a compensator $K(s)$ ($L(s) = I_m$ usually)) so that $\hat{G}(s)$ is diagonally dominant on a relatively large part (with respect to the poles and zeros of $G(s)$) of the imaginary axis $[0 \leq \omega \leq \omega_{max}]$. Then the Gershgorin discs (with respect to either the rows or columns of $\hat{G}(s)$) when plotted atop the Nyquist locus of the diagonal elements of $\hat{G}(s)$ sweep out a broad or "fuzzy" Nyquist locus for each separate input-output pair. If feedback compensation is designed for these fuzzy Nyquist loci using the classical Nyquist criterion, then overall system stability is guaranteed. Furthermore, using the result of Ostrowski, Rosenbrock in [1] shows that the Gershgorin discs shrink as a result of the overall closed loop structure by a quantifiable amount. The final reduced fuzzy Nyquist loci can then be used to define gain and phase margins for each of the decentralized single-input/single-output servomechanisms.

The main disadvantage of the INA method is observed during the design of the input compensator, $K(s)$, to achieve diagonal dominance. Although it is true that we can always find a dynamic precompensator $K(s)$ to achieve dominance no algorithm yet exists which will guarantee the realization of a $K(s)$ which achieves dominance under specific constraints on the dynamic order. For simplicity of realization, it is usually required that $K(s)$ be a matrix of constant real gains, K, which is often selected by the designer on an ad hoc basis. Algorithms have been reported in [9] and [26] which search for the required K to achieve diagonal dominance. Unfortunately, it is not guaranteed that these algorithms will converge to the globally "best" compensator or that they will in general find any compensator that achieves dominance.

(1.3) SUMMARY OF THE RESULTS OF THIS THESIS

The intent of this thesis is to apply the generalized Gershgorin circle theorem for partitioned matrices [20] to the problem of decentralized compensation using a development analogous to that of Rosenbrock [2, 3] described in the previous section. Specifically, let the system shown in figure 1.1 be compensated by decentralized feedback (i.e. F(s) is block-diagonal). We seek a measure of interaction between the diagonal subsystems defined by partitioning G(s) conformally with F(s) in terms of the frequency response of the subsystems and their interconnections.

Suppose that during a design process subsystem interactions are ignored (i.e. the off diagonal blocks of G(s), $G_{ij}(s)$ where $i \neq j$, are set to zero) and the diagonal blocks of F(s) are chosen in m separate designs. If certain bounds on the measure of interactions are satisfied (see section 3.2) the actual system, compensated by decentralized feedback, is guaranteed to be stable.

The specific frequency response measure of subsystem interaction used here is developed from the generalized Gershgorin theorems in [20]. These results permit the definition of <u>block diagonal dominance</u> of a partitioned rational matrix transfer function Z(s) on a closed contour D in the complex plane (see section 2.2-5). Then the stability of a system compensated by decentralized feedback can be determined via the results of this thesis when its frequency response can be described by a rational transfer function matrix which is block diagonally dominant on an appropriate closed contour D in the complex plane. The proposed design method is a three step process. First, a test is performed on the open loop plant transfer function matrix. If the test is satisfied, a decoupled design is pursued where the "diagonal" subsystems are assumed to be non-interacting. Complete freedom in available here as to the design technique employed for each of the decoupled subsystems. Finally, a test similar to that performed in the first step is performed on a matrix constructed from the plant and feedback matrices in a familiar way. If this test is satisfied, then the closed loop system is guaranteed to be stable. Furthermore, the tests performed in steps 1, and 3 can be performed graphically. The graphical tests will be shown to indicate restrictions on the choice of compensators for each of the decoupled designs of step 2 in a simple way (see section 3.2-2).

The significant contribution of this thesis is to provide an extension of the INA method to the broader context of decentralized feedback compensation. Consequently, we propose a technique for finding the natural decentralized structure of a system (if one exists) and we set aside the requirement for compensation to achieve diagonal dominance in this thesis. It shall be recognized however that the techniques developed here suggest a sequential construction of the series compensators (K(s), L(s)) to achieve weak coupling between subsystems. Additionally, we can remove the restriction that the feedback compensator always be diagonal which is clearly restrictive for many applications. Finally, it is worth emphasizing that the results available in [21]-[25] from linear algebra point to several useful alternatives for the Gershgorin circle theorem which lead to "INA type" designs and which may offer several advantages over the standard INA method.

REGULARITY THEOREMS FOR PARTITIONED MARTICES AND APPLICATIONS TO ANALYSIS OF DYNAMICAL SYSTEMS

(2.0) PRELIMINARIES AND NOTATION

In references [20]-[25] the authors develop, using results which establish the non-singularity of a square complex matrix A, theorems which establish bounds for the spectrum of A and bounds for the determinant of A. These results which can provide a variety of estimates for the eigenvalues of A are in fact generalizations of the Gershgorin circle theorem. Interestingly, these results can lead to even tighter estimates for the spectrum of A as seen in [20].

In this chapter we describe how these regularity theorems for partitioned matrices lead to estimates for the dynamic behavior of a linear time-invariant system described by a rational transfer function matrix. These estimates are derived in a way analogous to the approach of Rosenbrock in [1, 2], but we recognize that for our purposes any such regularity theorem can be used.

Finally, we state for reference the generalized form for partitioned matrices of several other well known regularity results. We will omit the parallel development of stability theorems using these other results in this thesis. It will be obvious though as the design technique is developed how these results may be substituted in the procedure. The question of the numerical complexity of these alternative techniques and the relative merits of each for obtaining tighter estimates for stability analysis will be the subject of further research.

We establish the following notation for consistency of presentation. Let A_{ij} denote the i,j^{th} submatrix of an n x n complex matrix A partitioned into m^2 submatrices where $1 \leq m \leq n$. Then the dimension of each block A_{ij} is $k_i \times k_j$ where $\sum_{i=1}^{m} k_i = m$. The determinant of a square matrix A will be written det A. The symbol $\sum_{j(i)}^{m}$ will replace $\sum_{\substack{i \neq j \\ j=1}}^{m}$ so that,

$$\sum_{j(i)}^{m} A_{ij} = \sum_{\substack{i \neq j \\ j=1}}^{m} A_{ij}.$$

Then establishing the notation, $\|x\|$, for the vector norm of $x \in \mathbb{C}^n$ we will denote the induced matrix norm of $A \in \mathbb{C}^{n \times n}$ by,

$$N(A) \triangleq \sup_{\substack{x \in X \\ x \neq 0}} \frac{\|Ax\|}{\|x\|} \tag{2.1}$$

where X is the subspace appropriate for x in this case $\mathbb{C}^n$. Then the induced infimum matrix norm for A [20, 22] is,

$$n(A) \triangleq \inf_{\substack{x \in X \\ x \neq 0}} \frac{\|Ax\|}{\|x\|} . \tag{2.2}$$

Clearly if A is non-singular then,

$$n(A) = N(A^{-1})^{-1} . \tag{2.3}$$

(2.1) BLOCK DIAGONAL DOMINANCE FOR PARTITIONED MATRICES

(2.1-1) Block Gershgorin Theorems

Following Feingold and Varga [20] we state,

Definition 2-1: Let the n x n matrix A be partitioned into m^2 sub-matrices. Then A is block diagonally dominant (BDD) if,

(i) each A_{ii} is non-singular for $i = 1, \ldots, m$

and

$$\text{either,} \quad n(A_{ii}) > \sum_{i(j)}^{m} N(A_{ij})$$

$$\text{(ii) or,} \quad n(A_{ii}) > \sum_{i(j)}^{m} N(A_{ji}) \qquad (2.4)$$

for each $i = 1, \ldots, m$.

The above reduces to the usual definition for diagonal dominance [1, 2] when the A_{ij} are 1 x 1 matrices.

We first have,

Theorem 2-2: If the partitioned matrix A as in (defn. 2-1) is BDD then A is non-singular.

Proof: Assume that A is BDD and is singular. Then there must exist some non-zero vector, w, such that $Aw = 0$. If w is partitioned conformally with the block structure of A, then w_i is the i^{th} subvector of w. Rewrite the assumption as,

$$\sum_{i(j)}^{m} A_{ij} w_j = -A_{ii} w_i, \quad \text{for each } i = 1, \ldots, m.$$

Now, since $w \neq 0$, we can normalize w so that $\|w_j\| \leq 1$ for each $j = 1, \ldots, m$ and for some k, $1 \leq k \leq m$, $\|w_k\| = 1$.

Now by definition of the induced norm we have,

$$\|A_{kk}w_k\| = \|\sum_{j(k)}^{m} A_{kj}w_j\| \le \sum_{j(k)}^{m} N(A_{kj})\|w_j\| \le \sum_{j(k)}^{m} N(A_{kj}) .$$

Since $\|w_k\| = 1$, the left-hand side can be written,

$$\|A_{kk}w_k\| = \frac{\|A_{kk}w_k\|}{\|w_k\|} \ge n(A_{kk}) ,$$

but since k is arbitrary, we have proved the theorem.

Then we can state immediately [20],

Theorem 2-3: For the partitioned matrix A in (2-1), each eigenvalue λ of A satisfies,

$$n(A_{jj} - \lambda I_j) \le \sum_{k(j)}^{m} N(A_{jk}) \quad (2.5)$$

$$\left(\text{or} \le \sum_{k(j)}^{m} N(A_{kj})\right) \quad \text{for at least one } 1 \le j \le m.$$

Proof: Immediate since assuming the contrary implies by theorem 2-2 that $A - \lambda I$ is non-singular.

Now theorem 2-3 defines inclusion regions for the spectrum of A in the complex plane which are analogous to the Gershgorin discs. We define,

$$S_{i,A} = \left\{\lambda \in \mathbb{C} : n(A_{ii} - \lambda I_i) \le \sum_{j(i)}^{m} N(A_{ij})\right\}$$

and (2.6)

$$S'_{i,A} = \left\{\lambda \in \mathbb{C} : n(A_{ii} - \lambda I_i) \le \sum_{j(i)}^{m} N(A_{ji})\right\}$$

which are each closed and bounded and; are therefore, compact sets in $\mathbb{C}$ for each $i = 1, \ldots, m$. From equation (2.5) we can see that each $S_{i,A}$ contains also the spectrum of A_{ii}, since $n(A_{ii} - \lambda I_i)$ attains its minimum for λ an eigenvalue of A_{ii}. If we also define,

$$S = \bigcup_{i=1}^{m} S_{i,A} \cap \bigcup_{i=1}^{m} S'_{i,A} \tag{2.7}$$

then we see that the spectrum of A is contained in the set S.

A useful result towards characterizing how many eigenvalues of A are in each set $S_{i,A}$ (following [20]),

Theorem 2-4: If the union $\mathbb{R} = \bigcup_{j=1}^{\ell} S_{j,A}$ of ℓ, $1 \leq \ell < m$ (2.8)

Gershgorin sets is disjoint from the $m - \ell$ remaining Gershgorin sets for a partitioned matrix A, then $\mathbb{R}$ contains precisely $\sum_{j=1}^{\ell} k_j$ eigenvalues of A.

The flexibility inherent in these block Gershgorin results is important for our application since the inclusion regions depend on the choice of the vector norm used on each individual subspace (different subspaces can have different norms) and on the partitioning of A. In fact, as in [20] tighter estimates can be obtained over the scalar Gershgorin theorems. These facts suggest considering various alternate combinations in order to achieve optimum bounds for the eigenvalues of A.

The shape of the generalized Gershgorin sets is not clearly defined by theorem 2-3. The generalization of Gershgorin's result where the $S_{i,A}$ are each circles is the following.

Theorem 2-5: Let the matrix A be partitioned such that its diagonal submatrices are each normal. If the Euclidean vector norm is used on each subspace X_i of $X, i = 1, \ldots, m$, then each Gershgorin set $S_{i,A}$ is the union of k_j circles.

Proof: Let the eigenvalues of A_{ii} be λ_ℓ, $1 \le \ell \le k_i$. Then A_{ii} normal implies that $n(A_{ii} - zI_{kj}) = \min |\lambda_\ell - z|$.

As it will be desired later to calculate lower bounds for the spectrum of a matrix A we seek a lower bound for λ an eigenvalue of A which is not an implicit function of the unknown parameter λ .

We have,

Lemma 2-6: The Gershgorin sets $S_{i,A}(S'_{i,A})$ of the partitioned matrix A are included in the tori

$$T_{i,A} = \left\{\lambda \in \mathbb{C}:\ n(A_{ii}) - \sum_{j(i)}^{m} N(A_{ij}) \le |\lambda| \le N(A_{ii}) + \sum_{j(i)}^{m} N(A_{ij})\right\}$$

$$\left(T_{i,A} = \left\{\lambda \in \mathbb{C}:\ n(A_{ii}) - \sum_{j(i)}^{m} N(A_{ji}) \le |\lambda| \le N(A_{ii}) + \sum_{j(i)}^{m} N(A_{ji})\right\}\right) \qquad (2.9)$$

Proof: First show $n(A - \lambda I) \ge n(A) - |\lambda|$. (a)

By the triangle inequality for vector norms,

$$\|Ax\| \le \|Ax - \lambda x\| + \|\lambda x\| = \|(A - \lambda I)x\| + |\lambda| ,$$

for any vector x such that $\|x\| = 1$.

Then by definition (eq. 2.2),

$$n(A) \le \|(A - \lambda I)x\| + |\lambda|$$

since the infimum can be taken equivalently over the unit ball $\|x\| = 1$.

$$\leq n(A - \lambda I) + |\lambda|$$

Then show that ,

$$n(A - \lambda I) \geq |\lambda| - N(A). \quad (b)$$

Again by the triangle inequality,

$$\|\lambda x\| \leq \|Ax - \lambda x\| + \|Ax\| .$$

Then for any x such that $\|x\| = 1$,

$$|\lambda| \leq \|(A - \lambda I)x\| + \|Ax\|,$$

or $$|\lambda| - \|Ax\| \leq \|(A - \lambda I)x\|$$

so $$|\lambda| - N(A) \leq n(A - \lambda I) .$$

Now from equations (a), (b) and (2.6) the result follows.

As will be seen in the subsequent development we are primarily interested in the maximum of the two inner radii of the tori $T_{i,A}$ and $T'_{i,A}$ for each i = 1, . . . , m, since whenever these are all strictly positive we have that A is non-singular. Various considerations indicate that the maximum of the two inner radii is in fact a tight estimate for the $\rho_{i,A}$ defined as,

$$\rho_{i,A} \triangleq \max \left\{ \min_{z \in S_{i,A}} |z| , \min_{z \in S'_{i,A}} |z| \right\} \tag{2.10}$$

for each i = 1, . . . , m where $S_{i,A}(S'_{i,A})$ are defined in (2.6) for A. For example from theorem 2-5, it can be seen that if the diagonal blocks of A, A_{ii}, are each normal then the two quantities are the same for each i = 1, . . . , m, since the $S_{i,A}(S'_{i,A})$ are the union of m circles in the complex plane.

(2.1-2) Block Ostrowski Theorems

Next we extend the results of Ostrowski utilized in [2, p. 27] for our block diagonal dominance results. We will need the following notation as in [1,2]. Let $\hat{A} \triangleq A^{-1}$. Then when we write $\hat{A}_{ii}^{-1}$ we mean the inverse of the i^{th} diagonal submatrix of the inverse of A.

We recognize that the definition of BDD in (2-1) can be rewritten as,

$$\theta_i \, n(A_{ii}) = \sum_{j(i)}^{m} N(A_{ij}) \ , \ i = 1, \ldots , m$$

$$\left(\text{or} \quad \theta_i' \, n(A_{ii}) = \sum_{j(i)}^{m} N(A_{ji}) \ , \ i = 1, \ldots , m \right) \tag{2.11}$$

where for each $i, 0 \leq \theta_i < 1$ (or $0 \leq \theta_i' < 1$).

Then we establish the result,

Theorem 2-7: If the partitioned matrix A is BDD, then A has an inverse $\hat{A} = A^{-1}$ which satisfies,

$$N(\hat{A}_{ji}) \leq \theta_j N(\hat{A}_{ii})$$

$$\left(\text{or} \quad N(\hat{A}_{ij}) \leq \theta_j' N(\hat{A}_{ii}) \right) \tag{2.12}$$

for each $i, j = 1, \ldots , m$ except $i = j$.

Proof: From theorem 2-2, since A is BDD it is non-singular and we can write,

$$\sum_{k=1}^{m} A_{jk} \hat{A}_{ki} = 0$$

for all $i, j = 1, \ldots , m$ except $i = j$

or,

$$\hat{A}_{ji} + A_{jj}^{-1} \sum_{k(j)}^{m} A_{jk} \hat{A}_{ki} = 0.$$

Taking norms we have,

$$N(\hat{A}_{ji}) \leq \max_{k \neq j} N(\hat{A}_{ki}) \sum_{k(j)}^{m} N(A_{jj}^{-1}) N(A_{jk}),$$

$$= \theta_j \max_{k \neq j} N(\hat{A}_{ki})$$

Since this holds for all j different from i and each $\theta_j < 1$

$$\max_{k \neq j} N(\hat{A}_{kj}) = N(\hat{A}_{ii}) ,$$

and the result follows. (The proof for BDD by columns, $0 \leq \theta_j' < 1$, follows similarly).

We now can show the important block Ostrowski result.

Theorem 2-8: Let the partitioned matrix A be BDD. Define,

$$\Phi_i \triangleq \max_{k \neq i} \theta_k \left(\text{or } \Phi_i' \triangleq \max_{k \neq i} \theta_k' \right).$$

Then,

$$n(\hat{A}_{ii}^{-1} - A_{ii}) < \theta_i \Phi_i \, n(A_{ii})$$
$$\left(\text{or} \quad < \theta_i' \Phi_i' \, n(A_{ii}) \right) \tag{2.13}$$

for each i = 1, . . . , m.

Proof: Again A is non-singular so we write,

$$\sum_{k=1}^{m} A_{ik} \hat{A}_{ki} = I, \; i = 1, \ldots, m,$$

or

$$(\hat{A}_{ii}^{-1} - A_{ii}) \hat{A}_{ii} = \sum_{k(i)}^{m} A_{ik} \hat{A}_{ki} \; .$$

Taking norms we have,

$$N\left[(\hat{A}_{ii}^{-1} - A_{ii})\,\hat{A}_{ii}\right] \leq \sum_{k(i)}^{m} N(A_{ik})N(\hat{A}_{ki}). \quad \text{(a)}$$

Now let us show that in general,

$$N(AB) \geq N(B) \cdot n(A) \quad \text{(b)}$$

Since for A non-singular we have,

$$B = A^{-1} A B .$$

Taking norms we see that,

$$N(B) \leq N(A^{-1}) \cdot N(AB)$$

or,

$$N(A^{-1})^{-1} \cdot N(B) \leq N(AB)$$

So from equation (2. 3) we have shown (b).

Now using (b) on (a) we can write,

$$n(\hat{A}_{ii}^{-1} - A_{ii}) \leq \sum_{k(i)}^{m} N(A_{ik})\, N(\hat{A}_{ki}) \;/\; N(\hat{A}_{ii})$$

$$\leq \sum_{k(i)}^{m} N(A_{ik}) \max_{k \neq i} \theta_k = \theta_i\, \Phi_i\, N(A_{ii}),$$

from equation (2. 12) and the definition of Φ_i,

$$\leq \left[\sum_{k(i)}^{m} N(A_{ik})\right] \max_{k \neq i} \left\{N(\hat{A}_{ki})\right\} / N(\hat{A}_{ii}).$$

Then from theorem 2-7,

$$\leq \left[\sum_{k(i)}^{m} N(A_{ik})\right] \max_{k \neq i} \theta_k .$$

So by definition of Φ_i and (eg. 3. 5) we have,

$$n(\hat{A}_{ii}^{-1} - A_{ii}) = \theta_i\, \Phi_i\, n(A_{ii}),$$

which completes the proof.

(Again, the proof assuming A is BDD by columns follows similarily).

This result will be useful for characterizing the behavior of the individual subsystems with interaction as will be seen in Chapter 3.

(2.2) CONSEQUENCES FOR BDD FOR DYNAMICAL SYSTEM ANALYSIS

(2.2-1) General Comments

In this section we prove a basic result which underlies the methods presented in the next chapter for stability analysis and design of decentralized control systems. First, we will define our conventions for stability analysis. We then review some basic complex variable theory which leads to the generalized Nyquist theorem. Then we prove the main result which will extend this classical theorem to the analysis of decentralized control systems.

(2.2-2) Basic Definitions

For the extent of this analysis we will adopt the conventions for multivariable stability analysis of Rosenbrock in [2]. We, therefore, define the poles of the multivariable system described by the rational transfer function matrix, G(s), to be the poles of det D(s), where $G(s) = N(s)\, D^{-1}(s)$ a polynomial matrix factorization.

(2. 2-3) Some Basic Results in Complex Analysis

We recall from complex analysis,

Lemma 2-9: Suppose that f(z) if analytic inside and on a closed elementary contour, D, in the complex plane except at a finite number of points (called poles) inside D. And, if in addition, f(z) does not vanish on D, then,

$$\frac{1}{2\pi i} \oint_D \frac{f'(z)}{f(z)} \, dz = Z - P \qquad (2.14)$$

where Z is the number of zeros of f inside D,

and P is the number of poles of f inside D.

Then the following corollary to the above lemma is commonly known as the principle of the argument.

Corollary 2-10: With the definition of D and f(z) above, let z trace once around the curve D in a clockwise direction. Correspondingly, let f(z) trace out the closed curve Γ_f. Then Γ_f encircles the origin in the complex plane Z-P times in the clockwise direction.

Proof: Using lemma 2-9, we see that,

$$Z - P = \frac{1}{2\pi i} \oint_D d(\log f(z))$$

$$= \frac{1}{2\pi i} \oint_D d(\log |f(z)| + i \arg f(z)).$$

Thus as z goes around D, $|f(z)|$ returns to its original value while arg f(z) increases by 2π times the number of encirclements of the origin by Γ_f.

(2.2-4) Extension of BDD to Partitioned Matrix Rational Functions

We next develop the main result of this chapter as a consequence of BDD. First we define,

Definition 2-11: Let A(s) be an n x n rational transfer function matrix partitoned into m^2 submatrices as in definition 2-1, and let D be a closed elementary contour in $\mathbb{C}$. Then A(s) is said to be block diagonally dominant on D (BDD on D) if:

(i) $A_{ii}(s)$ has no pole on D, $i = 1, \ldots, m$

and (ii) A(s) is block diagonally dominant for all s on D as in (defn. 2-1).

We then have the following generalization of Rosenbrock's result [2, Th. 1.9.4].

Theorem 2-12: Let A(s) be an n x n rational matrix partitioned as above, which is BDD on a closed elementary contour D in the complex plane. As s traces once around D in a clockwise direction, let det A(s) map D into the curve Γ_A which encircles the origin N_A times clockwise. Similarly, let det $A_{ii}(s)$ map D into Γ_i which encricles the origin N_i times clockwise for each $i = 1, \ldots, m$. Then,

$$N_A = \sum_{i=1}^{m} N_i. \qquad (2.15)$$

Proof: We follow here an appropriate generalization of the proof used in [2]. Since, by assumption of BDD on D (Definition 2-12) $A_{ii}(s)$

has no pole on D, it is finite on D, and so is det $A_{ii}(s)$. Again by BDD (Definition 2-1) det $A_{ii}(s)$ has no zero on D so $n(A_{ii}(s))$ is non-zero on D. Therefore, from equation (2.4) $N(A_{ij}(s))$ must be finite on D for $1 \leq i, j \leq m$, $i \neq j$. So there are no poles of $A_{ij}(s)$ on D, $1 \leq i, j \leq m$. Also, and theorem 2-2, A(s) is non-singular on D which implies that there is no zero of det A(s) on D.

Let $A(\alpha, s)$ be the partitioned matrix

$$A(\alpha, s) = \begin{cases} A_{ii}(\alpha, s) = A_{ii}(s) \\ A_{ij}(\alpha, s) = \alpha A_{ij}(s), \; i \neq j \end{cases} \quad \text{where } 0 \leq \alpha \leq 1. \tag{2.16}$$

Then for any $0 \leq \alpha \leq 1$ $A(\alpha, s)$ is finite on D and so is det $A(\alpha, s)$. Let

$$\beta(\alpha, s) = \frac{\det A(\alpha, s)}{\prod_{i=1}^{m} \det A_{ii}(s)} \tag{2.17}$$

where we see that $\beta(0, s) = 1$. Let $\beta(1, s)$ map D into Γ_β. For each s on D, $\beta(\alpha, s)$ defines a continuous curve joining $\beta(0, s) = 1$ and the point on Γ_β corresponding to s.

Now assume that Γ_β encircles the origin. Then there must exist some α, $0 \leq \alpha \leq 1$, such that for some s on D, $\beta(\alpha, s) = 0$. Then from (eqn. 2.17) det $A(\alpha, s) = 0$ or det $A_{ii}(s) \to \infty$ for some $i \in [1, m]$. However, since A(s) is BDD and $0 \leq \alpha \leq 1$ then all $A(\alpha, s)$ are BDD on D and they must all be non-singular. Likewise, there can be no pole of det $A_{ii}(s)$ on D by assumption. So we have shown that Γ_β cannot encircle the origin.

Therefore, we have using corollary 2-10 by counting encirclements of Γ_β that,

$$0 = N_A - \sum_{i=1}^{m} N_i \tag{2.19}$$

and this concludes the proof.

From the proof just completed, we see that the crucial use of block diagonal dominance, theorem 2-2, is in establishing the regularity of $A(\alpha, s)$ for $0 \leq \alpha \leq 1$. Then application of the principle of the argument gives the desired result. Thus any of a wide variety of regularity theorems for partitioned matrices as reported in [20]-[25] can be used to develop similar results.

A graphical test for BDD of A(s) on D is suggested by theorem 2-3. Clearly, if for every s on D the Gershgorin sets $S_{i,A}(S'_{i,A})$ $i=1, \ldots, m$ (eqn. (2.6)) exclude the origin then A(s) is BDD on D by definition 2-11. We emphasize the flexibility of our approach by choice of:

(1) partitioning for A(s)
(2) norms on individual subspaces.

This implies for the analysis of dynamical systems the following significant contributions.

(1) Tighter estimates of diagonal dominance are available.
(2) The "natural" decomposition of a system with respect to its frequency characteristics can be found.
(3) Improved estimates for stability may be given over the case of A(s) partitioned into 1 x 1 matrices as in [1, 2].

We recognize that the computational task of determining for A(s) the Gershgorin sets, $S_{i,A}(S'_{i,A})$ for each $i=1, \ldots, m$ and all s on D may be burdensome, but in order to establish BDD of A(s) on D we require only that the $S_{i,A}(s)$ $(S'_{i,A}(s))$ exclude the origin for all s on D and each $i=1, \ldots, m$. Define then,

$$\rho_{i,A}(s) \triangleq \min_{z \in S_{i,A}(s)} |z|, \quad i = 1, \ldots, m$$

$$\left(\rho'_{i,A(s)} \triangleq \min_{z \in S'_{i,A}(s)} |z|, \quad i = 1, \ldots, m \right) \tag{2.20}$$

where we can use min instead of inf because the $S_{i,A}(s)$ ($S'_{i,A}(s)$) are closed and bounded, and therefore, compact. Then the definition of BDD on D in (defn. 2-11) is equivalent to: for $A_{ii}(s)$ having no pole on D, and $A_{ii}(s)$ non-singular on D that,

$$\rho_D(a) = \min_{s \in D} \left\{ \max \left[\min_{i \in [1,m]} \rho_{i,A}(s), \min_{i \in [1,m]} \rho'_{i,A}(s) \right] \right\} > 0. \tag{2.21}$$

Equation (2.21) is the generalization of the result due to Rosenbrock [2, p. 143, eq. (15.4)].

Furthermore, it is not clear in our work to date how to use the envelope swept out by the $S_{i,A}(s)$($S'_{i,A}(s)$) as s goes around D for design. In the scalar case of [1,2], the envelope of the Gershgorin discs sweeps out a broad or "fuzzy" Nyquist locus which can be used to select compensators for the diagonal subsystems. But for the present generalization of the partitioned matrix A(s) we would need a relationship between the $n(A_{ii}(s))$, $N(A_{ij}(s))$ and det (A_{ii}) that would provide some analogous result. Although, in the present development following [20], the desired relationship is not available, the results of Brenner in [23,24] indicate bounds for determinants are available for partitioned matrices. The application of these results to the present problem will be the subject of further research.

To ease the computational burden of equation (2.21), we define (see lemma 2-6),

$$d_{A,i}(s) \stackrel{\Delta}{=} n(A_{ii}) - \sum_{j(i)}^{m} N(A_{ij})$$

$$\left(\text{or } d'_{A,i}(s) \stackrel{\Delta}{=} n(A_{ii}) - \sum_{j(i)}^{m} N(A_{ji})\right) \tag{2.22}$$

Then we can recognize immediately that A(s) is BDD on D if and only if,

$$d_D(A) = \min_{s \in D} \left\{ \max \left[\min_{i \in [1,m]} d_{A,i}(s), \min_{i \in [1,m]} d'_{A,i}(s) \right] \right\} > 0. \tag{2.23}$$

The computational aspects of the test described in (eqn. (2.23)) are much simpler than the test described in (eqn. (2.21)). As we will see in the next chapter, the graphical test suggested by (eqn. (2.23)) will also lead to useful information which will aid us in our choice of feedback compensation.

(2.3) SOME OTHER WELL KNOWN REGULARITY RESULTS

In this section we will review some well known regularity results for complex matrices which are partitioned. As we saw in the last section, these kinds of results are useful for the analysis of dynamical systems.

As in [20], we state the block extension of the ovals of Cassini.

Theorem 2-13: Let A be an n x n complex matrix partitioned into m^2 submatrices as before. Then the spectrum of A is included in the union of the $m(m-1)/2$ sets C_{ij} defined by:

$$C_{ij} \triangleq \left\{ z \in \mathbb{C} : n(A_{ii} - zI_i)\, n(A_{jj} - zI_j) \le \left(\sum_{k(i)}^{m} N(A_{ik}) \right) \left(\sum_{k(j)}^{m} N(A_{jk}) \right) \right\}$$

$$\text{for all } 1 \le i,\ j \le m \text{ and } i = j. \tag{2.24}$$

Also from [20] we have,

<u>Theorem 2-14:</u> Let the n x n matrix A be partitioned as above. Define,

$$R_j \triangleq \sum_{k(j)}^{m} N(A_{jk}) \qquad , \qquad C_j \triangleq \sum_{k(j)}^{m} N(A_{kj}) \, ,$$

$$j = 1, \ldots, m$$

Then, for any α with $0 \le \alpha \le 1$, each eigenvalue of A satisfies

$$n(A_{jj} - \lambda I_j) \le R_j^{\alpha} C_j^{1-\alpha} \tag{2.25}$$

for at least one $j = 1, \ldots, m$.

A similar result due to Fan and Hoffman [see 20], is generalized by Feingold and Varga in the following.

<u>Theorem 2-15:</u> Let A be an n x n matrix partitioned as before. Let $p > 1$, and $1/p + 1/q = 1$. If $k > 0$ satisfies,

$$\sum_{i=1}^{m} \left\{ \frac{\sum_{j(i)}^{m} N(A_{ij})}{\left(\sum_{j(i)}^{m} N(A_{ij})^p \right)^{q/p}} \right\} \le k^q (1 + k^q) \tag{2.26}$$

(where we agree that if 0/0 occurs we let 0/0 = 0), then each eigenvalue λ of A satisfies at least one of:

$$n(A_{jj}-\lambda I_j) \leq k \left(\sum_{k(j)}^{m} N(A_{jk})^p \right)^{1/p}, \quad j = 1, \ldots, m. \qquad (2.27)$$

Note that these results are even more flexible than that used in section (2.1). In [22] Johnston develops an algorithm which minimizes these inclusion regions for the spectrum of A by exploiting this flexibility. What is suggested for our purposes is that a procedure be developed for finding the maximum margin of regularity (eqn. (2.10)).

Of even more interest for our application is the results of Brenner in [23, 24]. Here a matrix A is partitioned as above. Then a second matrix B is constructed from the determinants of the blocks of A. B is m x m with elements,

$$b_{ij} = \det A_{ij} \qquad (2.28)$$

Then upper and lower bounds for det A are developed when B is diagonally dominant in the usual sense. The application of these results in the context of this paper promise some exciting results for the decentralized control problem. Further developments in this research will be reported elsewhere.

APPLICATION OF BDD TO DECENTRALIZED FEEDBACK COMPENSATION

(3.0) GENERAL COMMENTS

In this chapter we apply the concepts of the previous chapter to the design of decentralized compensation. We will first define the poles and zeros of a system for our stability analysis and review the basic structure of multivariable feedback compensation. Then we will establish several useful theorems for decentralized compensation. The key which unlocks these results is theorem 2-12. Next, we propose a design procedure for the decentralized control problem in which a graphical test is developed which guides the choice of the diagonal (decoupled) sub-compensators in the decentralized feedback compensator. Finally, we extend the Ostrowski result for the general partitioning considered here. Recall that in Rosenbrock's design technique [2, 3] the Ostrowski result gives a useful bound on the i^{th} subsystem dynamic response when the other loop designs are connected to the plant. Then by using the inverse matrix description of the system, this result gives tighter bounds for the subsystem performance in the closed loop structure. It is this result which is the primary motivation for using the inverse matrix transfer function representation [2, 3]. As we will see, our results in this regard are not quite as strong.

(3.1) FEEDBACK STRUCTURE FOR MULTIVARIABLE SYSTEMS

In figure 1.1, we depict the feedback control structure of a multivariable system where u(s), e(s), and y(s) are n-vectors of rational transfer functions. The relationship between u(s) and y(s) in figure 1.1 is established by,

$$y(s) = G(s)e(s) = G(s)[u(s) - F(s)y(s)]\,. \tag{3.1}$$

Therefore, the closed loop input-output response is described by,

$$H(s) = [I + G(s)F(s)]^{-1}G(s) = G(s)[I + F(s)G(s)]^{-1} \tag{3.2}$$

Following Rosenbrock [2, 3] we define the closed loop relationship using the inverse system notation where $\hat{H}(s) \triangleq H^{-1}(s)$ as,

$$\hat{H}(s) = F(s) + \hat{G}(s)\,. \tag{3.3}$$

Also, we recognize that by <u>decentralized compensation</u> we mean

$$F(s) = \text{block-diag.}\,\{F_1, F_2, \ldots, F_m\} \tag{3.4}$$

where $F_i(s)$ is $k_i \times k_i$ and $\sum_{i=1}^{m} k_i = n$. This structure for F(s) induces a partitioning of the vectors e(s), u(s), and y(s) into m subvectors each and G(s) is partitioned conformally into m^2 submatrices. The design procedure described in section 3.2-2 will allow the designer to associate m triples $(e_i(s),\ u_i(s),\ y_i(s))$ in m decoupled designs with the stability of the overall system. As such the problem statement formed here requires that $e_i(s)$, $u_i(s)$, and $y_i(s)$ are all k_i vectors for each $i = 1, \ldots, m$. This restriction is necessary for the theory we use here since the results are based on square matrices. In practice, this constraint may not be a problem since it represents problems of most interest [2, pp. 155].

With this structure for F(s) we see from (eq. 3.3) that,

$$\hat{H}_{ij}(s) = \begin{cases} \hat{G}_{ii}(s) + F_i(s), & i = j \\ \hat{G}_{ij}(s) & , i \neq j . \end{cases} \tag{3.5}$$

In discussing stability we will follow the conventions of Rosenbrock [2, pp. 1-27]. Thus the zeros (poles) of a matrix transfer function G(s) are the zeros of all numerator (denominator) polynomials in the McMillan form of G(s). Equivalently, if we have a state space realization of G(s),

$$G(s) = C[sI - A]^{-1}B \tag{3.6}$$

where the realization is not necessarily of minimal degree, then the system poles are the eigenvalues of the matrix A. For the closed loop system described by equation (2.2) if we write,

$$\begin{aligned} &G(s) = N(s)\, D^{-1}(s) \\ \text{and} \quad &F(s) = D_F^{-1}(s)\, N_F(s) , \end{aligned} \tag{3.7}$$

polynomial matrix factorizations for the transfer function matrices. Then from (eq. 3.2) we recognize as the <u>system poles</u> the zeros of $\det[I_n + G(s)\,F(s)]\,\det D(s)\,\det D_F(s)$. Often, the <u>poles of G(s)</u> are defined as the zeros of det D(s) in (eq. 3.7) except that the factorization is required to be right coprime. We argue for the purposes of design that we are not interested in ignoring pole-zero cancellations that can occur in forming the coprime factorization. If these unobservable and/or uncontrollable modes are confined to be stable then the non-minimal realization is usually called stabilizable/detectable. If the closed loop system has a pole-zero cancellation in the right half plane one can argue that the system is stable. But we recognize that only the model is stable and since the parameters of any real system can be modeled with only finite accuracy small uncertainties in these parameters will lead to instability.

From (eq. 3.2) we have the relation,

$$\det [I_n + G(s)F(s)] = \det [I_n + F(s)G(s)] \tag{3.8}$$

$$= \frac{\det G(s)}{\det H(s)} \quad = \frac{\det \hat{H}(s)}{\det \hat{G}(s)} .$$

From (eq. 3.5) we see the importance of the matrix $I_n + G(s)F(s)$, called the return difference matrix.

Then we develop the generalized Nyquist theorem for determining stability of a closed loop multivariable system. Following [2, p. 141] we define a closed elementary contour D in the complex plane consisting of a large part of the imaginary axis [-iR, iR] and a semicircle of radius R in the closed right half plane. Suppose the open loop system has p_o poles in the closed right half plane (p_o is the number of zeros of $\det D(s) \cdot \det D_F(s)$ in (eq. 3.7)). Choose R large enough so that D encloses the p_o open loop poles in the right half plane. If any open loop poles lie on the imaginary axis the curve D is modified by indenting D into the left half plane so as to enclose these poles. Let $\det (I_n + G(s)F(s))$, $\det \hat{G}(s)$, $\det \hat{H}(s)$ map D into the curves

Γ_{RD}, $\hat{\Gamma}_G$, $\hat{\Gamma}_H$ respectively which encircle the origin in the complex plane N_{RD}, $\hat{N}_G$, $\hat{N}_H$ times clockwise respectively. Then we have the generalization Nyquist theorem.

<u>Theorem 3-1:</u> The closed loop system shown in figure 1.1 and described by (eq. 3.1) is asymtotically stable if and only if,

$$\text{(a)} \qquad N_{RD} = - p_o \tag{3.9}$$

$$\text{(b)} \qquad \hat{N}_G - \hat{N}_H = p_o \tag{3.10}$$

<u>Proof of (a):</u> With p_o defined to be the number of zeros of $\det D(s) \cdot \det D_F(s)$ found in the closed right half plane (eq. 3.7) we can define from equation (3.2),

$$\Phi(s) = \det(I_n + G(s)F(s)) \det D(s) \det D_F(s). \tag{3.11}$$

Now on applying the principle of the argument (corollary 2-10) to $\det [I_n + F(s)G(s)]$ on the curve D as defined above, except that R may have to be increased so as to guarantee that D encloses all zeros of $\Phi(s)$ (there are z_{RD} in number), we find on counting the number of encirclements of Γ_{RD} that,

$$N_{RD} = z_{RD} - p_o. \tag{3.12}$$

Since the zeros of $\Phi(s)$ are the system poles we conclude asymtotic stability if and only if equation (3.9) is satisfied,

<u>Proof of (b):</u> We notice from equation (3.8) that

$$N_{RD} = \hat{N}_H - \hat{N}_G \tag{3.13}$$

therefore by substitution into equation (3.9) already proved we have equation (3.10).

(3.2) A DESIGN TECHNIQUE FOR DECENTRALIZED COMPENSATION

(3.2-1) Stability Results

We can now prove several theorems using theorem 2-12 which are generalizations of the results of Rosenbrock in [2, pp. 143-144]. Here we use the definition of D and p_o as in the previous section.

Theorem 3-2: Let G(s) and H(s) both be BDD on D. Let det $G_{ii}(s)$ (det $H_{ii}(s)$) map D into $\Gamma_{G,i}$ ($\Gamma_{H,i}$) closed curves which encircle the origin $N_{G,i}$ ($N_{H,i}$) times clockwise for each $i = 1, \ldots, m$. Then the closed loop system is asymtotically stable if and only if,

$$\sum_{i=1}^{m} N_{G,i} - \sum_{i=1}^{m} N_{H,i} = - p_o \qquad (3.14)$$

Proof: Let det G(det H) map D into Γ_G (Γ_H) which encircle the origin N_G (N_H) times clockwise. Then by theorem 3-1 (a) and (eq. 3.8) the system is asymtotically stable if and only if,

$$N_G - N_H = N_{RD} = - p_o .$$

But by the assumption of BDD of G(s) and H(s) on D we have by theorem 2-13,

$$N_G - N_H = \sum_{i=1}^{m} N_{G,i} - \sum_{i=1}^{m} N_{H,i}. \qquad (3.15)$$

And the result follows.

Also taking a slightly different approach we have,

Theorem 3-3: Suppose (as is usually the case) that F(s) represents an asymtotically stable compensator (i.e. that det $D_F(s)$ has no zeros in the closed right half plane). Let the matrix transfer function $[F^{-1}(s)+G(s)]$ be BDD on D. Let $\det[F_i^{-1}(s)+G_{ii}]$ map D into Γ_i which encircles the origin N_i times clockwise for each $i = 1, \ldots, m$. Then the closed loop system is asymtotically stable if and only if,

$$\sum_{i=1}^{m} N_i = - p_o \, . \tag{3.16}$$

Proof: We can write,

$$\det [I_n + G(s)F(s)] = \det [F^{-1}(s) + G(s)] \det F(s). \tag{3.17}$$

Then by the assumption if we ignor the zeros of det $D_F(s)$ we can still apply theorem 3-1 (a) and by theorem 2-12

$$\sum_{i=1}^{m} N_i = N_{RD}. \tag{3.18}$$

For thoroughness of presentation, we will develop next the stability theorems for decentralized control via the inverse matrix transfer function description. In [2, pp. 155], Rosenbrock concentrates on the inverse system representation for a variety of reasons. The most important of these reasons for the standard case is that in the final analysis tighter estimates of system performance are available by using the Ostrowski result with the inverse representation. In the present generalization, the result is not so meaningful - as we will see.

First, we show the stability result.

Theorem 3-4: Suppose that $\hat{G}(s)$ and $\hat{H}(s)$ are BDD on D. Let det $\hat{G}_{ii}$ (resp. det $\hat{H}_{ii}$) map D into $\hat{\Gamma}_{G,i}$ (resp. $\hat{\Gamma}_{H,i}$) which encircle the origin $\hat{N}_{G,i}$ (resp. $\hat{N}_{H,i}$) times clockwise for each $i = 1, \ldots, m$. Then the closed loop system is asymtotically stable if and only if,

$$\sum_{i=1}^{m} \hat{N}_{G,i} - \sum_{i=1}^{m} \hat{N}_{H,i} = p_o \tag{3.19}$$

Proof: Follows immediately from theorem 3-1(b) and theorem 2-12.

(3.2-2) The Design Procedure

We propose, based on the results of the previous section, the following three step design procedure.

(1) The plant to be controlled is described by either its direct or inverse matrix transfer function. We test for BDD of the matrix $G(s)$ (or $\hat{G}(s)$) on D by determining that $d_D(A) > 0$ (eq. 2.23) for s taken along the $j\omega$ axis $\omega \in [0, \omega_{max}]$. Here ω_{max} is chosen as was R previously and by symmetry this guarantees that $G(s)$ (resp. $\hat{G}(s)$) is BDD on D.

(2) We then proceed to design m separate compensators to stabilize the ideal decoupled subsystems of $G(s)$ (resp. $\hat{G}(s)$) (i.e. the diagonal blocks of $G(s)$ (resp. $\hat{G}(s)$).

(3) Finally, we test for BDD of either $F^{-1}(s) + G(s)$ or $\hat{H}(s)$ as appropriate and apply theorems 3-2, 3-3, or 3-4.

In order to aid the choice of compensators is step 2 so that the appropriate test for BDD is step 3 can be satisfied, we now develop sufficient conditions for the choice of each $F_i(s)$ so that BDD of the appropriate test matrix is guaranteed.

With respect to theorem 3-3, we establish a sufficient condition for the choice of $F_i(s)$ for $i = 1, \ldots, m$ to guarantee that $F_i^{-1} + G_{ii}(s)$ is BDD on D.

Corollary 3-5: If,

$$N(F_i(s))^{-1} > N(G_{ii}(s)) + \sum_{j(i)}^{m} N(G_{ij}(s)) \quad \left(\text{or} > N(G_{ii}(s)) + \sum_{j(i)}^{m} N(G_{ji}(s))\right) \tag{3.20}$$

or

$$n(F_i(s))^{-1} < n(G_{ii}(s)) - \sum_{i(j)}^{m} N(G_{ij}(s)) \quad \left(\text{or} < n(G_{ii}(s)) - \sum_{i(j)}^{m} N(G_{ji}(s))\right) \tag{3.21}$$

for each $i = 1, \ldots, m$ and for all s on D then $F^{-1}(s) + G(s)$ is BDD on D.

Proof: We need only prove the following two part lemma. Then the proof is immediate from the definition of BDD on D (defn. 2-11) and the block diagonal structure of F(s).

Lemma 3-6: For A, B n x n complex matrices,

(a) $$n(A+B) \geq n(A) - N(B) \tag{3.22}$$

(b) $$n(A+B) \geq n(B) - N(A) \tag{3.23}$$

Proof: (a) By the triangle inequality for vector norms,

$$\|Ax\| \leq \|(A+B)x\| + \|Bx\| \; . \tag{3.24}$$

Then since the induced infimum matrix norm (eqn. 2.2) can be defined equivalently on the unit ball as,

$$n(A) \stackrel{\Delta}{=} \inf_{\substack{\|x\| = 1 \\ x \in X}} \|Ax\|$$

then (3.24) implies

$$n(A) \leq \|(A+B)x\| + \|Bx\| \text{ for all } x \text{ such that } \|x\| = 1.$$

So

$$n(A) - \|Bx\| \leq \|(A+B)x\| \text{ where } \|x\| = 1$$

implies that

$$n(A) - N(B) \leq n(A+B) \, .$$

(b) follows equivalently from the proof in part (a) by the communitivity of A + B.

Corollary 3-5 leads to a simple graphical test which can determine the suitability of a candidate feedback compensator for the i^{th} decoupled subsystem design. In figure 3.1, we plot the curves,

$$N(G_{ii}(s)) + \sum_{j(i)}^{m} N(G_{ij}(s)) \tag{3.25}$$

and

$$n(G_{ii}(s)) - \sum_{j(i)}^{m} N(G_{ij}(s)) \tag{3.26}$$

where $s = i\omega$ for $\omega \in [0, \omega_b]$ and where ω_b is chosen as a practical upper limit on the bandwidth of the system.

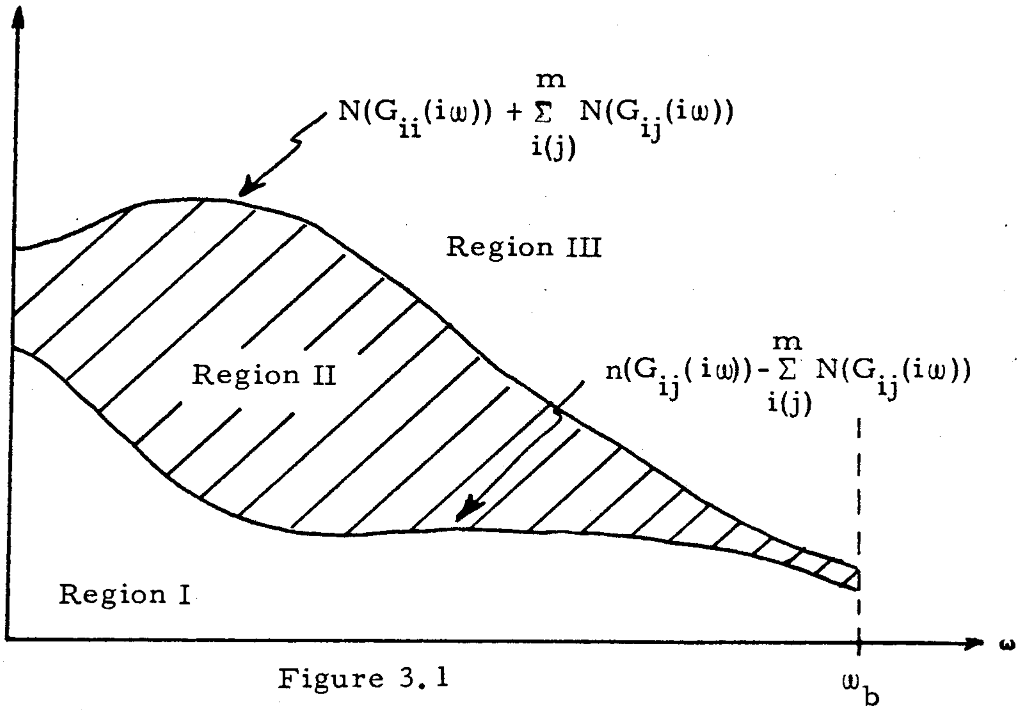


Figure 3.1

Then if we pick for the i^{th} local compensator $F_i(s)$ such that either $N(F_i(i\omega))^{-1}$ is in region III for $\omega \in [0, \omega_b]$ or $n(F_i(i\omega))^{-1}$ is in region I for $\omega \in [0, \omega_b]$ for each $i = 1, \ldots, m$, corollary 3-5 guarantees that the matrix $F^{-1}(s) + G(s)$ will be BDD on D. Then theorem 3-3 can be applied to determine stability of the closed loop decentralized control system.

Similarly, with respect to theorem 3-4, we establish the result,

Corollary 3-7: If

(a) $$n(F_i(s)) > N(\hat{G}_{ii}(s)) + \sum_{j(i)}^{m} N(\hat{G}_{ij}(s))$$

$$\left(\text{or} > N(\hat{G}_{ii}(s)) + \sum_{j(i)}^{m} N(\hat{G}_{ji}(s))\right) \tag{3. 27}$$

(b) $$N(F_i(s)) < n(\hat{G}_{ii}(s)) - \sum_{j(i)}^{m} N(\hat{G}_{ij}(s))$$

$$\left(\text{or} < n(G_{ii}(s)) - \sum_{j(i)}^{m} N(G_{ji}(s)\right) \tag{3. 28}$$

for each i = 1, . . . , m and for all s on D,

then $\hat{H}(s)$ is BDD on D.

Proof: From lemma 3-6 and (eq. 3. 5) the result follows.

Again the simple graphical test described above can be applied where we plot the curves,

$$N(\hat{G}_{ii}(s) + \sum_{j(i)}^{m} N(\hat{G}_{ij}(s)) \tag{3. 29}$$

and

$$n(\hat{G}_{ii}(s)) - \sum_{j(i)}^{m} N(\hat{G}_{ij}(s)). \tag{3. 30}$$

If we pick each $F_i(s)$ such that $n(F_i(s))$ is in region III or $N(F_i(s))$ is in region I of the graph in figure 3.2, then corollary 3-6 guarantees that if G(s) is also BDD on D, then theorem 3-4 can be used to determine the stability of the closed loop design.

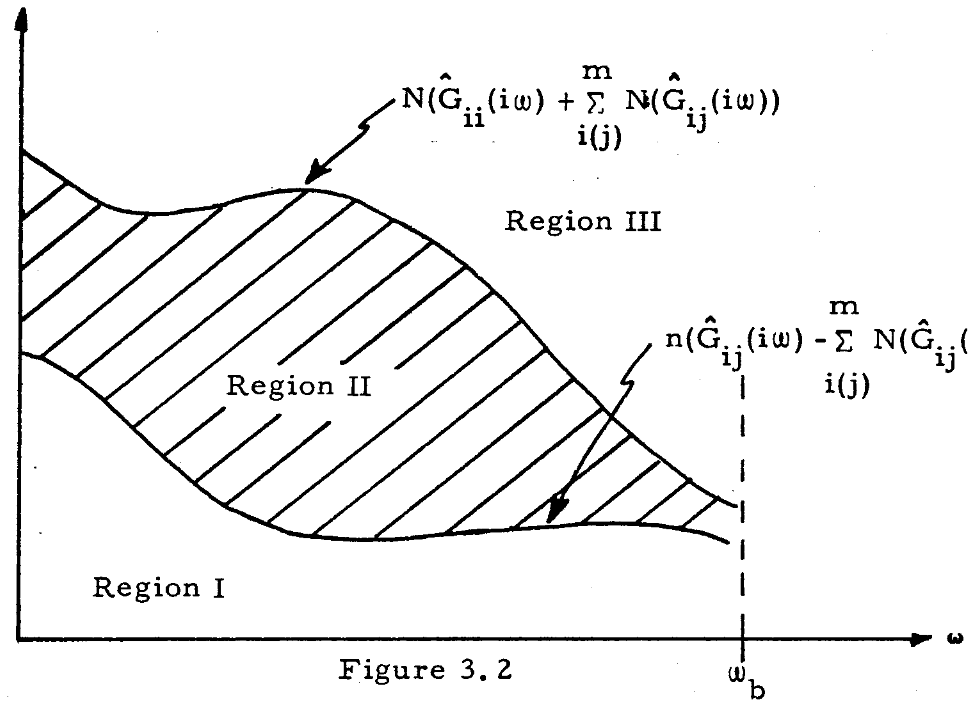


Figure 3.2

Notice that in both cases the graphical criteria are based on the BDD of the test matrix rows. We can of course guarantee the result equivalently by column BDD on D. What can be done is to plot the optimum bound based on the two measures for all $\omega \in [0, \omega_{max}]$. We recognize though that the other regularity results discussed in section 2.3 may be incorporated in the design procedure here to establish optimum estimates for these bounds. A design example using these techniques appears in the next chapter.

(3.3) PERFORMANCE OF THE ITH SUBSYSTEM WITH INTERACTION

Once decentralized compensation has been applied to a multivariable system, we are often interested in the performance of the i^{th} local subsystem (i.e. the triple $(e_i(s), u_i(s), y_i(s))$). The question may be posed: How well does the i^{th} controller perform as compared to the ideal decoupled design developed in step 2 of the design process ? Rosenbrock [2, pp. 149-152] developed bounds for this performance question for the case where m = n. We will now follow his approach in using the Ostrowski results and explore the result in the generalized framework of BDD. We restrict ourselves to the inverse matrix transfer function representation for reasons explained before and by Rosenbrock [2, pp. 155].

From theorem 2-8 we find,

Theorem 3-7: Let $\hat{H}(s) = F(s) + \hat{G}(s)$ be BDD on D. Then for each s on D,

$$n\left[H_{ii}^{-1}(s) - (F_i(s) + \hat{G}_{ii}(s))\right] < \hat{\Phi}_i(s)\hat{\theta}_i(s)n(\hat{G}_{ii}(s)) < \hat{\theta}_i(s)n(\hat{G}_{ii}(s))$$

$$\text{or} < \hat{\Phi}_i'(s)\hat{\theta}_i'(s)n(\hat{G}_{ii}(s)) < \hat{\theta}_i'(s)n(\hat{G}_{ii}(s)) \qquad (3.31)$$

where

$$\hat{\theta}_i(s) \triangleq \sum_{i(j)}^{m} \frac{N(\hat{G}_{ij}(s))}{n(\hat{G}_{ii}(s))} \quad \left(\hat{\theta}_i'(s) \triangleq \sum_{i(j)}^{m} \frac{N(\hat{G}_{ji}(s))}{n(\hat{G}_{ii}(s))}\right) \qquad (3.32)$$

and

$$\hat{\Phi}_i(s) \triangleq \max_{i \neq j} \hat{\theta}_j(s) \left(\hat{\Phi}_i'(s) \triangleq \max_{i \neq j} \hat{\theta}_j'(s)\right). \qquad (3.33)$$

Proof: By substitution of $\hat{H}(s)$ (see eq. 3.5) into theorem 2-8.

Then with the notation established in theorem 3-7 we can state,

Corollary 3-8: If $\hat{H}(s)$ is BDD on D then,

$$n\left\{H_{ii}^{-1}(s) - F_i(s)\right\} < N(\hat{G}_{ii}(s)) + \hat{\theta}_i(s)\,\hat{\Phi}_i(s) n(\hat{G}_{ii}(s)) \tag{3.34}$$

$$N\left\{H_{ii}^{-1}(s) - F_i(s)\right\} > n(\hat{G}_{ii}(s)) - \hat{\theta}_i(s)\,\hat{\Phi}_i(s) n(\hat{G}_{ii}(s)) \tag{3.35}$$

for $i = 1, \ldots, m$.

Let us drop temporarily the Laplace variable s from our notation.

Define,

$$H[F_i, G] \triangleq G[I_n + \tilde{F}G]^{-1} \tag{3.36}$$

where,

$$\tilde{F} = \text{block-diag}\,\{F_1, F_2, \ldots, F_{i-1}, 0, F_{i+1}, \ldots, F_m\}. \tag{3.39}$$

Then clearly,

$$\hat{H}_{ii}[F_i, G] + F_i = H_{ii}^{-1}\ .$$

So we see that the term, $H_{ii}^{-1}(s) - F_i(s)$ in corollary 3-8, is the i^{th} diagonal block of the inverse of the transfer function matrix for the closed loop system when the i^{th} feedback compensator has been disconnected. This is the plant seen by the i^{th} feedback compensator when it is connected to the closed loop system. Corollary 3-8 describes the effect of the other closed loop controllers on the i^{th} local plant.

Comparing corollary 3-8 with the analogous result of Rosenbrock [2, pp. 150, th. 6.2] we are somewhat dissatisified with our result. Rosenbrock [2, pp. 149-152] uses his result to bound the Nyquist locus of the $h_{ii}^{-1}(s) - f_i$ using the Gershgorin circles and can therefore calculate estimates for the gain and phase margins for the i^{th} loop design in the closed loop system. But in our more generalized framework the result is not so strong.

Work in this area is continuing and the theorems of Brenner

[23, 24] are promising since he develops bounds on determinants of the diagonal submatrices.

To characterize our results to date on the performance of the subsystems we offer a simple result which supplies an estimate for the gain margin of the i^{th} subsystem. By gain margin we take here the simplest definition possible for a multivariable system.

Definition 3-8: (Multivariable Gain Margin) With a stable multivariable closed loop system described by,

$$H(s) = G(s)[I_n + F(s)G(s)]^{-1}$$

where F(s) is the n x n dynamic compensator which stabilizes the plant we define the gain margin as the largest positive real scalar (loop gain increase) β_m such that for all $\beta \epsilon [1, \beta_m]$ the closed loop system,

$$H_\beta(s) = G(s)[I_n + \beta F(s)G(s)]^{-1}$$

is asymtotically stable.

With this definition we see that β_m is a positive real scalar or gain applied equally to all n inputs (or outputs) to the compensator F(s).

Next, we define the margin of dominance.

Definition 3-9: Let the matrix $F^{-1}(s) + G(s)$ be BDD on D. Then define d_{mi}, the margin of dominance for the i^{th} block row (or block column) of $F^{-1}(s)+G(s)$, as the largest positive real scalar such that $d_i \epsilon [1, d_{mi}]$ implies

$$n\left[(d_iF_i(s))^{-1} + G_{ii}(s)\right] > \sum_{i(j)}^{m} N(G_{ij}(s))$$

$$\left(\text{or} > \sum_{i(j)}^{m} N(G_{ij}(s))\right)$$

for each $i = 1, \ldots, m$ and for all s on D.

Then the following theorem will relate the margin of dominance and the gain margin for the i^{th} subsystem.

Theorem 3-10: Assume that $F^{-1}(s) + G(s)$ is BDD on D. Let $\det(F_i^{-1}(s) + G_{ii}(s))$ map D into Γ_i which encircles the origin N_i times clockwise. Let $\det[(d_iF_i(s)))^{-1} + G_{ii}(s)]$ map D into Γ_{di} which encircles the origin N_{di} times clockwise where d_i is any positive real scalar such that $d_i \in [1, d_{mi}]$ and d_{mi} is the margin of dominance defined above. Then,

$$N_i = N_{di}$$

Proof: Choose, d_i^* such that $1 \le d_i^* \le d_{mi}$ and consider

$$\alpha(s, d_i^*) = \frac{\det[(d_i^* F_i)^{-1} + G_{ii}(s)]}{\det[F_i^{-1}(s) + G_{ii}(s)]}$$

and let $\alpha(s, d_i^*)$ map D into Γ_α. Assume Γ_α encircles the origin (as in the proof of theorem 2-13). But by BDD of $F^{-1}(s) + G_{ii}(s)$ on D, we know $\det[F_i^{-1}(s) + G_{ii}(s)]$ is bounded on D and non-zero on D. Also by the definition of margin of dominance d_{mi} the same hold for $\det[d_iF_i(s)^{-1} + G_{ii}(s)]$ for any $d_i \in [1, d_i^*]$. Therefore, by contradiction we have shown that Γ_α does not encircle the origin. Applying the principle of the argument (cor. 2-10) to $\alpha(s, d_i^*)$ and since this is true for any $d_i^* \in [1, d_{mi}]$ we see that,

$$0 = N_{di} - N_i .$$

The result is proven.

Then corresponding to theorem 3-4, we can state the analogous result of theorem 3-10 as a corollary.

<u>Corollary 3-11</u>: Assume that $F(s)+\hat{G}(s)$ is BDD on D. Let $\det(F_i(s)+\hat{G}_{ii}(s))$ map D into Γ_i which encircles the origin $\hat{N}_i$ times clockwise. Let $\det(\hat{d}_i F_i(s)+\hat{G}_{ii}(s))$ map D into $\hat{\Gamma}_{di}$ which encircles the origin $\hat{N}_{di}$ times clockwise where $\hat{d}_i$ is any positive real scalar such that $\hat{d}_i \in [1, \hat{d}_{mi}]$ and $\hat{d}_{mi}$ is the margin of dominance for the test matrix $\hat{d}_i F(s)+\hat{G}(s)$ replacing that in definition 3-9. Then,

$$\hat{N}_i = \hat{N}_{di}.$$

<u>Proof:</u> Follows exactly as that in theorem 3-10.

These results allow us to say the following about subsystem performance for the closed loop decentralized design. If the design procedure proposed here is used to design a decentralized control system and either theorem 3-3 or theorem 3-4 is used to determine stability for the system then necessarily the appropriate test matrix $F^{-1}(s)+G(s)$ (or $F(s)+\hat{G}(s)$) must be BDD on D. Then the margin of dominance d_{mi} (or $\hat{d}_{mi}$) is a lower bound for the gain margin of the i^{th} local controller in the closed loop design while all other loops are fixed. This follows immediately from the assumption that at the end of our design procedure we have a stable system (eg. $\sum_{i=1}^{m} N_i = -p_o$). Then apply theorem 3-10.

A DESIGN EXAMPLE

We pose the following design problem:

given the unstable plant,

$$G(s) = \left[\begin{array}{cc|cc} \frac{15}{s-5} & \frac{-5.25}{s+3.5} & 0 & \frac{2.5}{s+1} \\ \frac{5.25}{s+3} & \frac{21}{s-6} & \frac{2.5}{s+1} & 0 \\ \hline 0 & \frac{5}{s+2} & \frac{18}{s-6} & \frac{-4.5}{s+3} \\ \frac{5}{s+2} & 0 & \frac{7}{s+4} & \frac{17.5}{s-5} \end{array}\right]$$

it is desired to stabilize the plant using constant decentralized feedback. We recognize that from our definition in chapter 3 that the plant has four unstable poles in the closed right half plane.

The simpliest form for a decentralized constant feedback compensator would be,

$$F = \text{diag}\,\{f_1, f_2, f_3, f_4\}.$$

Let us, therefore, attempt the design using the Rosenbrock procedure, the Direct Nyquist Array. We plot in figures 4.1 and 4.2 the map of each $g_{ii}(s)$ for $s = j\omega$ and $\omega \in [0, \omega_{max}]$ where we choose here ω_{max} = 25 rad./sec. Then we plot atop the Nyquist locus for each $g_{ii}(s)$ the Gershgorin circles found from the row criteria. Since the circles include the origin, we check for diagonal dominance by columns in figures 4.3 and 4.4. But here again, the requirement of diagonal dominance is not satisfied for all s on D.

At this point, the usual Direct Nyquist Array technique [2] would require the design of a series compensator, in this case a 4 x 4 rational matrix (not in general diagonal), which achieves diagonal dominance for the open loop plant in cascade with the compensator. When such a compensator is found, the design of the decentralized feedback compensator can proceed. But two problems arise in this case. First, the resulting design is not truly decentralized. This can be seen from the design structure shown in figure 1.1 Here the i^{th} input correction to the plant plant u(s) is a function of all the outputs y(s) to be controlled and all the input commands v(s). Second, the design of these series compensators remains at best an ad hoc process requiring a great deal of skill and intuition on the part of the designer.

Now let us try the design procedure proposed in chapter 3 based on theorem 3-3. We plot in figure 4.5 and 4.6 the graphical test shown in figure 3.2 by columns. Here we plot based on the euclidean vector norm the associated induced supremum and infimum matrix norms for the 2 x 2 block partitioning of G(s) shown above as in figure 3.2. Then in figures 4.7 and 4.8, we plot the same curves for the block row test. Here we see that the plant G(s) is block diagonally dominant by the 2 x 2 partitioning shown. So we can proceed to step 2 of the design process.

In step 2, we seek choices for the diagonal blocks of F which will both achieve BDD of the test matrix $F^{-1} + G(s)$ and stabilize the plant in the context of theorem 3-3. First, we proceed to design compensators for the diagonal blocks of G(s). In figure 4.9, we plot the Nyquist loci and associated Gershgorin circles for the rows of,

$$G_{11}(s) = \begin{bmatrix} \frac{15}{s-5} & \frac{-5.25}{s+3.5} \\ \frac{5.25}{s+3} & \frac{21}{s-6} \end{bmatrix}$$

To stabilize this decoupled plant requires a feedback,

$$F_1 = \text{diag}\{f_1, f_2\}$$

where $f_1^{-1} < 1.5, \; f_2^{-1} < 1.8.$

But from figure 4.7 and corollary 3-5, we require that,

$$n(F_1) < n(G_{11}(s)) - N(G_{12}(s))$$

for all s on D

or from the figure,

$$n\begin{pmatrix} f_1 & 0 \\ 0 & f_2 \end{pmatrix}^{-1} < 0.352.$$

Since we are using here the euclidean vector norm, the induced infimum matrix norm implies

$$\min (f_1, f_2)^{-1} < 0.352$$

or

$$\min (f_1, f_2) > 2.84.$$

So we pick $f_1 = f_2 = 3.0$.

Similarily, from figure 4.10 the Nyquist locus and Gershgorin circles for

$$G_{22}(s) = \begin{bmatrix} \frac{18}{s-6} & \frac{-4.5}{s+3} \\ \frac{7}{s+4} & \frac{17.5}{s-5} \end{bmatrix}$$

We see that to stabilize the decoupled plant requires,

$$F_2 = \text{diag}\{f_3, f_4\}$$

where

$$f_3^{-1} < 1.5,\ f_4^{-1} < 1.8.$$

But from figure 4.8 BDD of $F^{-1} + G(s)$ on D can be guaranteed if

$$n \begin{pmatrix} f_3 & 0 \\ 0 & f_4 \end{pmatrix}^{-1} < 0.372$$

or

$$\min(f_3, f_4) > 2.69.$$

Thus a choice of $F = \text{diag}\{3, 3, 3, 3\}$ will stabilize the plant by decentralized feedback using non-dynamic compensation. Also, by our definition of gain margin (defn. 3-8) the closed loop system has infinite gain margin (infinite margin of dominance (defn. 3-9)). See figure 4.7 and 4.8.

CONCLUSIONS AND RECOMMENDATIONS

In this thesis, we have developed, using some basic theorems establishing the regularity of a complex matrix, results on the stability of a multivariable system compensated by decentralized feedback. A design procedure has been proposed which incorporates a graphical test which can aid in the choice of feedback compensation to achieve the decentralized design. The techniques posed use the classical frequency domain description of the dynamic input-output behavior of a system. Several important conclusions can be drawn and recommendations made for further research.

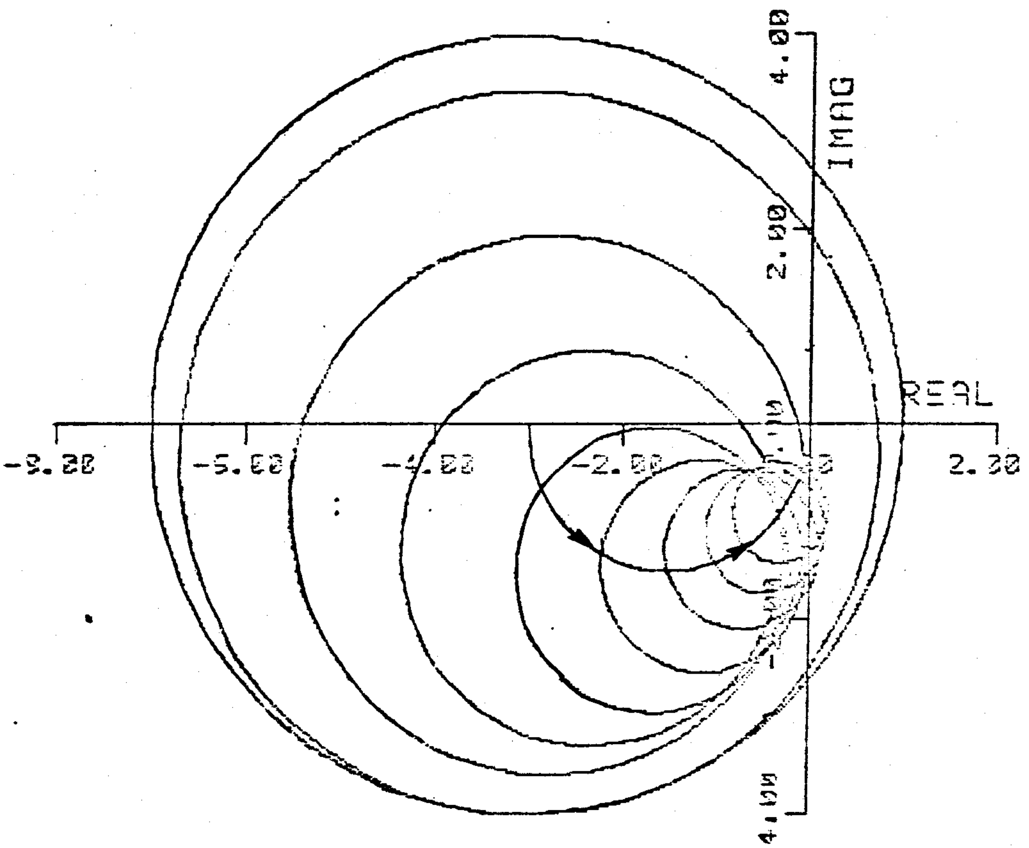
The frequency varying norms employed here provide a much more subtle characterization of the relative autonomy of subsystems in the presence of interconnections than the results obtained in [16]-[18]. Moreover the results show how the design process can be reduced to a series of more standard problems with some additional requirements on each compensator. The additional requirements on the design can be tested graphically; implying a computer-aided design approach is appropriate.

The flexibility of the block diagonal dominance theorems utilized here is emphasized. In particular, the choice of partitioning leads to natural frequency domain decompositions for a multivariable system. Thus questions like the appropriateness of a decentralized design for a plant can be considered. The choice of norms in our result and the choices available from the other theorems establishing regularity of a complex matrix discussed in section 2.3 imply that optimal bounds for the regularity of a complex matrix can be calculated. The question of numerical complexity in these calculations and the extension of these results using the theorems in section 2-3 is recommended for further research. Again the calculations involved dictate a computer-aided design is appropriate. The hierarchy of subpartitions of a partitioned matrix transfer function as illustrated in the design example in chapter 4 points up another way in which the technique provides flexibility for design and analysis of large scale problems.

A further recommencation for research is concentrated on the extension of the present design technique but using the results of Brenner [23] and [24]. These theorems promise even more subtle results since upper and lower bounds for the determinant of the matrix are found. In particular, characterization of subsystem performance in the presence of interactions may be available in a stronger way than from the present generalized approach. Of course, these questions cannot be divorced from the question of the numerical complexity in the required calculations. Further investigation is certainly warranted.

Finally, we single out from these results potential application for the general multivariable design problem as an area for further research. In particular, the hierarchical decomposition of a rational matrix into partitions and sub-partitions as illustrated in the design problem may lead to algorithms which can achieve the design of series compensators which provide approximate subsystem decoupling as in the approach of Rosenbrock [2, pp. 156-173]. In general, the question of designing compensators for the Inverse Nyquist Array technique which contain elements of some restricted dynamic degree (usually constant) is unanswered. But the work of Leininger [26] indicates that for 2 x 2 matrix transfer functions the solution is relatively easy. The hierarchical partitioning discussed here may pave the way for developing algorithms which sequentially realize the required compensator. At each step a 2 x 2 problem can be considered.

DIRECT NYQUIST ARRAY



ROW 1

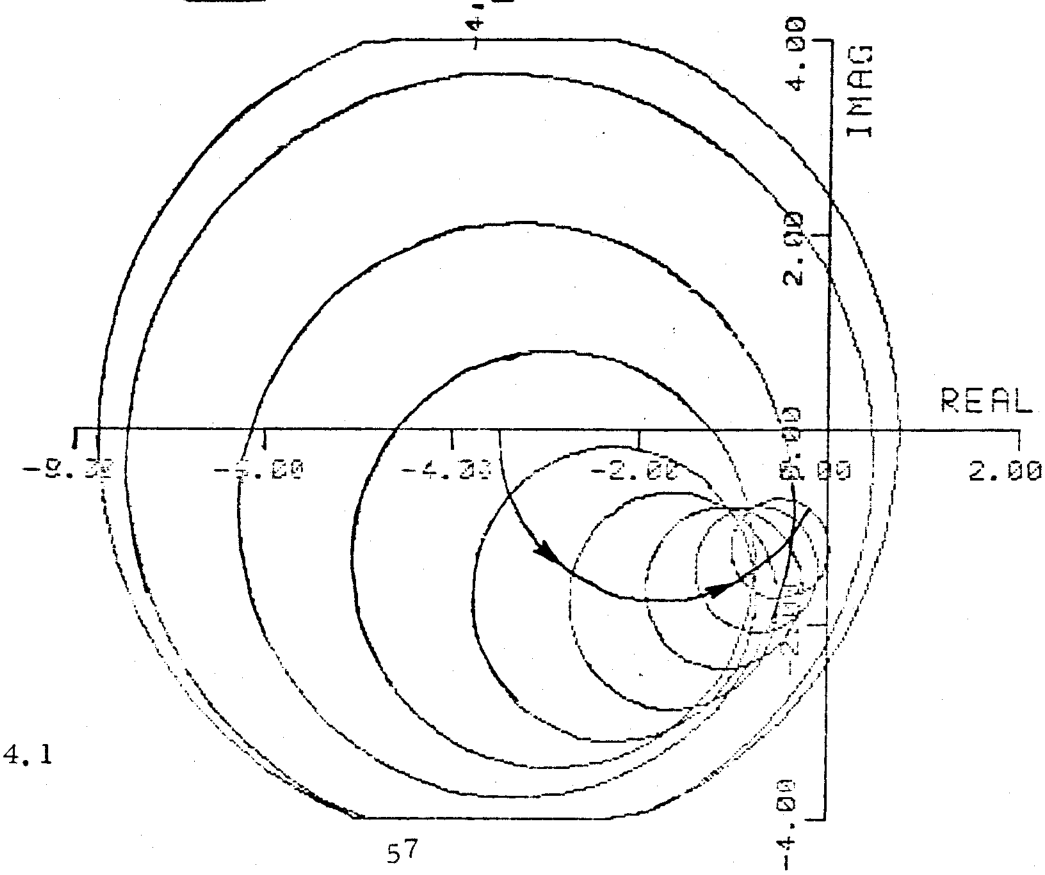


Figure 4.1

ROW 2

DIRECT NYQUIST ARRAY

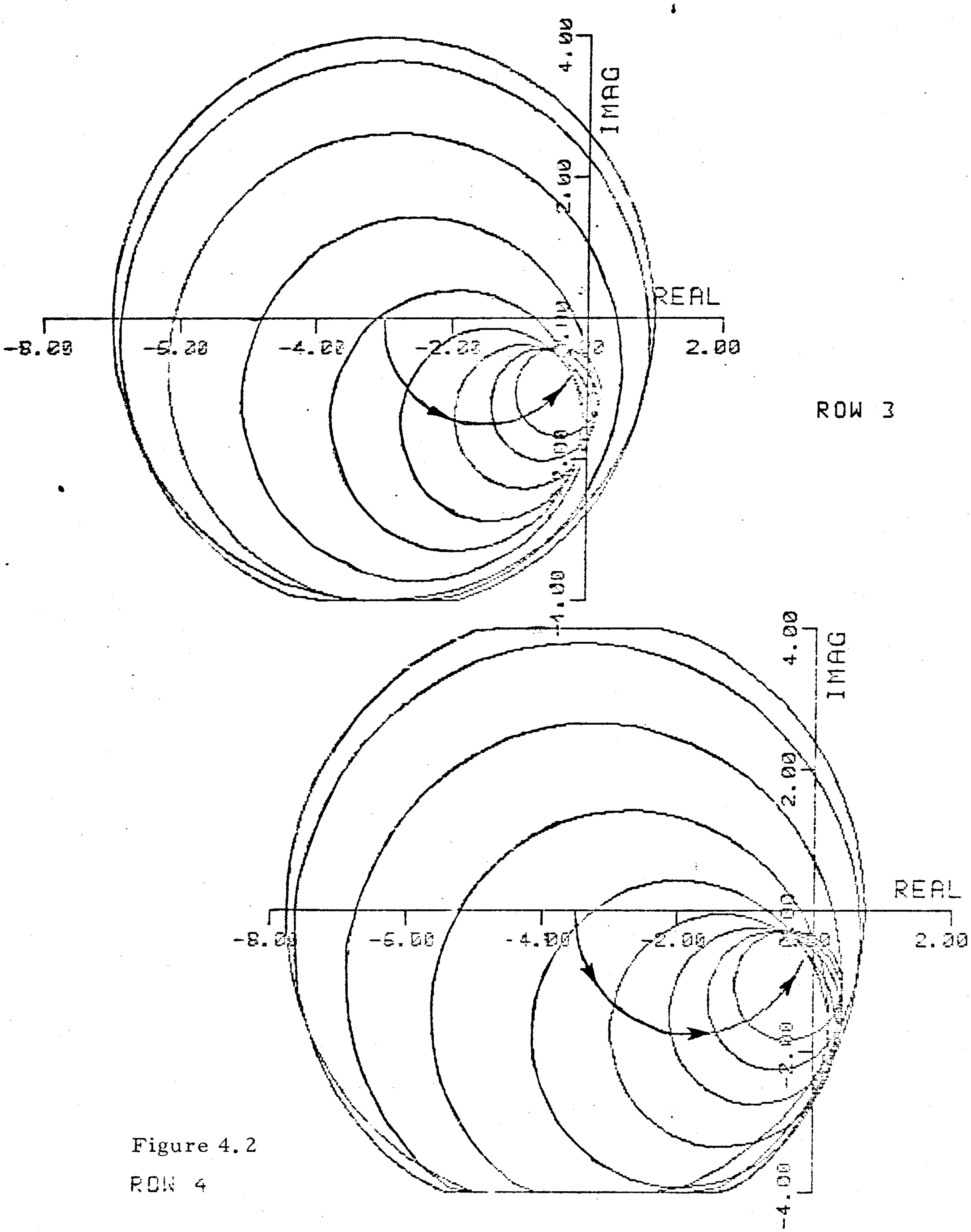
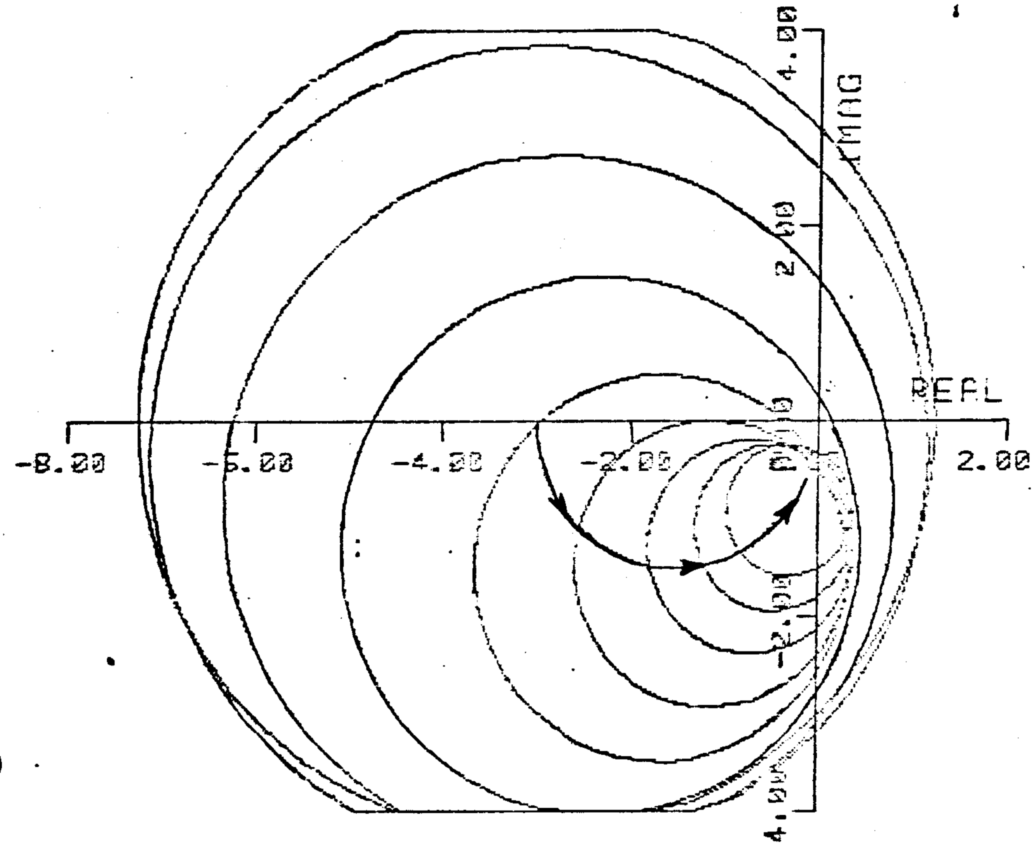


Figure 4.2

DIRECT NYQUIST ARRAY



COLUMN 1

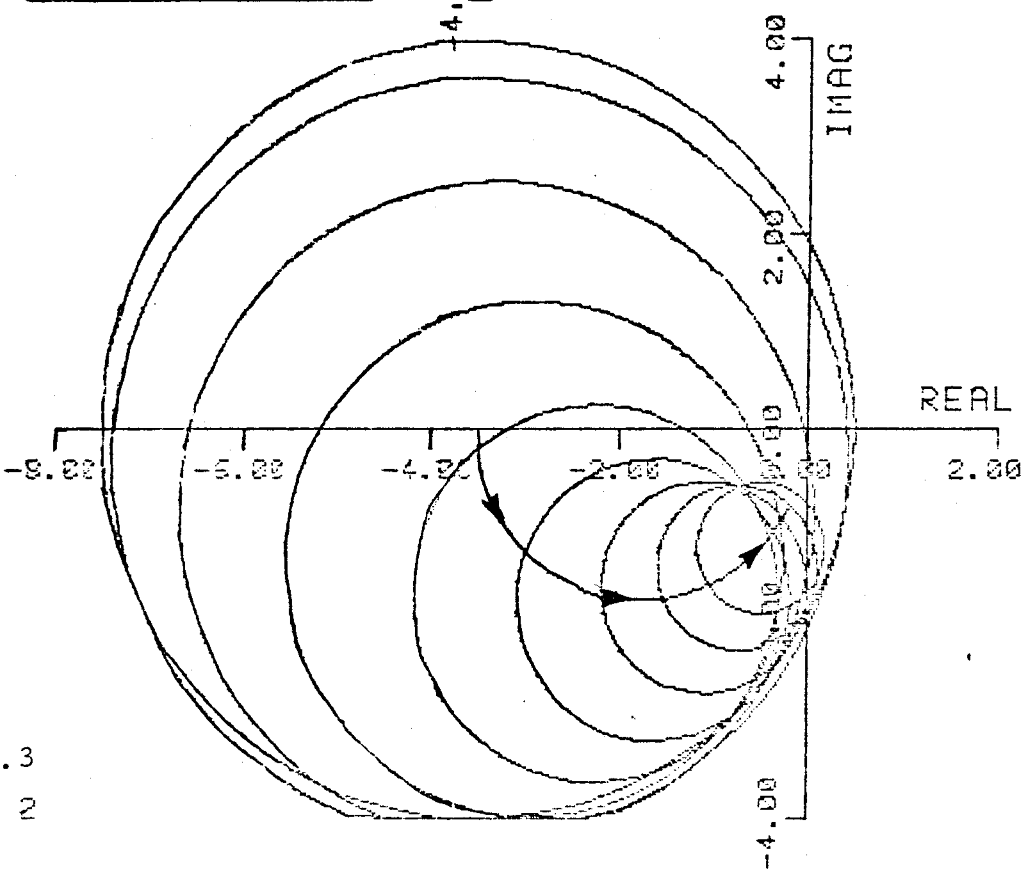
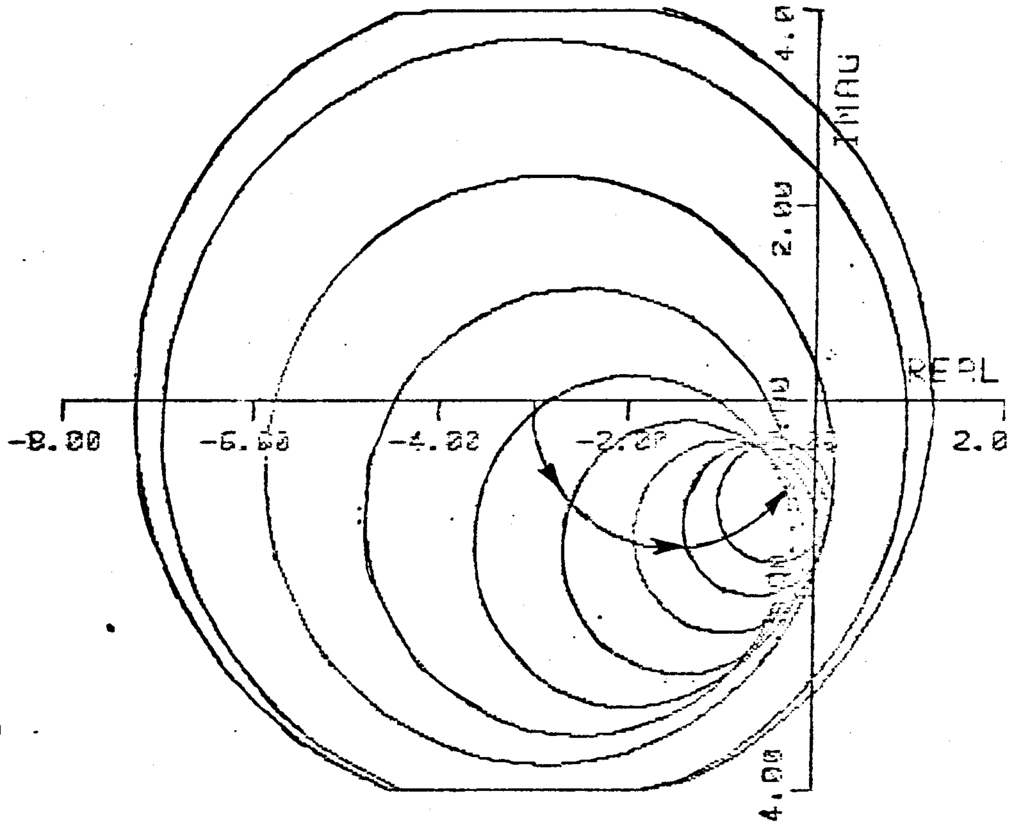


Figure 4.3

COLUMN 2

DIRECT NYQUIST ARRAY



COLUMN 3

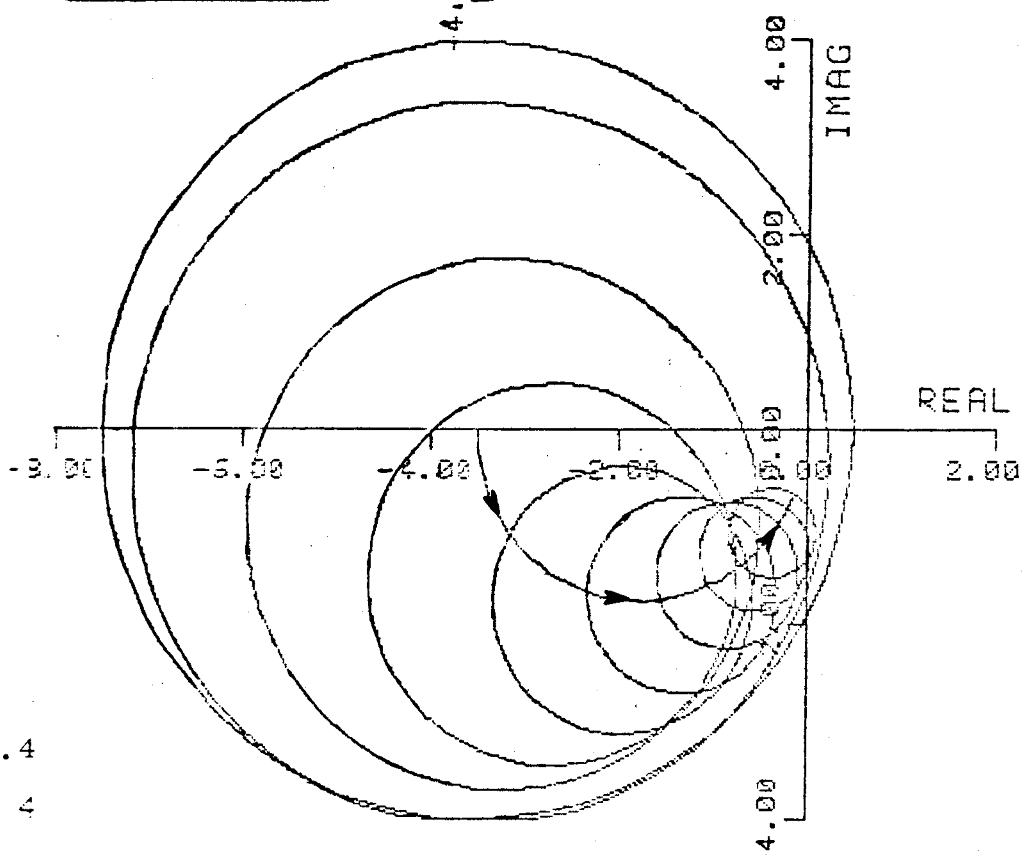
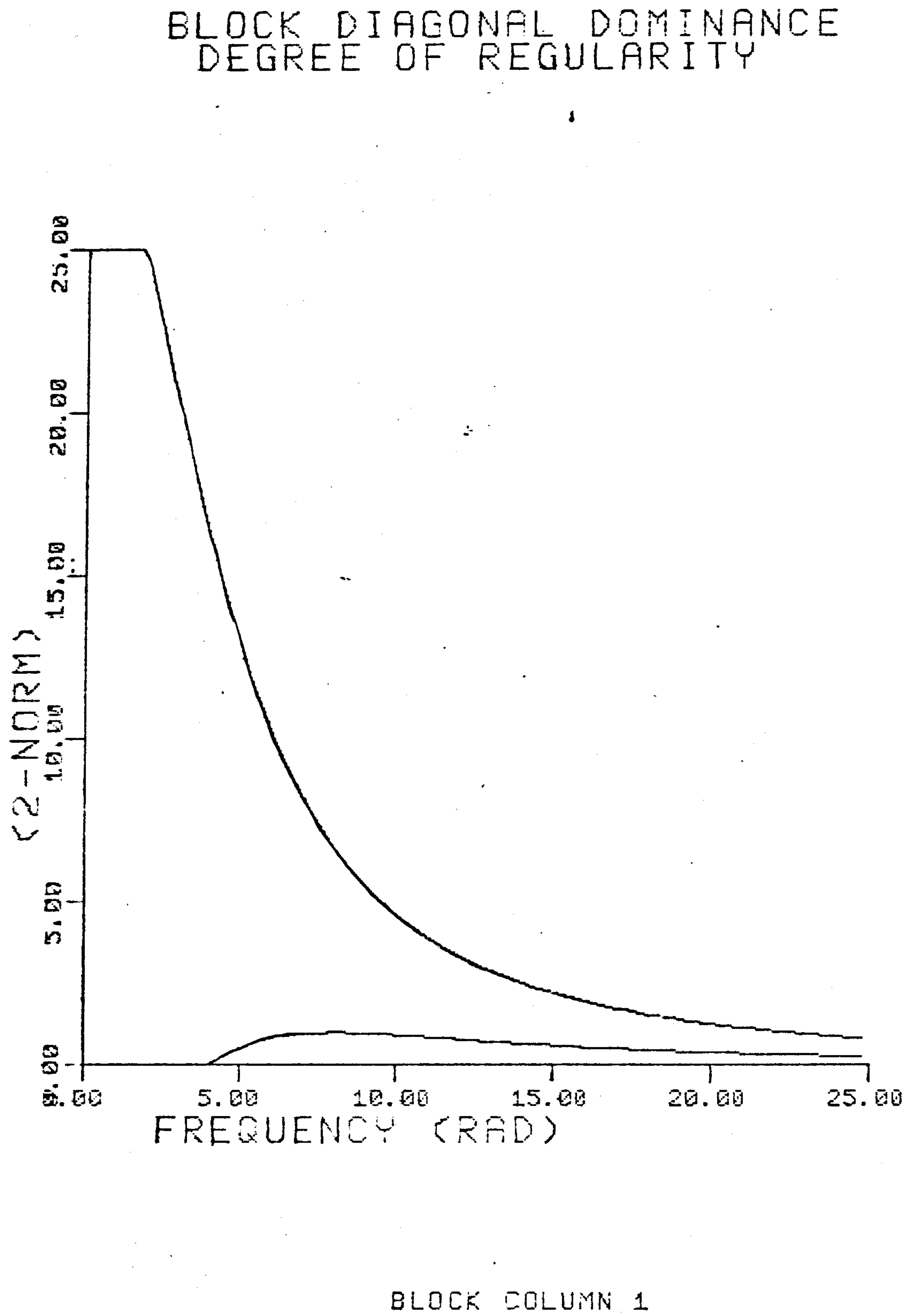


Figure 4.4

COLUMN 4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(2-NORM)
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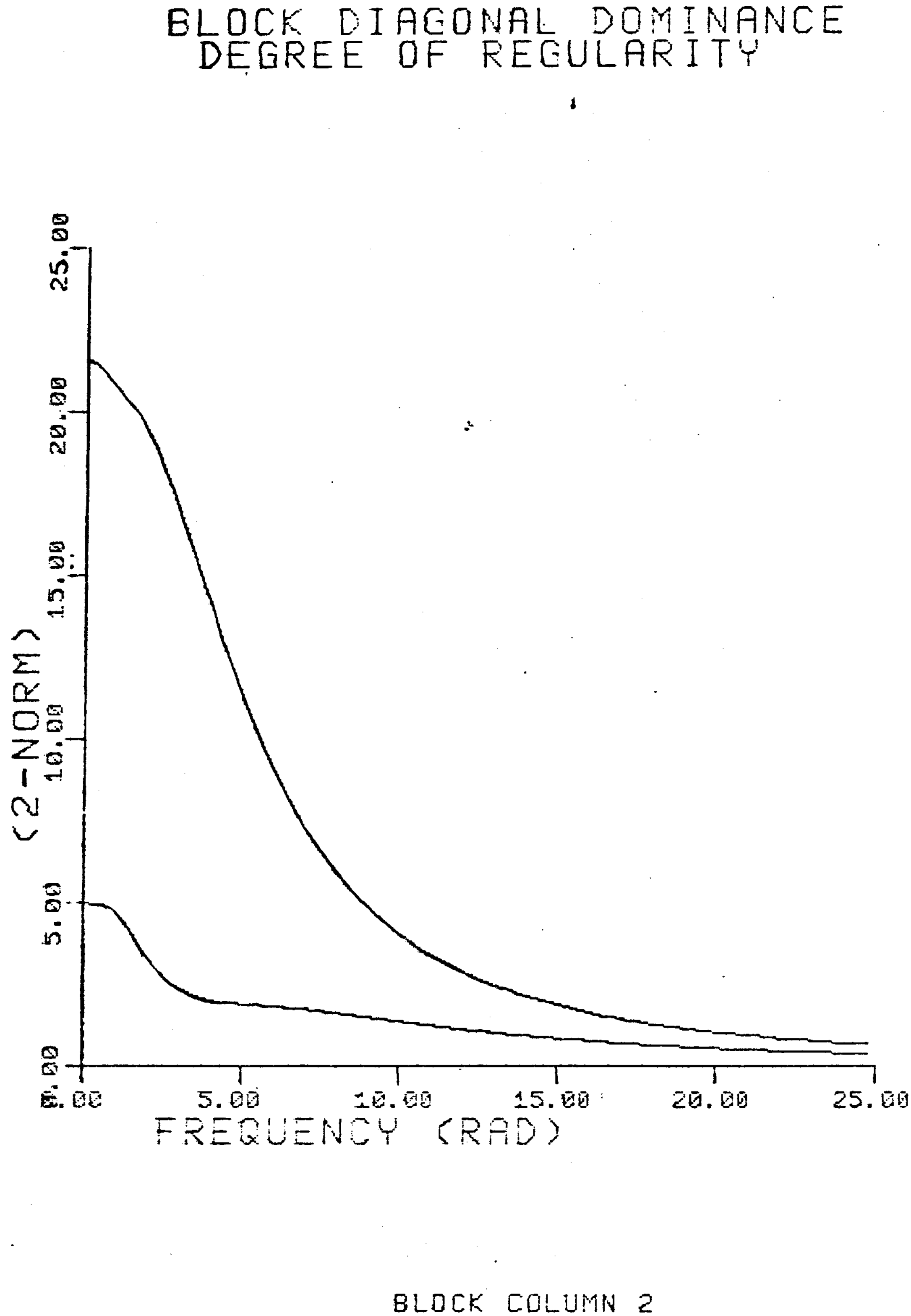
20.00
15.00
10.00
5.00
0.00
0.00
5.00
10.00
15.00
20.00
25.00
FREQUENCY (RAD)
BLOCK COLUMN 1

Figure 4.5


BLOCK DIAGONAL DOMINANCE
DEGREE OF REGULARITY
(2-NORM)
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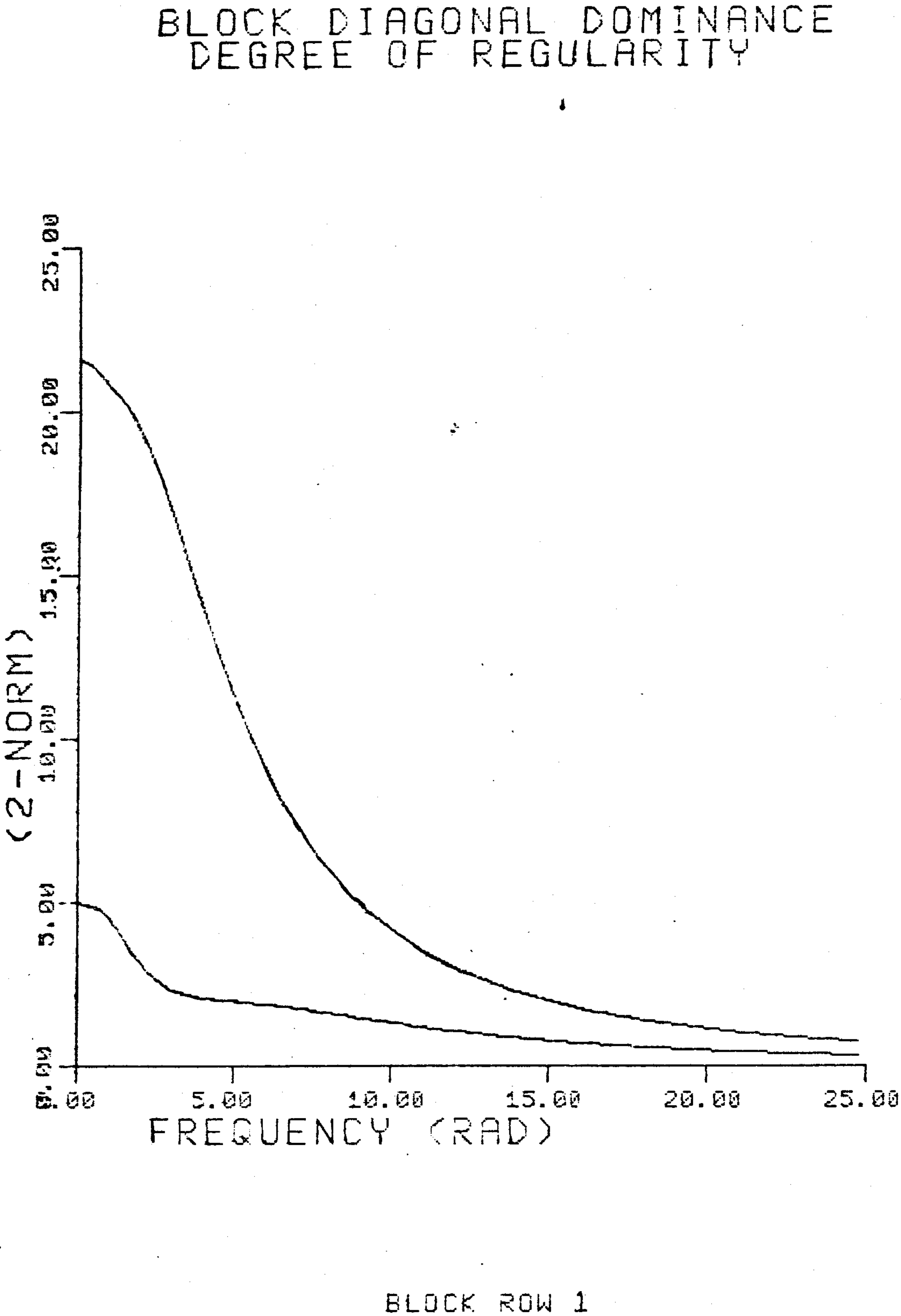
FREQUENCY (RAD)
BLOCK COLUMN 2

Figure 4.6


BLOCK DIAGONAL DOMINANCE
DEGREE OF REGULARITY
(2-NORM)
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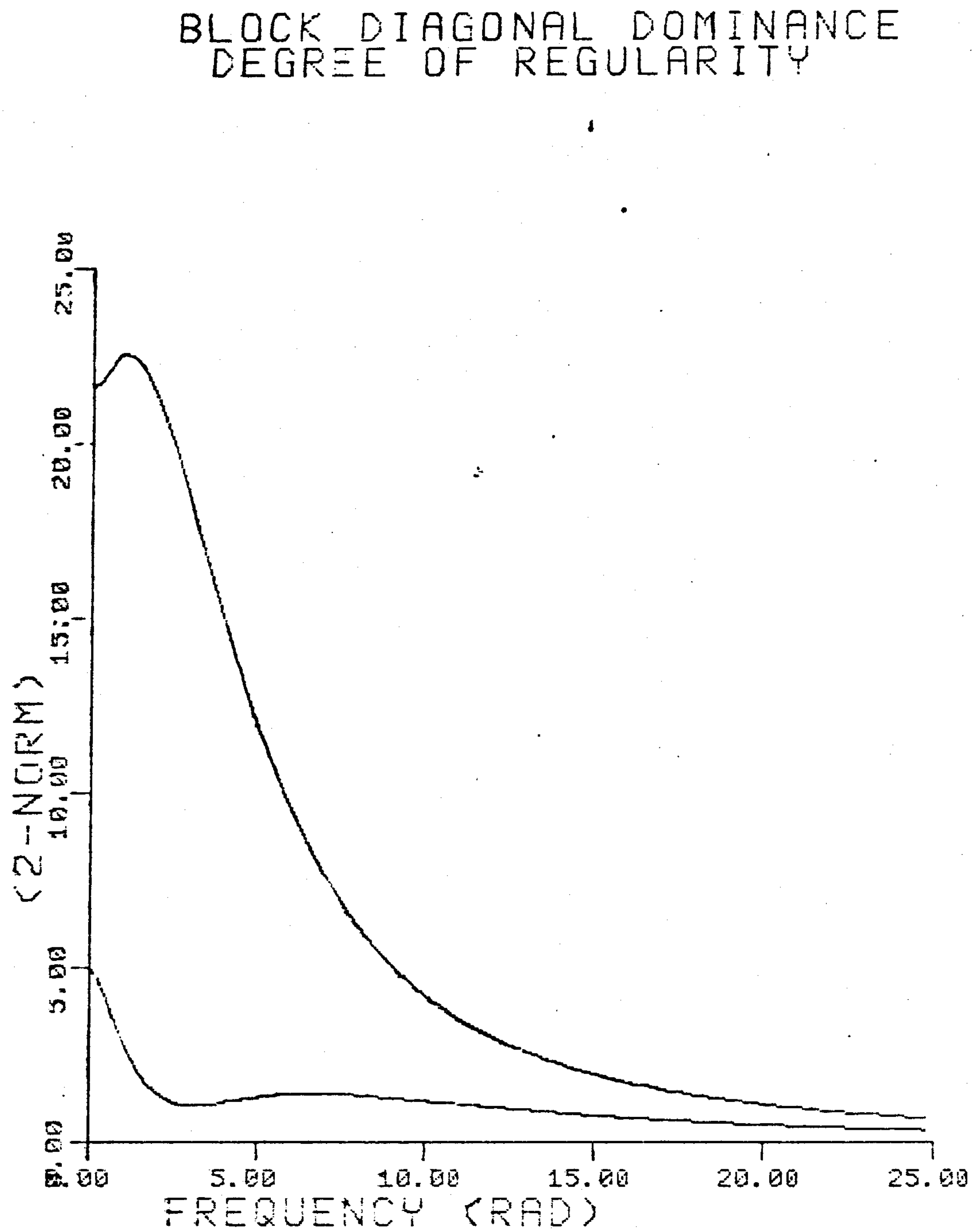
10.00
15.00
20.00
25.00
FREQUENCY (RAD)
BLOCK ROW 1

Figure 4.7



BLOCK ROW 2

Figure 4.8

DIRECT NYQUIST ARRAY

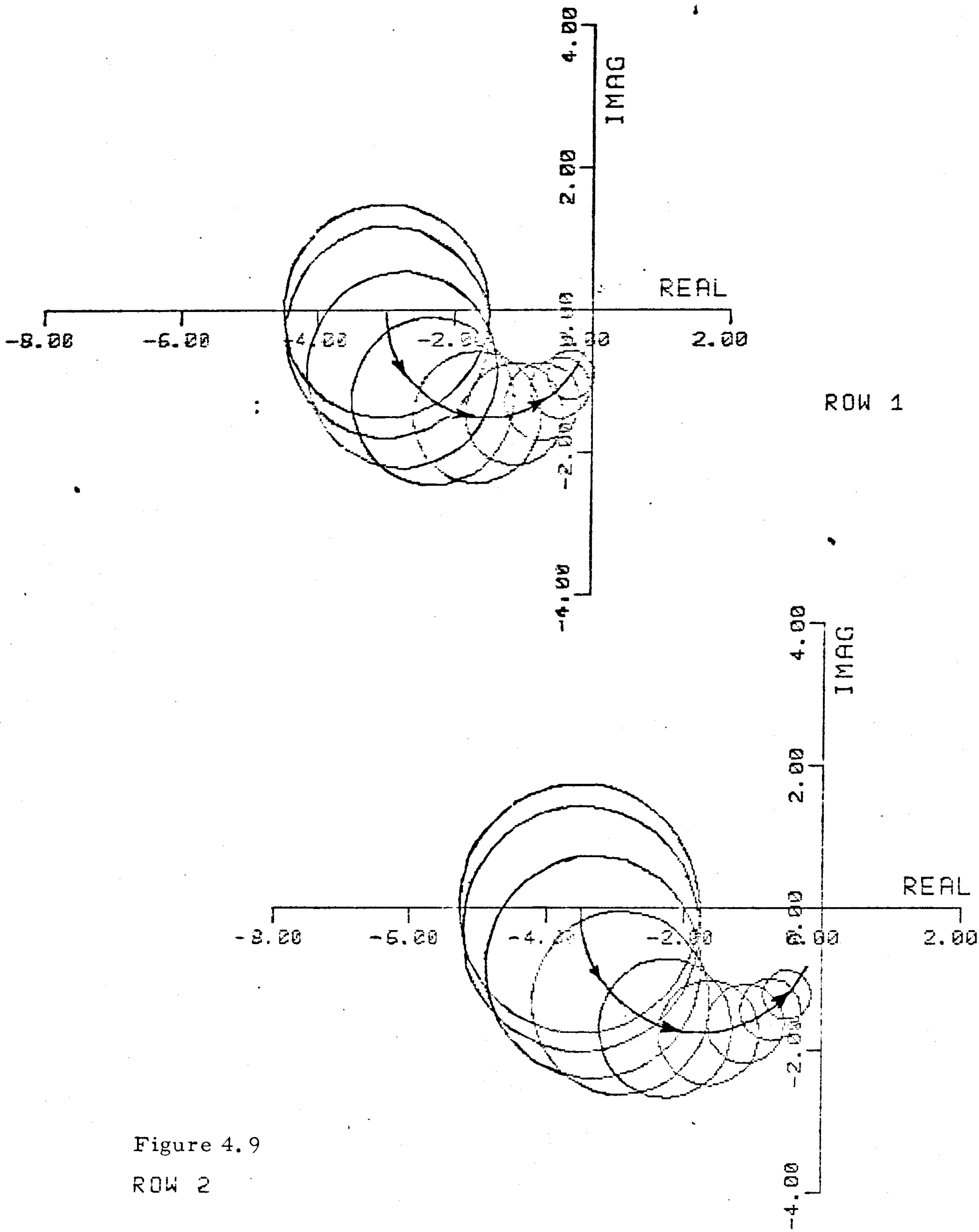
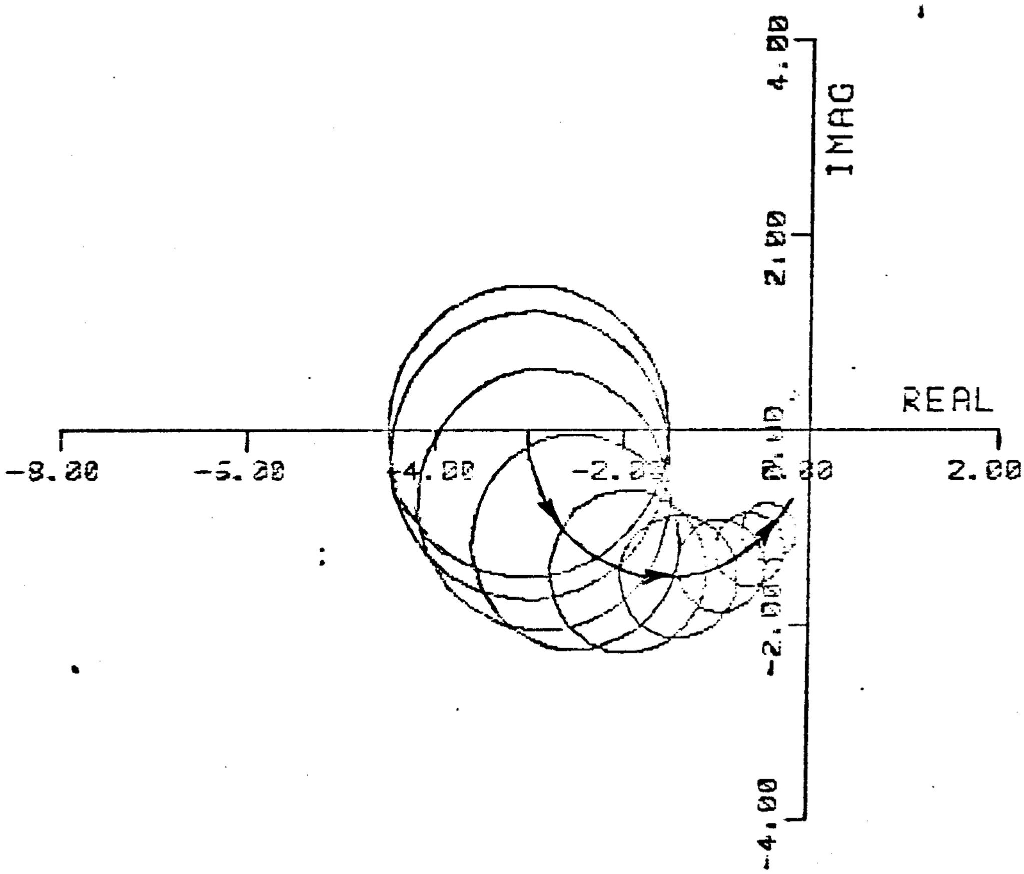


Figure 4.9

ROW 2

DIRECT NYQUIST ARRAY



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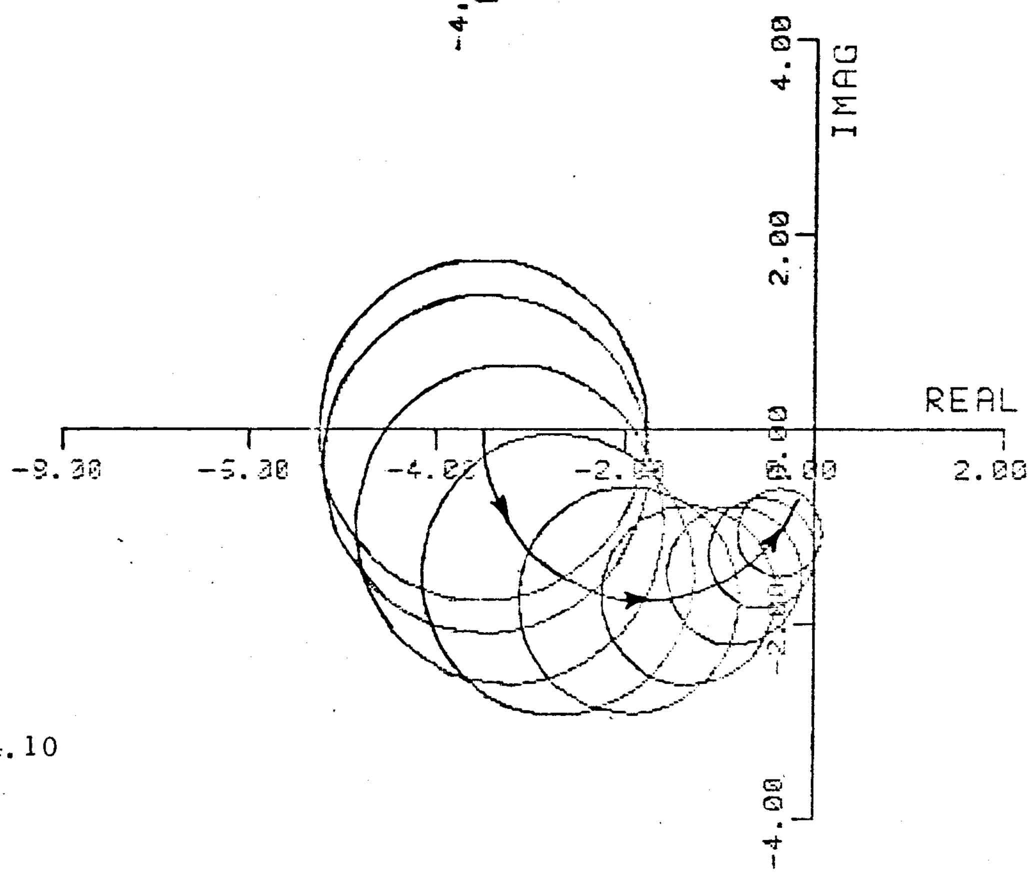

Figure 4.10

ROW 4

References

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